Chapter 1

# 1 LIFE MATHEMATICS

## 1.1 Introduction

In most of our daily activities like following a recipe or decorating our home or calculating our daily expenses we are unknowingly using mathematical principles. People have been using these principles for thousands of years, across countries and continents. Whether you're sailing a boat off the coast of Chennai or building a house in Ooty, you are using mathematics to get things done.

How can mathematics be so universal? First human beings did not invent mathematical concepts, we discovered them. Also the language of mathematics is numbers, not English or German or Russian. If we are well versed in this language of numbers, it can help us make important decisions and perform everyday tasks. Mathematics can help us shop wisely, remodel a house within a budget, understand population growth, invest properly and save happily.

Let us learn some basic mathematical concepts that are used in real life situations.

## 1.2 Revision - Ratio and Proportion

Try and recollect the definitions and facts on Ratio and Proportion and complete the following statements using the help box:

1. The comparison of two quantities of the same kind by means of division is termed as _________.
2. The two quantities to be compared are called the _________ of the ratio.
3. The first term of the ratio is called the _________ and the second term is called the _________.
4. In ratio, only quantities in the _________ units can be compared.
5. If the terms of the ratio have common factors, we can reduce it to its lowest terms by cancelling the _________.
6. When both the terms of a ratio are multiplied or divided by the same number (other than zero) the ratio remains _________.The obtained ratios are _________.

7. In a ratio the order of the terms is very important. (Say True or False)

8. Ratios are mere numbers. Hence units are not needed. (Say True or False)

9. Equality of two ratios is called a ________. If $a, b; c, d$ are in proportion, then $a : b :: c : d$.

10. In a proportion, the product of extremes =________

**Help Box:**

| | | |
|---|---|---|
| 1) Ratio | 2) terms | 3) antecedent, consequent |
| 4) same | 5) common factors | 6) unchanged, equivalent ratios |
| 7) True | 8) True | 9) proportion |
| 10) product of means | | |

***Example 1.1:***

Find 5 equivalent ratios of 2:7

***Solution:*** 2 : 7 can be written as $\frac{2}{7}$.

Multiplying the numerator and the denominator of $\frac{2}{7}$ by 2, 3, 4, 5, 6

we get $\frac{2\times 2}{7\times 2} = \frac{4}{14}$, $\frac{2\times 3}{7\times 3} = \frac{6}{21}$, $\frac{2\times 4}{7\times 4} = \frac{8}{28}$

$\frac{2\times 5}{7\times 5} = \frac{10}{35}$, $\frac{2\times 6}{7\times 6} = \frac{12}{42}$

4 : 14, 6 : 21, 8 : 28, 10 : 35, 12 : 42 are equivalent ratios of 2 : 7.

***Example 1.2:***

Reduce 270 : 378 to its lowest term.

***Solution:***

$$270:378 = \frac{270}{378}$$

Dividing both the numerator and the denominator by 2, we get

$$\frac{270 \div 2}{378 \div 2} = \frac{135}{189}$$

**Aliter:**

Factorizing 270,378 we get

$$\frac{270}{378} = \frac{2\times 3\times 3\times 3\times 5}{2\times 3\times 3\times 3\times 7}$$

$$= \frac{5}{7}$$

by 3, we get

$$\frac{135 \div 3}{189 \div 3} = \frac{45}{63}$$

by 9, we get

$$\frac{45 \div 9}{63 \div 9} = \frac{5}{7}$$

270 : 378 is reduced to 5 : 7

### *Example 1.3*

Find the ratio of 9 months to 1 year

**Solution:** 1 year = 12 months

Ratio of 9 months to 12 months = 9 : 12

9 : 12 can be written as $\frac{9}{12}$

$= \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

$= 3 : 4$

Quantities in the same units only can be compared in the form of a ratio. So convert year to months.

### *Example 1.4*

If a class has 60 students and the ratio of boys to girls is 2:1, find the number of boys and girls.

**Solution:**

Number of students = 60

Ratio of boys to girls = 2 : 1

Total parts = 2 + 1 = 3

Number of boys = $\frac{2}{3}$ of 60

$= \frac{2}{3} \times 60 = 40$

Number of boys = 40

Number of girls = Total Number of students – Number of boys

= 60 – 40

= 20

Number of girls = 20

**[OR]**

Number of girls

$= \frac{1}{3}$ of 60 $= \frac{1}{3} \times 60$

= 20

### *Example 1.5*

A ribbon is cut into 3 pieces in the ratio 3: 2: 7. If the total length of the ribbon is 24 m, find the length of each piece.

***Solution:***

$$\text{Length of the ribbon} = 24\text{m}$$

$$\text{Ratio of the 3 pieces} = 3 : 2 : 7$$

$$\text{Total parts} = 3 + 2 + 7 = 12$$

$$\text{Length of the first piece of ribbon} = \frac{3}{12} \text{ of } 24$$

$$= \frac{3}{12} \times 24 = 6 \text{ m}$$

$$\text{Length of the second piece of ribbon} = \frac{2}{12} \text{ of } 24$$

$$= \frac{2}{12} \times 24 = 4 \text{ m}$$

$$\text{Length of the last piece of ribbon} = \frac{7}{12} \text{ of } 24$$

$$= \frac{7}{12} \times 24 = 14 \text{ m}$$

So, the length of the three pieces of ribbon are 6 m, 4 m, 14 m respectively.

### *Example 1.6*

The ratio of boys to girls in a class is 4 : 5. If the number of boys is 20, find the number of girls.

***Solution:*** Ratio of boys to girls = 4 : 5

Number of boys = 20

Let the number of girls be $x$

The ratio of the number of boys to the number of girls is 20 : $x$

4 : 5 and 20 : $x$ are in proportion, as both the ratios represent the number of boys and girls.

(i.e.) 4 : 5 :: 20 : $x$

$$\text{Product of extremes} = 4 \times x$$

$$\text{Product of means} = 5 \times 20$$

In a proportion, product of extremes = product of means

$$4 \times x = 5 \times 20$$

$$x = \frac{5 \times 20}{4} = 25$$

$$\text{Number of girls} = 25$$

***Example 1.7***

If A : B = 4 : 6, B : C = 18 : 5, find the ratio of A : B : C.

***Solution:***

A : B = 4 : 6

B : C = 18 : 5

L.C.M. of 6, 18 = 18

A : B = 12 : 18

B : C = 18 : 5

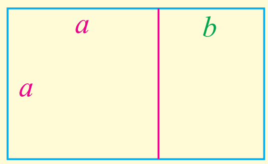
A : B : C =12 : 18 : 5

**HINT**

To compare 3 ratios as given in the example, the consequent (2nd term) of the 1st ratio and the antecedent (1st term) of the 2nd ratio must be made equal.

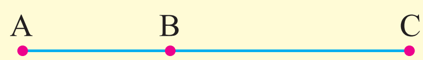
**Do you Know?**

**Golden Ratio:** Golden Ratio is a special number approximately equal to 1.6180339887498948482⋯. We use the Greek letter Phi (Φ) to refer to this ratio. Like Phi the digits of the Golden Ratio go on forever without repeating.



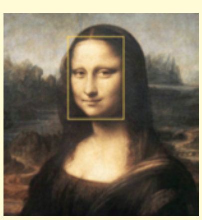

**Golden Rectangle:** A Golden Rectangle is a rectangle in which the ratio of the length to the width is the Golden Ratio. If width of the Golden Rectangle is 2 ft long, the other side is approximately = 2 (1.62) = 3.24 ft

**Golden segment:** It is a line segment divided into 2 parts. The ratio of the length of the 2 parts of this segment is the Golden Ratio




$$\frac{AB}{BC} = \frac{BC}{AC}$$

**Applications of Golden Ratio:**



**Think!**

1. Use the digits 1 to 9 to write as many proportions as possible. Each digit can be used only once in a proportion. The numbers that make up the proportion should be a single digit number.
   Eg: $\frac{1}{2} = \frac{3}{6}$
2. Suppose the ratio of zinc to copper in an alloy is 4 : 9, is there more zinc or more copper in the alloy?
3. A bronze statue is made of copper, tin and lead metals. It has $\frac{1}{10}$ of tin, $\frac{1}{4}$ of lead and the rest copper. Find the part of copper in the bronze statue.

## 1.3 Variation

Cutting down of trees → RESULTS → Increase in temperature

Usage of more vehicles → RESULTS → Increase of pollutants

You walk faster → RESULTS → You take less time

More vehicles on the road → RESULTS → Less road space

These are some changes.

What happens when......

You study well? → You score more marks

You eat more? → You become fatter

You shout in class? → Class becomes noisy

In all the above cases we see that a change in one factor brings about a change in the related factor. Such changes are termed as variation.

Now, **try and match the answers to the given questions:**

What happens when............

| | |
|---|---|
| You buy more pens? | More number of teachers |
| Number of students are more? | Costs you more |
| You travel less distance? | Weight of bag is less |
| Number of books are reduced? | Time taken is less |

The above examples are interdependent quantities that change numerically.

We observe that, an increase (↑) in one quantity brings about an increase (↑) in the other quantity and similarly a decrease (↓) in one quantity brings about a decrease (↓) in the other quantity .

Now, look at the following tables:

| Cost of 1 pen (₹) | Cost of 10 pens (₹) |
|---|---|
| 5 | $10 \times 5 = 50$ |
| 20 | $10 \times 20 = 200$ |
| 30 | $10 \times 30 = 300$ |

As the number of pens increases, the cost also increases correspondingly.

| Cost of 5 shirts (₹) | Cost of 1 shirt (₹) |
|---|---|
| 3000 | $\frac{3000}{5} = 600$ |
| 1000 | $\frac{1000}{5} = 200$ |

As the number of shirts decreases, the cost also decreases correspondingly.

Thus we can say, if an increase ($\uparrow$) [decrease ($\downarrow$)] in one quantity produces a proportionate increase ($\uparrow$) [decrease ($\downarrow$)] in another quantity, then the two quantities are said to be in **direct variation**.

Now, let us look at some more examples:

i) When the speed of the car increases, do you think that the time taken to reach the destination will increase or decrease?

ii) When the number of students in a hostel decreases, will the provisions to prepare food for the students last longer or not?

We know that as the speed of the car increases, the time taken to reach the given destination definitely decreases.

Similarly, if the number of students decreases, the provisions last for some more number of days.

Thus, we find that if an increase ($\uparrow$) [decrease ($\downarrow$)] in one quantity produces a proportionate decrease ($\downarrow$) [increase ($\uparrow$)] in another quantity, then we say that the two quantities are in **inverse variation**.

Try these

Identify the direct and inverse variations from the given examples.

1. Number of pencils and their cost
2. The height of poles and the length of their shadows at a given time
3. Speed and time taken to cover a distance
4. Radii of circles and their areas
5. Number of labourers and the number of days taken to complete a job
6. Number of soldiers in a camp and weekly expenses
7. Principal and Interest
8. Number of lines per page and number of pages in a book

Look at the table given below:

| Number of pens | $x$ | 2 | 4 | 7 | 10 | 20 |
|---|---|---|---|---|---|---|
| Cost of pens (₹) | $y$ | 100 | 200 | 350 | 500 | 1000 |

We see that as '$x$' increases ($\uparrow$) '$y$' also increases ($\uparrow$).

We shall find the ratio of number of pens to cost of pens

$$\frac{\text{Number of pens}}{\text{Cost of pens}} = \frac{x}{y}, \text{ to be } \frac{2}{100}, \frac{4}{200}, \frac{7}{350}, \frac{10}{500}, \frac{20}{1000}$$

and we see that each ratio = $\frac{1}{50}$ = Constant.

Ratio of number of pens to cost of pens is a constant.

$$\therefore \frac{x}{y} = \text{constant}$$

It can be said that **when two quantities vary directly the ratio of the two given quantities is always a constant**.

Now, look at the example given below:

| Time taken (Hrs) | $x_1 = 2$ | $x_2 = 10$ |
|---|---|---|
| Distance travelled (km) | $y_1 = 10$ | $y_2 = 50$ |

We see that as time taken increases (↑), distance travelled also increases (↑).

$$X = \frac{x_1}{x_2} = \frac{2}{10} = \frac{1}{5}$$

$$Y = \frac{y_1}{y_2} = \frac{10}{50} = \frac{1}{5}$$

$$X = Y = \frac{1}{5}$$

From the above example, it is clear that in **direct variation, when a given quantity is changed in some ratio then the other quantity is also changed in the same ratio**.

Now, study the relation between the given variables and find *a* and *b*.

| Time taken (hrs) | $x$ | 2 | 5 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| Distance travelled (Km) | $y$ | 120 | 300 | $a$ | 480 | 600 | $b$ |

Here again, we find that the ratio of the time taken to the distance travelled is a constant.

$$\frac{\text{Time taken}}{\text{Distance travelled}} = \frac{2}{120} = \frac{5}{300} = \frac{10}{600} = \frac{8}{480} = \frac{1}{60} = \text{Constant}$$

(i.e.) $\frac{x}{y} = \frac{1}{60}$. Now, we try to find the unknown

$$\frac{1}{60} = \frac{6}{a}$$

$$\frac{1 \times \boxed{6}}{60 \times \boxed{6}} = \frac{6}{360}$$

$$a = 360$$

$$\frac{1}{60} = \frac{12}{b}$$

$$\frac{1 \times \boxed{12}}{60 \times \boxed{12}} = \frac{12}{720}$$

$$b = 720$$

Look at the table given below:

| Speed (Km / hr) | $x$ | 40 | 48 | 60 | 80 | 120 |
|---|---|---|---|---|---|---|
| Time taken (hrs) | $y$ | 12 | 10 | 8 | 6 | 4 |

Here, we find that as $x$ increases (↓) y decreases (↑)

$$xy = 40 \times 12 = 480$$

$$= 48 \times 10 = 60 \times 8 = 80 \times 6 = 120 \times 4 = 480$$

$$\therefore xy = \text{constant}$$

It can be stated that **if two quantities vary inversely, their product is a constant**.

Look at the example below:

| Speed (Km/hr) | $x_1 = 120$ | $x_2 = 60$ |
|---|---|---|
| Time taken (hrs) | $y_1 = 4$ | $y_2 = 8$ |

As speed increases (↑), time taken decreases (↓).

$$X = \frac{x_1}{x_2} = \frac{120}{60} = 2$$

$$Y = \frac{y_1}{y_2} = \frac{4}{8} = \frac{1}{2} \quad 1/Y = 2$$

$$X = \frac{1}{Y}$$

Thus, it is clear that in inverse variation, when a given quantity is changed in some ratio the other quantity is changed in inverse ratio.

Now, study the relation between the variables and find a and b.

| No of men | $x$ | 15 | 5 | 6 | b | 60 |
|---|---|---|---|---|---|---|
| No of days | $y$ | 4 | 12 | a | 20 | 1 |

We see that, $xy = 15 \times 4 = 5 \times 12 = 60 = \text{constant}$

$$xy = 60$$

$$6 \times a = 60$$

$$6 \times 10 = 60$$

$$a = 10$$

$$xy = 60$$
$$b \times 20 = 60$$
$$3 \times 20 = 60$$
$$\boxed{b = 3}$$

Try these

1. If $x$ varies directly as $y$, complete the given tables:

(i)

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 1 | 3 | | | 9 | 15 |
| $y$ | 2 | | 10 | 16 | | |

(ii)

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 2 | 4 | 5 | | |
| $y$ | 6 | | | 18 | 21 |

2. If $x$ varies inversely as $y$, complete the given tables:

(i)

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 20 | 10 | 40 | 50 | |
| $y$ | | | 50 | | 250 |

(ii)

| | | | | | |
|---|---|---|---|---|---|
| $x$ | | 200 | 8 | 4 | 16 |
| $y$ | 10 | | 50 | | |

***Example 1.8***

If the cost of 16 pencils is ₹48, find the cost of 4 pencils.

***Solution:***

Let the cost of four pencils be represented as '$a$'.

| **Number of pencils** | **Cost (₹)** |
|---|---|
| $x$ | $y$ |
| 16 | 48 |
| 4 | $a$ |

As the number of pencils decreases (↓), the cost also decreases (↓). Hence the two quantities are in **direct variation**.

We know that, in direct variation, $\frac{x}{y}$ = constant

$$\frac{16}{48} = \frac{4}{a}$$

$$16 \times a = 48 \times 4$$

$$a = \frac{48 \times 4}{16} = 12$$

Cost of four pencils = ₹12

**Aliter:**

Let the cost of four pencils be represented as ‘a’ .

| Number of pencils | Cost (₹) |
|---|---|
| $x$ | $y$ |
| 16 | 48 |
| 4 | $a$ |

As number of pencils decreases (↓), cost also decreases (↓), **direct variation (Same ratio)**.

$$\frac{16}{4} = \frac{48}{a}$$

$$16 \times a = 4 \times 48$$

$$a = \frac{4 \times 48}{16} = 12$$

Cost of four pencils = ₹12.

***Example 1.9***

A car travels 360 km in 4 hrs. Find the distance it covers in 6 hours 30 mins at the same speed.

***Solution:***

Let the distance travelled in $6\frac{1}{2}$ hrs be $a$

| Time taken (hrs) | Distance travelled (km) |
|---|---|
| $x$ | $y$ |
| 4 | 360 |
| $6\frac{1}{2}$ | $a$ |

30 mins $= \frac{30}{60}$ hrs
$= \frac{1}{2}$ of an hr
6 hrs 30 mins = $6\frac{1}{2}$ hrs

As time taken increases (↑), distance travelled also increases (↑), **direct variation.**

In direct variation, $\frac{x}{y}$ = constant

$$\frac{4}{360} = \frac{6\frac{1}{2}}{a}$$

$$4 \times a = 360 \times 6\frac{1}{2}$$

$$4 \times a = 360 \times \frac{13}{2}$$

$$a = \frac{360 \times 13}{4 \times 2} = 585$$

Distance travelled in $6\frac{1}{2}$ hrs = 585 km

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**Aliter:** Let the distance travelled in $6\frac{1}{2}$ hrs be $a$

| Time taken (hrs) | Distance travelled (km) |
|---|---|
| 4 | 360 |
| $6\frac{1}{2}$ | $a$ |

As time taken increases (↑), distance travelled also increases (↑), **direct variation (same ratio).**

$$\frac{4}{6\frac{1}{2}} = \frac{360}{a}$$
$$4 \times a = 360 \times 6\tfrac{1}{2}$$
$$4 \times a = 360 \times \frac{13}{2}$$
$$a = \frac{360}{4} \times \frac{13}{2} = 585$$

Distance travelled in $6\frac{1}{2}$ hrs = 585 km.

***Example 1.10***

7 men can complete a work in 52 days. In how many days will 13 men finish the same work?

***Solution:*** Let the number of unknown days be $a$.

| Number of men | Number of days |
|---|---|
| $x$ | $y$ |
| 7 | 52 |
| 13 | $a$ |

As the number of men increases (↑), number of days decreases (↓), **inverse variation**

In inverse variation, $xy$ = constant

$$7 \times 52 = 13 \times a$$
$$13 \times a = 7 \times 52$$
$$a = \frac{7 \times 52}{13} = 28$$

13 men can complete the work in 28 days.

**Aliter:**

Let the number of unknown days be $a$.

| Number of men | Number of days |
|---|---|
| 7 | 52 |
| 13 | $a$ |

As number of men increases (↑), number of days decreases (↓), **inverse variation (inverse ratio).**

$$\frac{7}{13} = \frac{a}{52}$$

$$7 \times 52 = 13 \times a$$

$$13 \times a = 7 \times 52$$

$$a = \frac{7 \times 52}{13} = 28$$

13 men can complete the work in 28 days

***Example 1.11***

A book contains 120 pages. Each page has 35 lines . How many pages will the book contain if every page has 24 lines per page?

***Solution:*** Let the number of pages be $a$.

| Number of lines per page | Number of pages |
|---|---|
| 35 | 120 |
| 24 | $a$ |

As the number of lines per page decreases (↓) number of pages increases (↑) it is in **inverse variation (inverse ratio)**.

$$\frac{35}{24} = \frac{a}{120}$$

$$35 \times 120 = a \times 24$$

$$a \times 24 = 35 \times 120$$

$$a = \frac{35 \times 120}{24}$$

$$a = 35 \times 5 = 175$$

If there are 24 lines in one page, then the number of pages in the book = 175

## Exercise 1.1

1. Choose the correct answer

i) If the cost of 8 kgs of rice is ₹160, then the cost of 18 kgs of rice is

(A) ₹480 (B) ₹180 (C) ₹360 (D) ₹1280

ii) If the cost of 7 mangoes is ₹35, then the cost of 15 mangoes is

(A) ₹75 (B) ₹25 (C) ₹35 (D) ₹50

iii) A train covers a distance of 195 km in 3 hrs. At the same speed, the distance travelled in 5 hours is

(A) 195 km. (B) 325 km. (C) 390 km. (D) 975 km.

iv) If 8 workers can complete a work in 24 days, then 24 workers can complete the same work in

(A) 8 days (B) 16 days (C) 12 days (D) 24 days

v) If 18 men can do a work in 20 days, then 24 men can do this work in

(A) 20 days (B) 22 days (C) 21 days (D) 15 days

2. A marriage party of 300 people require 60 kg of vegetables. What is the requirement if 500 people turn up for the marriage?

3. 90 teachers are required for a school with a strength 1500 students. How many teachers are required for a school of 2000 students?

4. A car travels 60 km in 45 minutes. At the same rate, how many kilo metres will it travel in one hour?

5. A man whitewashes 96 sq.m of a compound wall in 8 days. How many sq.m will be white washed in 18 days?

6. 7 boxes weigh 36.4 kg. How much will 5 such boxes weigh?

7. A car takes 5 hours to cover a particular distance at a uniform speed of 60 km / hr. How long will it take to cover the same distance at a uniform speed of 40 km / hr?

8. 150 men can finish a piece of work in 12 days. How many days will 120 men take to finish the same work?

9. A troop has provisions for 276 soldiers for 20 days. How many soldiers leave the troop so that the provisions may last for 46 days?

10. A book has 70 pages with 30 lines of printed matter on each page. If each page is to have only 20 lines of printed matter, how many pages will the book have?

11. There are 800 soldiers in an army camp. There is enough provisions for them for 60 days. If 400 more soldiers join the camp, for how many days will the provisions last?

**Just for fun**

If an owl builds a nest in 1 second , then what time will it take if there were 200 owls?
Owls don't build their own nests. They simply move into an old hawk's nest or rest in ready made cavities.

**Try these**

Read the questions. Recollect the different methods that you have learnt earlier. Try all the different methods possible and solve them.

1. A wheel makes 48 revolutions in 3 seconds. How many revolutions does it make in 30 seconds?
2. A film processor can develop 100 negatives in 5 minutes. How many minutes will it take to develop 1200 negatives?
3. There are 36 players in 2 teams. How many players are there in 5 teams?

# Points to Remember

1. Two quantities are said to be in direct variation if the increase (decrease) in one quantity results in a proportionate increase (decrease) in the other quantity.

2. Two quantities are said to be in inverse variation if the increase (decrease) in one quantity results in a proportionate decrease (increase) in the other quantity.

3. In direct proportion, the ratio of one quantity is equal to the ratio of the second quantity.

4. In indirect proportion, the ratio of one quantity is equal to the inverse ratio of the second quantity.