locations. The expression (1.48) is same if the charges are brought to their positions in any other order. Since the Coulomb force is a conservative force, the electrostatic potential energy is independent of the manner in which the configuration of charges is arrived at.

Electrostatic potential energy of a dipole in a uniform electric field

- Consider a dipole placed in the uniform electric field $\vec{E}$ as shown in the Figure 1.31. A dipole experiences a torque when kept in an uniform electric field $\vec{E}$. This torque rotates the dipole to align it with the direction of the electric field. To rotate the dipole (at constant angular velocity) from its initial angle to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

- The work done by the external torque to rotate the dipole from angle to θ at constant angular velocity is

$$W = \int_{\theta'}^{\theta} \tau_{ext} \, d\theta \qquad (1.49)$$

Since $\vec{\tau}_{ext}$ is equal and opposite to $\vec{\tau}_E = \vec{p} \times \vec{E}$, we have

$$|\vec{\tau}_{ext}| = |\vec{\tau}_E| = |\vec{p} \times \vec{E}| \qquad (1.50)$$

Substituting equation (1.50) in equation (1.49), we get

$$W = \int_{\theta'}^{\theta} pE \sin\theta \, d\theta$$

$$W = pE(\cos\theta' - \cos\theta)$$

This work done is equal to the potential energy difference between the angular positions θ and θ'.

$$U(\theta) - U(\theta') = \Delta U = -pE\cos\theta + pE\cos\theta'$$

If the initial angle is $\theta' = 90^\circ$ and is taken as reference point, then $U(\theta') = pE\cos 90^\circ = 0$.

The potential energy stored in the system of dipole kept in the uniform electric field is given by

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E} \qquad (1.51)$$

- In addition to p and E, the potential energy also depends on the orientation θ of the electric dipole with respect to the external electric field.

- The potential energy is maximum when the dipole is aligned anti-parallel (θ = π) to the external electric field and minimum when the dipole is aligned parallel (θ = 0) to the external electric field.

Gauss Law and its Applications

Electric Flux

- The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. It is usually denoted by the Greek letter and its unit is N m2 C-1. Electric flux is a scalar quantity and it can be positive or negative. For a simpler understanding of electric flux, the following Figure 1.32 is useful.

- The electric field of a point charge is drawn in this figure. Consider two small rectangular area elements placed normal to the field at regions A and B. Even though these elements have the same area, the number of electric field lines crossing the element in region A is more than that crossing the element in region B. Therefore the electric flux in region A is more than that in region B. The electric field strength for a point charge decreases as the distance increases, then for a point charge electric flux also decreases as the distance increases. The above discussion gives a qualitative idea of electric flux. However a precise definition of electric flux is needed.

Electric flux for uniform Electric field

- Consider a uniform electric field in a region of space. Let us choose an area A normal to the electric field lines as shown in Figure 1.33 (a). The electric flux for this case is

$$\Phi_E = EA \qquad (1.52)$$

- Suppose the same area A is kept parallel to the uniform electric field, then no electric field lines pierce through the area A , as shown in Figure 1.33(b). The electric flux for this case is zero.

$$\Phi_E = 0 \qquad (1.53)$$

- If the area is inclined at an angle θ with the field, then the component of the electric field perpendicular to the area alone contributes to the electric flux. The electric field component parallel to the surface area will not contribute to the electric flux. This is shown in Figure 1.33 (c). For this case, the electric flux

$$\Phi_E = (E\cos\theta)\,A \qquad (1.54)$$

- Further, θ is also the angle between the electric field and the direction normal to the area. Hence in general, for uniform electric field, the electric flux is defined as

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta \qquad (1.55)$$

Here, note that $\vec{A}$ is the area vector $\vec{A} = A\hat{n}$. Its magnitude is simply the area A and the direction is along the unit vector $\hat{n}$ perpendicular to the area as shown in Figure 1.33. Using this definition for flux, $\Phi_E = \vec{E} \cdot \vec{A}$, equations (1.53) and (1.54) can be obtained as special cases.

In Figure 1.33 (a), $\theta = 0^\circ$ so $\Phi_E = \vec{E} \cdot \vec{A} = EA$

In Figure 1.33 (b), $\theta = 90^\circ$ so $\Phi_E = \vec{E} \cdot \vec{A} = 0$

Electric flux in a non uniform electric field and an arbitrarily shaped area

- Suppose the electric field is not uniform and the area A is not flat (Figure 1.34), then the entire area is divided into n small area segments

$\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3 \ldots\ldots\ldots \Delta\vec{A}_n$ such that each area element is almost flat and the electric field over each area element is considered to be uniform.

The electric flux for the entire area A is approximately written as

$$\Phi_E = \vec{E}_1 \cdot \Delta\vec{A}_1 + \vec{E}_2 \cdot \Delta\vec{A}_2 + \vec{E}_3 \cdot \Delta\vec{A}_3 \ldots\ldots \vec{E}_n \cdot \Delta\vec{A}_n$$
$$= \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i \qquad (1.56)$$

- By taking the limit $\Delta Ai \rightarrow 0$ (for all i) the summation in equation (1.56) becomes integration. The total electric flux for the entire area is given by

$$\Phi_E = \int \vec{E} \cdot d\vec{A} \qquad (1.57)$$

From Equation (1.57), it is clear that the electric flux for a given surface depends on both the electric field pattern on the surface area and orientation of the surface with respect to the electric field.

Electric flux for closed surfaces

- In the previous section, the electric flux for any arbitrary curved surface is discussed. Suppose a closed surface is present in the region of the non-uniform electric field as shown in Figure 1.35 (a).

The total electric flux over this closed surface is written as

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} \qquad (1.58)$$

(1.58) is a closed surface integration and for each areal element, the outward normal is the direction $d\vec{A}$ of as shown in the Figure 1.35(b). The total electric flux over a closed surface can be negative, positive or zero. In the Figure 1.35(b), it is shown that in one area element, the angle between $d\vec{A}$ and $\vec{E}$ is less than 90o, then the electric flux is positive and in another areal element, the angle between $d\vec{A}$ and $\vec{E}$ is greater than 90o, then the electric flux is negative.

In general, the electric flux is negative if the electric field lines enter the closed surface and positive if the electric field lines leave the closed surface.

Gauss law

A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in Figure 1.36. We can calculate the total

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\, dA \cos\theta$$

electric flux through the closed surface of the sphere using the equation (1.58).

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and θ=°0

$$\Phi_E = \oint E\,dA \qquad \text{since } \cos 0^\circ = 1 \qquad (1.59)$$

E is uniform on the surface of the sphere,

$$\Phi_E = E\oint dA \qquad (1.60)$$

Substituting for $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ in equation 1.60, we get

$$\Phi_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\times 4\pi r^2 = 4\pi\frac{1}{4\pi\varepsilon_0}Q$$

$$\Phi_E = \frac{Q}{\varepsilon_0} \qquad (1.61)$$

.

The equation (1.61) is called as Gauss's law.

The remarkable point about this result is that the equation (1.61) is equally true for any arbitrary shaped surface which encloses the charge Q and as shown in the Figure 1.37. It is seen that the total electric flux is the same for closed surfaces A1, A2 and A3 as shown in the Figure 1.37.

Gauss's law states that if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ_E through the closed surface is

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \qquad (1.62)$$

Here Q_{encl} denotes the charges inside the closed surface.

Discussion of Gauss law

(i) The total electric flux through the closed surface depends only on the

$_{\rightarrow}E$

charges enclosed by the surface and the charges present outside the surface will not contribute to the flux and the shape of the closed surface which can be chosen arbitrarily.

(ii) The total electric flux is independent of the location of the charges inside the closed surface.

(iii) To arrive at equation (1.62), we have chosen a spherical surface. This imaginary surface is called a Gaussian surface. The shape of the Gaussian surface to be chosen depends on the type of charge configuration and the kind of symmetry existing in that charge configuration. The electric field is spherically symmetric for a point charge, therefore spherical Gaussian surface is chosen. Cylindrical and planar Gaussian surfaces can be chosen for other kinds of charge configurations.

(iv) In the LHS of equation (1.62), the electric field E is due to charges present inside and outside the Gaussian surface but the charge Q_{encl} denotes the charges which lie only inside the Gaussian surface.

(v) The Gaussian surface cannot pass through any discrete charge but it can pass through continuous charge distributions. It is because, very close to the discrete charges, the electric field is not well defined.

(vi) Gauss law is another form of Coulomb's law and it is also applicable to the charges in motion. Because of this reason, Gauss law is treated as much more general law than Coulomb's law.

$\Phi_{E} q = 3\varepsilon_{\circ}$

(i) In figure (a), calculate the electric flux through the closed areas A1 and A2.

(ii) In figure (b), calculate the electric flux through the cube

Solution

(i) In figure (a), the area A1 encloses the charge Q. So electric flux through this closed surface A_1 is $\frac{Q}{\varepsilon}$. But the closed surface A2 contains no charges inside, so electric flux through A2 is zero.

(ii) In figure (b), the net charge inside the cube is 3q and the total electric flux in the cube is therefore $\Phi_E = \frac{3q}{\varepsilon_0}$

Note that the charge -10 q lies outside the cube and it will not contribute the total flux through the surface of the cube.

Applications of Gauss law

Electric field due to any arbitrary charge configuration can be calculated using Coulomb's law or Gauss law. If the charge configuration possesses some kind of symmetry, then Gauss law is a very efficient way to calculate the electric field. It is illustrated in the following cases.

(i) Electric field due to an infinitely long charged wire

Consider an infinitely long straight wire having uniform linear charge density λ. Let P be a point located at a perpendicular distance r from the wire (Figure 1.38(a)).

The electric field at the point P can be found using Gauss law. We choose two small charge elements A1 and A2 on the wire which are at equal distances from the point P. The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r. This is shown in the Figure 1.38(b). From this property, we can infer that the charged wire possesses a cylindrical symmetry.

Let us choose a cylindrical Gaussian surface of radius r and length L as shown in the Figure 1.39.

The total electric flux in this closed surface is calculated as follows.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$
$$= \int_{\substack{\text{Curved}\\ \text{surface}}} \vec{E}\cdot d\vec{A} + \int_{\substack{\text{top}\\ \text{surface}}} \vec{E}\cdot d\vec{A} + \int_{\substack{\text{bottom}\\ \text{surface}}} \vec{E}\cdot d\vec{A} \quad (1.63)$$

It is seen from Figure (1.39) that for the curved surface, $\vec{E}$ is parallel to $\vec{A}$ and $\vec{E}\cdot d\vec{A} = E\,dA$. For the top and bottom surfaces, $\vec{E}$ is perpendicular to $\vec{A}$ and $\vec{E}\cdot d\vec{A} = 0$

Substituting these values in the equation (1.63) and applying Gauss law to the cylindrical surface, we have

$$\Phi_E = \int_{\substack{\text{Curved}\\ \text{surface}}} E\,dA = \frac{Q_{encl}}{\varepsilon_0} \qquad (1.64)$$

Since the magnitude of the electric field for the entire curved surface is constant, *E* is taken out of the integration and *Qencl*mis given by *QLencl*=λ.

$$E\int_{\substack{Curved\\surface}} dA = \frac{\lambda L}{\varepsilon_0} \quad (1.65)$$

Here $\Phi_E = \int_{\substack{Curved\\surface}} dA$ = total area of the curved surface = $2\pi rL$. Substituting this in equation (1.65), we get

$$E\cdot 2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r} \quad (1.66)$$

In vector form $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}$ (1.67)

The electric field due to the infinite charged wire depends on $\frac{1}{r}$ rather than $\frac{1}{r^2}$ for a point charge.

Equation (1.67) indicates that the electric field is always along the perpendicular direction r Gaussiansurface) to wire. In fact, if λ > 0 then

Electric field due to charged infinite planar sheetpoints perpendicular outward () from the wire and if λ < 0, then points perpendicular inward r.

The equation (1.67) is true only for an infinitely long charged wire. For a charged wire of finite length, the electric field need not be radial at all points. However, equation (1.67) for such a wire is taken approximately true around the mid-point of the wire and far away from the both ends of the wire

(ii) Electric field due to charged infinite plane sheet

Consider an infinite plane sheet of charges with uniform surface charge density σ. Let P be a point at a distance of r from the sheet as shown in the Figure 1.40.

Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points. A cylindrical shaped Gaussian surface of length 2r and area A of the flat

surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.
Applying Gauss law for this cylindrical surface,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$
$$= \int_{\text{Curved surface}} \vec{E} \cdot d\vec{A} + \int_P \vec{E} \cdot d\vec{A} + \int_{P'} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad (1.68)$$

The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and P′ (Figure 1.40). Then,

$$\Phi_E = \int_P E\,dA + \int_{P'} E\,dA = \frac{Q_{encl}}{\varepsilon_0} \quad (1.69)$$

Since the magnitude of the electric field at these two equal surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get

$$2E \int_P dA = \frac{\sigma A}{\varepsilon_0}$$

The total area of surface either at P or P′

$$\int_P dA = A$$

$$\text{Hence } 2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0} \quad (1.70)$$

$$\text{In vector form, } \vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n} \quad (1.71)$$

Here n is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance r.

The electric field will be the same at any point farther away from the charged plane. Equation (1.71) implies that if σ > 0 the electric field at any point P is outward perpendicular to the plane and if σ < 0 the electric field points inward perpendicularly (-n) to the plane.

For a finite charged plane sheet, equation (1.71) is approximately true only in the middle region of the plane and at points far away from both ends.

(iii) Electric field due to two parallel charged infinite sheets

Consider two infinitely large charged plane sheets with equal and opposite charge densities +σ and -σ which are placed parallel to each other as shown in the Figure 1.41.

The electric field between the plates and outside the plates is found using Gauss law. The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\varepsilon_o}$ and it points perpendicularly outward if σ > 0 and points inward if σ < 0.

At the points P_2 and P_3, the electric field due to both plates are equal in magnitude and opposite in direction (Figure 1.41). As a result, electric field at a point outside the plates is zero. But inside the plate, electric fields are in same direction i.e., towards the right, the total electric field at a point P_1

$$E_{inside} = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o} \qquad (1.72)$$

The direction of the electric field inside the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere inside the plate.

(iv) Electric field due to a uniformly charged spherical shell

Consider a uniformly charged spherical shell of radius R and total charge Q as shown in Figure 1.42. The electric field at points outside and inside the sphere is found using Gauss law.

Case (a) At a point outside the shell (r > R)

Let us choose a point P outside the shell at a distance r from the center as shown in Figure 1.42 (a). The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0. So we choose a spherical Gaussian surface of radius r is chosen and the total charge enclosed by this Gaussian surface is Q. Applying Gauss law

$$\oint_{\text{Gaussian surface}} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o} \quad (1.73)$$

The electric field $\vec{E}$ and $d\vec{A}$ point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of $\vec{E}$ is also the same at all points due to the spherical symmetry of the charge distribution.

$$\text{Hence } E \oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA = \frac{Q}{\varepsilon_o} \qquad (1.74)$$

But $\oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA =$ total area of Gaussian surface $= 4\pi r^2$. Substituting this value in equation (1.74)

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_o} \text{ (or) } E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

$$\text{In vector form} \quad \vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\hat{r} \qquad (1.75)$$

The electric field is radially outward if $Q > 0$ and radially inward if $Q < 0$. From equation (1.75), we infer that the electric field at a point outside the shell will be same as if the entire charge Q is concentrated at the center of the spherical shell. (A similar result is observed in gravitation, for gravitational force due to a spherical shell with mass M)

Case (b): At a point on the surface of the spherical shell (r = R) The electrical field at points on the spherical shell (r = R) is given by

$$\vec{E} = \frac{Q}{4\pi\varepsilon_o R^2}\hat{r} \qquad (1.76)$$

Since Gaussian surface encloses no charge, So Q = 0. The equation (1.77) becomes

$$E = 0 \qquad (r < R) \qquad (1.78)$$

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell. A graph is plotted between the electric field and radial distance. This is shown in Figure 1.43.

ELECTROSTATICS OF CONDUCTORS AND DIELECTRICS

Conductors at electrostatic equilibrium

An electrical conductor has a large number of mobile charges which are free to move in the material. In a metallic conductor, these mobile charges are free electrons which are not bound to any atom and therefore are free to move on the surface of the conductor. When there is no external electric field, the free electrons are in continuous random motion in all directions. As a result, there is no net motion of electrons along any particular direction which implies that the conductor is in electrostatic equilibrium. Thus at electrostatic equilibrium, there is no net current in the conductor. A conductor at electrostatic equilibrium has the following properties.

(i) The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.

This is an experimental fact. Suppose the electric field is not zero inside the metal, then there will be a force on the mobile charge carriers due to this electric field. As a result, there will be a net motion of the mobile charges, which contradicts the conductors being in electrostatic equilibrium. Thus the electric field is zero everywhere inside the conductor. We can also understand this fact by applying an external uniform electric field on the conductor. This is shown in Figure 1.44.

Before applying the external electric field, the free electrons in the conductor are uniformly distributed in the conductor. When an electric field is applied, the free electrons accelerate to the left causing the left plate to be negatively charged and the right plate to be positively charged as shown in Figure 1.44.

Due to this realignment of free electrons, there will be an internal electric field created inside the conductor which increases until it nullifies the external electric field. Once the external electric field is nullified the conductor is said to be in electrostatic equilibrium. The time taken by a conductor to reach electrostatic equilibrium is in the order of 10-16s, which can be taken as almost instantaneous.

(ii) There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.

We can prove this property using Gauss law. Consider an arbitrarily shaped conductor as shown in Figure 1.45.

A Gaussian surface is drawn inside the conductor such that it is very close to the surface of the conductor. Since the electric field is zero everywhere inside

the conductor, the net electric flux is also zero over this Gaussian surface. From Gauss's law, this implies that there is no net charge inside the conductor. Even if some charge is introduced inside the conductor, it immediately reaches the surface of the conductor.

(iii) The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of where σ is the surface charge density at that point.

If the electric field has components parallel to the surface of the conductor, then free electrons on the surface of the conductor would experience acceleration (Figure 1.46(a)). This means that the conductor is not in equilibrium. Therefore at electrostatic equilibrium, the electric field must be perpendicular to the surface of the conductor. This is shown in Figure 1.46 (b).

We now prove that the electric field has magnitude $\frac{\sigma}{\varepsilon_0}$ just outside the conductor's surface. Consider a small cylindrical Gaussian surface, as shown in the Figure 1.47. One half of this cylinder is embedded inside the conductor

Since electric field is normal to the surface of the conductor, the curved part of the cylinder has zero electric flux. Also inside the conductor, the electric field is zero. Hence the bottom flat part of the Gaussian surface has no electric flux.

Therefore the top flat surface alone contributes to the electric flux. The electric field is parallel to the area vector and the total charge inside the surface is σA. By applying Gaus's law,

$$EA = \frac{\sigma A}{\varepsilon_o}$$

In vector form, $$\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{n} \qquad (1.79)$$

Here n represents the unit vector outward normal to the surface of the conductor. Suppose $\sigma < 0$, then electric field points inward perpendicular to the surface.

(iv) The electrostatic potential has the same value on the surface and inside of the conductor.

We know that the conductor has no parallel electric component on the surface which means that charges can be moved on the surface without doing any work. This is possible only if the electrostatic potential is constant at all points on the surface and there is no potential difference between any two points on the surface.
Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

Electrostatic shielding

Using Gauss law, we proved that the electric field inside the charged spherical shell is zero, Further, we showed that the electric field inside both hollow and solid conductors is zero. It is a very interesting property which has an important consequence.

Consider a cavity inside the conductor as shown in Figure 1.48 (a). Whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside the cavity is zero. A sensitive electrical instrument which is to be protected from external electrical disturbance is kept inside this cavity. This is called electrostatic shielding.

Faraday cage is an instrument used to demonstrate this effect. It is made up of metal bars configured as shown in Figure1.48 (b). If an artificial lightning jolt is created outside, the person inside is not affected.

During lightning accompanied by a thunderstorm, it is always safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, since the electric field inside is zero. During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

Electrostatic induction

In section 1.1, we have learnt that an object can be charged by rubbing using an appropriate material. Whenever a charged rod is touched by another conductor, charges start to flow from charged rod to the conductor. Is it possible to charge a conductor without any contact? The answer is yes. This type of **charging without actual contact is called electrostatic induction.**

(i) Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it, as shown in Figure 1.49(a).

The negative charge of the rod repels the electrons in the conductor to the opposite side. As a result, positive charges are induced near the region of the charged rod while negative charges on the farther side.
Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero. Once the charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod. But the total charge is zero.

(ii) Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding. Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere. Note that positive charges will not flow to the ground because they are attracted by the negative charges of the rod (Figure 1.49(b)).

(iii) When the grounding wire is removed from the conductor, the positive charges remain near the charged rod (Figure 1.49(c))

(iv) Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor (Figure 1.49 (d)). By this process, the neutral conducting sphere becomes positively charged.

For an arbitrary shaped conductor, the intermediate steps and conclusion are the same except the final step. The distribution of positive charges is not uniform for arbitrarily-shaped conductors. Why is it not uniform? The reason for it is discussed in the section 1.9

E X A M P L E

A small ball of conducting material having a charge +q and mass m is thrown upward at an angle θ to horizontal surface with an initial speed vo as shown in the figure. There exists an uniform electric field E downward along with the gravitational field g. Calculate the range, maximum height and time of flight in the motion of this charged ball. Neglect the effect of air and treat the ball as a point mass.

If the conductor has no net charge, then its motion is the same as usual projectile motion of a mass m which we studied in Kinematics (unit 2, vol-1 XI physics). Here, in this problem, in addition to downward gravitational force, the charge also will experience a downward uniform electrostatic force.

The acceleration of the charged ball due to
gravity $= -g\hat{j}$

The acceleration of the charged ball due to
uniform electric field $= -\frac{qE}{m}\hat{j}$

The total acceleration of charged ball in downward direction $\vec{a} = -\left(g + \frac{qE}{m}\right)\hat{j}$

It is important here to note that the acceleration depends on the mass of the object. Galileo's conclusion that all objects fall at the same rate towards the Earth is true only in a uniform gravitational field. When a uniform electric field is included, the acceleration of a charged object depends on both mass and charge.

But still the acceleration a= ($g+\frac{qE}{m}$) is constant throughout the motion. Hence we use kinematic equations to calculate the range, maximum height and time of flight. In fact we can simply replace g by $g+\frac{qE}{m}$ in the usual expressions of range, maximum height and time of flight of a projectile.

	Without charge	With the charge +q
Time of flight T	$\frac{2v_y\sin\theta}{g}$	$\frac{2v_y\sin\theta}{\left(g+\frac{qE}{m}\right)}$
Maximum height h_{max}	$\frac{v^2\sin^2\theta}{2g}$	$\frac{v^2\sin^2\theta}{2\left(g+\frac{qE}{m}\right)}$
Range R	$\frac{v^2\sin 2\theta}{g}$	$\frac{v^2\sin 2\theta}{\left(g+\frac{qE}{m}\right)}$

Note that the time of flight, maximum height, range are all inversely proportional to the acceleration of the object. Since $\left(g+\frac{qE}{m}\right)>g$ for charge +q, the quantities T, h_{max}, and R will decrease when compared to the motion of an object of mass m and zero net charge. Suppose the charge is −q, then $\left(g-\frac{qE}{m}\right)<g$, and the quantities T, h_{max} and

R will increase. Interestingly the trajectory is still parabolic .

Dielectrics or insulators

A dielectric is a non-conducting material and has no free electrons. The electrons in a dielectric are bound within the atoms. Ebonite, glass and mica are some examples of dielectrics. When an external electric field is applied, the electrons are not free to move anywhere but they are realigned in a specific way. A dielectric is made up of either polar molecules or non-polar molecules.

Non-polar molecules

A non-polar molecule is one in which centers of positive and negative charges coincide. As a result, it has no permanent dipole moment. Examples of non-polar molecules are hydrogen (H_2), oxygen (O_2), and carbon dioxide (CO_2) etc.

When an external electric field is applied, the centers of positive and negative charges are separated by a small distance which induces dipole moment in the direction of the external electric field. Then the dielectric is said to be polarized by an external electric field. This is shown in Figure 1.50.

In polar molecules, the centers of the positive and negative charges are separated even in the absence of an external electric field. They have a permanent dipole moment. Due to thermal motion, the direction of each dipole moment is oriented randomly (Figure 1.51(a)). Hence the net dipole moment is zero in the absence of an external electric field. Examples of polar molecules are H_2O, N_2O, HCl, NH_3.

When an external electric field is applied, the dipoles inside the polar molecule tend to align in the direction of the electric field. Hence a net dipole moment is induced in it. Then the dielectric is said to be polarized by an external electric field (Figure 1.51(b)).

Polarisation

In the presence of an external electric field, the dipole moment is induced in the dielectric material. **Polarisation is defined as the total dipole moment per unit volume of the dielectric.** For most dielectrics (linear isotropic), the Polarisation is directly proportional to the strength of the external electric field. This is written as

$$\vec{P} = \chi_e \vec{E}_{ext} \quad (1.80)$$

where χ_e is a constant called the electric susceptibility which is a characteristic of each dielectric.

Induced Electric field inside the dielectric

When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which cancels the external electric field. But in the case of a dielectric, which has no free electrons, the external electric field only realigns the charges so that an internal electric field is produced. The magnitude of the internal electric field is smaller than that of external electric field.

Therefore the net electric field inside the dielectric is not zero but is parallel to an external electric field with magnitude less than that of the external electric field. For example, let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor) as shown in the Figure 1.52(b).

The uniform electric field between the plates acts as an external electric field which polarizes the dielectric placed between plates. The positive charges are induced on one side surface and negative charges are induced on the other side of surface.

But inside the dielectric, the net charge is zero even in a small volume. So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in conductors. This is shown in the Figure 1.52(b).

For example, the charged balloon after rubbing sticks onto a wall. The reason is that the negatively charged balloon is brought near the wall, it polarizes opposite charges on the surface of the wall, which attracts the balloon. This is shown in Figure 1.53.

Dielectric strength

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown. The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength. For example, the dielectric strength of air is 3 × 106 V m-1. If the applied electric field increases beyond this, a spark is produced in the air. The dielectric strengths of some dielectrics are given in the Table 1.1.

Substance	**Dielectric strength (Vm-1)**
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Air	3×10^6
Pyrex glass	14×10^6

CAPACITORS AND CAPACITANCE

Capacitors

Capacitor is a device used to store electric charge and electrical energy. It consists of two conducting objects (usually plates or sheets) separated by some distance. Capacitors are widely used in many electronic circuits and have applications in many areas of science and technology.
A simple capacitor consists of two parallel metal plates separated by a small distance as shown in Figure 1.54 (a).

When a capacitor is connected to a battery of potential difference V, the electrons are transferred from one plate to the other plate by battery so that one plate becomes negatively charged with a charge of –Q and the other plate positively charged with +Q. The potential difference between the plates is equivalent to the battery's terminal voltage. This is shown in Figure 1.54(b). If the battery voltage is increased, the amount of charges stored in the plates also increase. In general, the charge stored in the capacitor is proportional to the potential difference between the plates.

$$Q \propto V$$

so that Q = CV

where the C is the proportionality constant called capacitance. **The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors.**

$$C = \frac{Q}{V} \quad (1.81)$$

The SI unit of capacitance is *coulomb per volt* or *farad (F)* in honor of Michael Faraday. Farad is a very large unit of capacitance. In practice, capacitors are available in the range of microfarad (1μF = 10^{-6} F) to picofarad (1pf = 10^{-12} F). A capacitor is represented by the symbol -||- or -|(- . Note that the total charge stored in the capacitor is zero (Q - Q = 0). When we say the capacitor stores charges, it means the amount of charge that can be stored in any one of the plates.

Nowadays there are capacitors available in various shapes (cylindrical, disk) and types (tantalum, ceramic and electrolytic), as shown in Figure 1.55. These capacitors are extensively used in various kinds of electronic circuits.

Capacitance of a parallel plate capacitor

Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d as shown in Figure 1.56

The electric field between two infinite parallel plates is uniform and is given by.

$E = \frac{\sigma}{\varepsilon_0}$ where σ is the surface charge density on the plates $\left(\sigma = \frac{Q}{A}\right)$. If the separation distance d is very much smaller than the size of the plate ($d^2 << A$), then the above result is used even for finite-sized parallel plate capacitor.

The electric field between the plates is

$$E = \frac{Q}{A\varepsilon_0} \quad (1.82)$$

Since the electric field is uniform, the electric potential between the plates having separation d is given by

$$V = Ed = \frac{Qd}{A\varepsilon_0} \quad (1.83)$$

Therefore the capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{\varepsilon_0 A}{d} \quad (1.84)$$

From equation (1.84), it is evident that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates. This can be understood from the following.

(i) If the area of cross-section of the capacitor plates is increased, more charges can be distributed for the same potential difference. As a result, the capacitance is increased.

(ii) If the distance d between the two plates is reduced, the potential difference between the plates (*V* = *Ed*) decreases with *E* constant. As a result, voltage difference between the terminals of the battery increases which in turn leads to an additional flow of charge to the plates from the battery, till the voltage on the capacitor equals to the battery's terminal voltage. Suppose the distance is increased, the capacitor voltage increases and becomes greater than the battery voltage. Then, the charges flow from capacitor plates to battery till both voltages becomes equal.

E X A M P LE

A parallel plate capacitor has square plates of side 5 cm and separated by a distance of 1 mm. (a) Calculate the capacitance of this capacitor. (b) If a 10 V battery is connected to the capacitor, what is the charge stored in any one of the plates? (The value of

$\varepsilon_o = 8.85 \times 10^{-12}\ Nm^2 C^{-2}$)

Solution

(a) The capacitance of the capacitor is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{1\times10^{-3}}$$

$$= 221.2\times10^{-13} F$$

$$C = 22.12\times10^{-12} F = 22.12 pF$$

(b) The charge stored in any one of the plates is Q = CV, Then

$$Q = 22.12\times10^{-12}\times10 = 221.2\times10^{-12} C = 221.2 pC$$

Sometimes we notice that the ceiling fan does not start rotating as soon as it is switched on. But when we rotate the blades, it starts to rotate as usual. Why it is so? We know that to rotate any object, there must be a torque applied on the object. For the ceiling fan, the initial torque is given by the capacitor widely known as a condenser. If the condenser is faulty, it will not give sufficient initial torque to rotate the blades when the fan is switched on.

Energy stored in the capacitor

Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge -Q are transferred from one plate to the other plate. To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor.

To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

$$dW = V\,dQ \qquad (1.85)$$

$$\text{where } V = \frac{Q}{C}$$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C}\,dQ = \frac{Q^2}{2C} \qquad (1.86)$$

This work done is stored as electrostatic potential energy (UE) in the capacitor.

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \quad (\because Q = CV) \qquad (1.87)$$

where Q = CV is used. This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor. But where is this energy stored in the capacitor? To understand this question, the equation (1.87) is rewritten as follows using the

results $C = \frac{\varepsilon_0 A}{d}$ and V = Ed

$$U_E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 (Ad)E^2 \qquad (1.88)$$

where Ad = volume of the space between the capacitor plates. **The energy stored per unit volume of space is defined as energy**

density $u_E = \frac{U}{Volume}$ From equation (1.88), we get

$$u_E = \frac{1}{2}\varepsilon_o E^2 \quad (1.89)$$

From equation (1.89), we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.
It is important to note that the energy density depends only on the electric field and not on the size of the plates of the capacitor. In fact, expression (1.89) is true for the electric field due to any type of charge configuration.

Applications of capacitors

Capacitors are used in various electronics circuits. A few of the applications.

(a) Most people are now familiar with the digital camera. The flash which comes from the camera when we take photographs is due to the energy released from the capacitor, called a flash capacitor

(b) During cardiac arrest, a device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function. This defibrillator uses a capacitor of 175 μF charged to a high voltage of around 2000 V.

(c) Capacitors are used in the ignition system of automobile engines to eliminate sparking

(d) Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

However, capacitors have disadvantage as well. Even after the battery or power supply is removed, the capacitor stores charges and energy for some time. For example if the TV is switched off, it is always advisable to not touch the back side of the TV panel.

Effect of dielectrics in capacitors

In earlier discussions, we assumed that the space between the parallel plates of a capacitor is either empty or filled with air. Suppose dielectrics like mica, glass or paper are introduced between the plates, then the capacitance of the capacitor is altered. The dielectric can be inserted into the plates in two different ways. (i) when the capacitor is disconnected from the battery. (ii) when the capacitor is connected to the battery.

(i) when the capacitor is disconnected from the battery

Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V0 and the charge stored is Q0. The capacitance of the capacitor without the dielectric is

$$C_0 = \frac{Q_0}{V_0} \qquad (1.90)$$

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates.

The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified electric field is given by

$$E = \frac{E_0}{\varepsilon_r}$$

Here E_o is the electric field inside the capacitors when there is no dielectric and εr is the relative permeability of the dielectric or simply known as the dielectric constant. Since εr > 1, the electric field $E < E_o$.

As a result, the electrostatic potential difference between the plates (V = Ed) is also reduced. But at the same time, the charge Q_o will remain constant once the battery is disconnected.

Hence the new potential difference is

$$V = Ed = \frac{E_0}{\varepsilon_r}d = \frac{V_0}{\varepsilon_r}$$

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases.

Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q_0}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0$$

Since εr > 1, we have C > Co. Thus insertion of the dielectric constant εr increases the capacitance.

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

where ε = εr εo is the permittivity of the dielectric medium.
The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$$

After the dielectric is inserted, the charge Q_0 remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\frac{Q_0^2}{\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r}$$

Since εr > 1 we get U < Uo. There is a decrease in energy because,when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor

Let us now consider what happens when the battery of voltage V0 remains connected to the capacitor when the dielectric is inserted into the capacitor. This is shown in Figure1.59.

The potential difference V_0 across the plates remains constant. But it is found experimentally (first shown by Faraday) that when dielectric is inserted, the charge stored in the capacitor is increased by a factor ε_r.

$$\mathbf{Q = \varepsilon_r Q_o}$$

Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q}{V_0} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0$$

However the reason for the increase in capacitance in this case when the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

$$\text{Now,} \quad C_0 = \frac{\varepsilon_0 A}{d}$$

$$\text{and} \quad C = \frac{\varepsilon A}{d}$$

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2} C_0 V_0^2$$

Note that here we have not used the expression $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$ because here, both

charge and capacitance are changed, whereas in equation (1.100), Vo remains constant.

After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$$U = \frac{1}{2}CV_0^2 = \frac{1}{2}\varepsilon_r C_0 V_0^2 = \varepsilon_r U_0 \qquad (1.101)$$

Since $\varepsilon_r > 1$ we have $U > U_0$.

It may be noted here that since voltage between the capacitor V_0 is constant, the electric field between the plates also remains constant.

The energy density is given by

$$u = \frac{1}{2}\varepsilon E_0^2 \qquad (1.102)$$

When the key is pressed, the separation between the plates decreases leading to an increase in the capacitance. This in turn triggers the electronic circuits in the computer to identify which key is pressed.

S. No	Dielectric is inserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	Decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

E X A M P L E 1.21

A parallel plate capacitor filled with mica having εr = 5 is connected to a 10 V battery. The area of the parallel plate is 6 m2 and separation distance is 6 mm.

(a) Find the capacitance and stored charge.

(b) After the capacitor is fully charged, the battery is disconnected and the dielectric is removed carefully.

Calculate the new values of capacitance, stored energy and charge.

Solution

(a) The capacitance of the capacitor the presence of dielectric is

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{5 \times 8.85 \times 10^{-12} \times 6}{6 \times 10^{-3}}$$

$$= 44.25 \times 10^{-9} F = 44.25 nF$$

The stored charge is

$$Q = CV = 44.25 \times 10^{-9} \times 10$$

$$= 442.5 \times 10^{-9} C = 442.5 nC$$

The stored energy is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 44.25C \times 10^{-9} \times 100$$

$$= 2.21 \times 10^{-6} J = 2.21 \mu J$$

(b) After the removal of the dielectric, since the battery is already disconnected the total charge will not change. But the potential difference between the plates increases. As a result, the capacitance is decreased.

New capacitance is

When the dielectric is removed, it experiences an inward pulling force due to the plates. To remove the dielectric, an external agency has to do work on the dielectric which is stored as additional energy. This is the source for the extra energy 8.84 μJ.

$$C_0 = \frac{C}{\varepsilon_r} = \frac{44.25 \times 10^{-9}}{5}$$

$$= 8.85 \times 10^{-9} F = 8.85 nF$$

The stored charge remains same and 442.5 nC. Hence newly stored energy is

$$U_0 = \frac{Q^2}{2C_0} = \frac{Q^2 \varepsilon_r}{2C} = \varepsilon_r U$$

$$= 5 \times 2.21 \mu J = 11.05 \mu J$$

The increased energy is

$$\Delta U = 11.05 \mu J - 2.21 \mu J = 8.84 \mu J$$

Capacitor in series and parallel
(i) Capacitor in series

Consider three capacitors of capacitance C1, C2 and C3 connected in series with a battery of voltage V as shown in the Figure 1.60 (a).
As soon as the battery is connected to the capacitors in series, the electrons of charge

–Q are transferred from negative terminal to the right plate of C3 which pushes the electrons of same amount -Q from left plate of C3 to the right plate of C2 due to electrostatic induction. Similarly, the left plate of C2 pushes the charges of –Q to the right plate of C1 which induces the positive charge +Q on the left plate of C1. At the same time, electrons of charge –Q are transferred from left plate of C1 to positive terminal of the battery.

By these processes, each capacitor stores the same amount of charge Q. The capacitances of the capacitors are in general different, so that the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively.

The total voltage across each capacitor must be equal to the voltage of the battery.

$$V = V_1 + V_2 + V_3 \qquad (1.103)$$

Since, $Q = CV$, we have $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \qquad (1.104)$$

If three capacitors in series are considered to form an equivalent single capacitor Cs shown in Figure 1.60(b), then we have

$V = \frac{Q}{C_S}$. Substituting this expression into equation (1.104), we get

$$\frac{Q}{C_S} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad (1.105)$$

Thus, the inverse of the equivalent capacitance CS of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance CS is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V as shown in Figure 1.61 (a).

Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage. Since capacitance of the capacitors is different, the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q1, Q2, and Q3 respectively. According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$$Q = Q_1 + Q_2 + Q_3 \quad (1.106)$$

Now, since Q=CV, we have

$$Q = C_1V + C_2V + C_3V \quad (1.107)$$

If these three capacitors are considered to form a single capacitance CP which stores the total charge Q as shown in the Figure 1.61(b), then we can write Q = CPV. Substituting this in equation (1.107), we get

$$C_PV = C_1V + C_2V + C_3V$$

$$C_P = C_1 + C_2 + C_3 \quad (1.108)$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances.

The equivalent capacitance CP in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

E X A M P L E 1.22

Find the equivalent capacitance between P and Q for the configuration shown below in the figure (a).

Solution

The capacitors 1 μF and 3μF are connected in parallel and 6μF and 2 μF are also separately connected in parallel. So these parallel combinations reduced to equivalent single capacitances in their respective positions, as shown in the figure (b).

$$C_{eq} = 1\mu F + 3\mu F = 4\mu F$$

$$C_{eq} = 6\mu F + 2\mu F = 8\mu F$$

From the figure (b), we infer that the two 4 μF capacitors are connected in series and the two 8 μF capacitors are connected in series. By using formula for the series, we can reduce to their equivalent capacitances as shown in figure (c).

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \qquad \Rightarrow C_{eq} = 2\,\mu F$$

and

$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \qquad \Rightarrow C_{eq} = 4\,\mu F$$

From the figure (c), we infer that 2μF and 4μF are connected in parallel. So the equivalent capacitance is given in the figure (d).

$$C_{eq} = 2\mu F + 4\mu F = 6\mu F$$

Thus the combination of capacitances in figure (a) can be replaced by a single capacitance 6 μF.

DISTRIBUTION OF CHARGES IN A CONDUCTOR AND ACTION AT POINTS

Distribution of charges in a conductor

Consider two conducting spheres A and B of radii r1 and r2 respectively connected to each other by a thin conducting wire as shown in the

Figure 1.62. The distance between the spheres is much greater than the radii of either spheres.

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium. Let q1 be the charge residing on the surface of sphere A and q2 is the charge residing on the surface of sphere B such that Q = q1 + q2. The charges are distributed only on the surface and there is no net charge inside the conductor.

The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\epsilon_o}\frac{q_1}{r_1}$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\epsilon_o}\frac{q_2}{r_2}$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

$$V_A = V_B$$

$$\text{or} \quad \frac{q_1}{r_1} = \frac{q_2}{r_2}$$

Let us take the charge density on the surface of sphere A is σ_1 and charge density on the surface of sphere B is σ_2. This implies that $q_1 = 4\pi r_1^2\sigma_1$ and

$q_2 = 4\pi r_2^2\sigma_2$. Substituting these values into equation (1.112), we get

$$\sigma_1 r_1 = \sigma_2 r_2 \quad (1.113)$$

from which we conclude that

$$\sigma r = \text{constant} \quad (1.114)$$

Thus the surface charge density σ is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

EXAMPLE 1.23

Two conducting spheres of radius $r_1 = 8$ cm and $r_2 = 2$ cm are separated by a distance much larger than 8 cm and are connected by a thin conducting wire as shown in the figure. A total charge of Q = +100 nC is placed on one of the spheres. After a fraction of a second, the charge Q is redistributed and both the spheres attain electrostatic equilibrium.

(a) Calculate the charge and surface charge density on each sphere.

(b) Calculate the potential at the surface of each sphere.

Solution

(a) The electrostatic potential on the surface of the sphere A is $V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$

The electrostatic potential on the surface of the sphere A is $V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$

Since $V_A = V_B$. We have

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow q_1 = \left(\frac{r_1}{r_2}\right)q_2$$

But from the conservation of total charge, $Q = q_1 + q_2$, we get $q_1 = Q - q_2$. By substituting this in the above equation,

$$Q - q_2 = \left(\frac{r_1}{r_2}\right)q_2$$

so that $$q_2 = Q\left(\frac{r_2}{r_1 + r_2}\right)$$

Therefore,

$$q_2 = 100\times10^{-9}\times\left(\frac{2}{10}\right) = 20nC$$

and $q_1 = Q - q_2 = 80nC$

The electric charge density for sphere A is

$$\sigma_1 = \frac{q_1}{4\pi r_1^2}$$

The electric charge density for sphere B is

$$\sigma_2 = \frac{q_2}{4\pi r_2^2}$$

Therefore,

$$\sigma_1 = \frac{80\times10^{-9}}{4\times64\times10^{-4}} = 0.99\times10^{-6}\,Cm^{-2}$$

and

$$\sigma_2 = \frac{20\times10^{-9}}{4\pi\times4\times10^{-4}} = 3.9\times10^{-6}\,Cm^{-2}$$

Note that the surface charge density is greater on the smaller sphere compared to the larger sphere ($\sigma_2 \approx 4\sigma_1$) which confirms the result $\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$.

The potential on both spheres is the same. So we can calculate the potential on any one of the spheres.

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} = \frac{9\times10^9\times80\times10^{-9}}{8\times10^{-2}} = 9kV$$

Action at points or Corona discharge

- Consider a charged conductor of irregular shape as shown in Figure 1.63

We know that smaller the radius of curvature, the larger is the charge density. The end of the conductor which has larger curvature (smaller radius) has a large charge accumulation as shown in Figure 1.63 (b).
As a result, the electric field near this edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action at points or corona discharge.

Lightning arrester or lightning conductor

- This is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge.

- This device consists of a long thick copper rod passing from top of the building to the ground. The upper end of the rod has a sharp spike or a sharp needle as shown in

- The lower end of the rod is connected to the copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building, it induces

- a positive charge on the spike. Since the induced charge density on thin sharp spike is large, it results in a corona discharge. This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud. The negative charge pushed to the spikes passes through the copper rod and is safely diverted to the Earth. The lightning arrester does not stop the lightning; rather it diverts the lightning to the ground safely.

Van de Graaff Generator

- In the year 1929, Robert Van de Graaff designed a machine which produces a large amount of electrostatic potential difference, up to several million volts (107 V). This Van de Graff generator works on the principle of electrostatic induction and action at points.

- A large hollow spherical conductor is fixed on the insulating stand as shown in Figure 1.65. A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb shaped metallic conductors E and D are fixed near the pulleys.

- The comb D is maintained at a positive potential of 104 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

- Due to the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.

- outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of 107 which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

- The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

E X A M P L E

Dielectric strength of air is 3 × 106 V m-1. Suppose the radius of a hollow sphere in the Van de Graff generator is R = 0.5 m, calculate the maximum potential difference created by this Van de Graaff generator. When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the

The electric field on the surface of the sphere(by Gauss law) is given by

$$E = \frac{1}{4\pi\epsilon_o}\frac{Q}{R^2}$$

The potential on the surface of the hollow metallic sphere is given by

$$V = \frac{1}{4\pi\epsilon_o}\frac{Q}{R} = ER$$

with $V_{max} = E_{max}R$

Here $E_{max} = 3\times10^6\frac{V}{m}$. So the maximum potential difference created is given by

$V_{max} = 3 \times 10^6 \times 0.5$

$= 1.5 \times 10^6$ V (or) 1.5 million volt

12TH STANDARD
UNIT 2
CURRENT ELECTRICITY

INTRODUCTION

- In unit 1, we studied the properties of charges when it is at rest. In reality, the charges are always moving within the materials. For example, the electrons in a copper wire are never at rest and are continuously in random motion. Therefore it is important to analyse the behaviour of charges when it is at motion. The motion of charges is called 'electric current'. Current electricity is the study of flow of electric charges. It owes its origin to Alessandro Volta (1745-1827), who invented the electric battery which produced the first steady flow of electric current. Modern world depends heavily on the use of electricity. It is used to operate machines, communication systems, electronic devices, home appliances etc., In this unit, we will study about the electric current, resistance and related phenomenon in materials.

ELECTRIC CURRENT

- Matter is made up of atoms. Each atom consists of a positively charged nucleus with negatively charged electrons moving around the nucleus. Atoms in metals have one or more electrons which are loosely bound to the nucleus. These electrons are called free electrons and can be easily detached from the atoms. The substances which have an abundance of these free electrons are called conductors. These free electrons move at random throughout the conductor at a given temperature. In general due to this random motion, there is no net transfer of charges from one end of the conductor to other end and hence no current. When a potential difference is applied by the battery

across the ends of the conductor, the free electrons drift towards the positive terminal of the battery, producing a net electric current. This is easily understandable from the analogy given in the Figure 2.1.

- In the XI Volume 2, unit 6, we studied, that the mass move from higher gravitational potential to lower gravitational potential. Likewise, positive charge flows from higher electric potential to lower electric potential and negative charge flows from lower electric potential to higher electric potential. So battery or electric cell simply creates potential difference across the conductor.

- The electric current in a conductor is defined as the rate of flow of charges through a given cross-sectional area A. It is shown in the Figure 2.2.

- If a net charge Q passes through any cross section of a conductor in time t, then the current is defined as I = $\frac{Q}{t}$ But charge flow is not always constant. Hence current can more generally be defined as

$$I_{avg} = \frac{\Delta Q}{\Delta t} \qquad (2.1)$$

- Where ΔQ is the amount of charge that passes through the conductor at any cross section during the time interval Δt. If the rate at which charge flows changes in time, the current also changes. The instantaneous current I is defined as the limit of the average current, as $\Delta t \rightarrow 0$

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad (2.2)$$

The SI unit of current is the ampere (A)

1A= $\frac{1C}{1s}$

- That is, 1A of current is equivalent to 1 Coulomb of charge passing through a perpendicular cross section in 1second. The electric current is a scalar quantity.

E X A M P L E 2.1

- Compute the current in the wire if a charge of 120 C is flowing through a copper wire in 1 minute.

Solution

- The current (rate of flow of charge) in the wire is

$$I = \frac{Q}{t} = \frac{120}{60} = 2A$$

Conventional Current

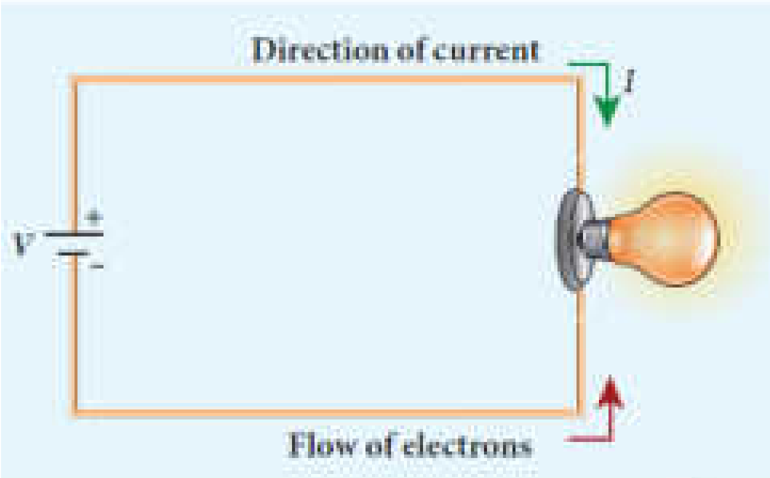


Figure 2.3 Direction of conventional current and electron flow

- In an electric circuit, arrow heads are used to indicate the direction of flow of current. By convention, this flow in the circuit should be from the positive terminal of the battery to the negative terminal. This current is called the conventional current or simply current and is in the direction in which a positive test charge would move. In typical circuits the charges that flow are actually electrons, from the negative terminal of the battery to the positive. As a result, the flow of electrons and the direction of conventional current points in opposite direction as shown in Figure 2.3. Mathematically, a transfer of positive charge is the same as a transfer of negative charge in the opposite direction.

Electric current is not only produced by batteries. In nature, lightning bolt produces enormous electric current in a short time. During lightning, very high potential difference is created between the clouds and ground so charges flow between the clouds and ground.

Drift velocity

- In a conductor the charge carriers are free electrons. These electrons move freely through the conductor and collide repeatedly with the positive ions. If there is no electric field, the electrons move in random directions, so the directions of their velocities are also completely random direction. On an average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction. As a result, there is no net flow of electrons in any direction and hence there will not be any current. Suppose a potential difference is set across the conductor by connecting a battery, an electric field E is created in the conductor. This electric field exerts a force on the electrons, producing a current. The electric field accelerates the electrons, while ions scatter the electrons and change the direction of motion. Thus, we have zigzag paths of electrons. In addition to the zigzag motion due to the collisions, the electrons move slowly along the conductor in a direction opposite to that of E as shown in the Figure 2.4.

Ions

- Any material is made up of neutral atoms with equal number of electrons and protons. If the outermost electrons leave the atoms, they become free electrons and are responsible for electric current. The atoms after losing their outer most electrons will have more positive charges and hence are called positive ions. These ions will not move freely within the material like the free electrons. Hence the positive ions will not give rise to current.

This velocity is called drift velocity $\dot{v}_d$. The drift velocity is the average velocity acquired by the electrons inside the conductor when it is subjected to an electric field. The average time between successive collisions is called the mean free time denoted by τ. The acceleration a experienced by the electron in an electric field E is given by

$$\vec{a} = \frac{-e\vec{E}}{m} \qquad \left(\text{since } \vec{F} = -e\vec{E}\right) \qquad (2.3)$$

The drift velocity $\vec{v}_d$ is given by

$$\vec{v}_d = \vec{a}\ \tau$$

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E} \qquad (2.4)$$

$$\vec{v}_d = -\mu\vec{E} \qquad (2.5)$$

Here $\mu = \frac{e\tau}{m}$ is the mobility of the electron and it is defined as the magnitude of the drift velocity per unit electric field.

$$\mu = \frac{|\vec{v}_d|}{|\vec{E}|} \qquad (2.6)$$

The SI unit of mobility is $\frac{m^2}{V\ s}$.

The typical drift velocity of electrons in the wire is 10^{-4} m s^{-1}. If an electron drifts with this speed, then the electrons leaving the battery will take hours to reach the light bulb. Then how electric bulbs glow as soon as we switch on the battery? When battery is switched on, the electrons begin to move away from the negative terminal of the battery and this electron exerts force on the nearby electrons. This process creates a propagating influence (electric field) that travels through the wire at the speed of light. In other words, the energy is transported from the battery to light bulb at the speed of light through propagating influence (electric field). Due to this reason, the light bulb glows as soon as the battery is switched on.

If an electric field of magnitude 570 NC^{-1}, is applied in the copper wire, find the acceleration experienced by the electron.

Solution:

$E = 570\ N\ C^{-1}$, $e = 1.6 \times 10^{-19}\ C$,
$m = 9.11 \times 10^{-31}$ kg and $a = ?$

$F = ma = eE$

$$a = \frac{eE}{m} = \frac{570 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}$$

$$= \frac{912 \times 10^{-19} \times 10^{31}}{9.11}$$

$$= 1.001 \times 10^{14}\ m\ s^{-2}$$

Misconception

(i) There is a common misconception that the battery is the source of electrons. It is not true. When a battery is connected across the given wire, the electrons in the closed circuit resulting the current. Battery sets the potential difference (electrical energy) due to which these electrons in the conducting wire flow in a particular direction. The resulting electrical energy is used by electric bulb, electric fan etc. Similarly the electricity board is supplying the electrical energy to our home.

(ii) We often use the phrases like 'charging the battery in my mobile' and 'my mobile phone battery has no charge' etc. These sentences are not correct.

When we say 'battery has no charge', it means, that the battery has lost ability to provide energy or provide potential difference to the electrons in the circuit. When we say 'mobile is charging', it implies that the battery is receiving energy from AC power supply and not electrons.

Microscopic model of current

Consider a conductor with area of cross section A and an electric field E applied from right to left. Suppose there are n electrons per unit volume in the conductor and assume that all the electrons move with the same drift velocity $\dot{v}_d$ as shown in Figure 2.5.

The drift velocity of the electrons = v_d

The electrons move through a distance dx within a small interval of dt

$$v_d = \frac{dx}{dt}; \quad dx = v_d\, dt \qquad (2.7)$$

Since A is the area of cross section of the conductor, the electrons available in the volume of length dx is

= volume × number per unit volume

$$= A dx \times n \qquad (2.8)$$

Substituting for dx from equation (2.7) in (2.8)

= (A v_d dt) n

Total charge in volume element dQ = (charge) × (number of electrons in the volume element)

$$dQ = (e)(Av_d dt)n$$

Hence the current $I = \frac{dQ}{dt} = \frac{neAv_d dt}{dt}$

$$I = neAv_d. \qquad (2.9)$$

Current density (J)

The current density (J) is defined as the current per unit area of cross section of the conductor.

$$J = \frac{I}{A}$$

The S.I unit of current density is $\frac{A}{m^2}$ (or) A m-2

$J = \frac{neAv_d}{A}$ (from equation 2.9)

$$J = nev_d \qquad (2.10)$$

The above expression is valid only when the direction of the current is perpendicular to the area A. In general, the current density is a vector quantity and it is given by

$$\vec{J} = ne\vec{v}_d$$

Substituting $\vec{v}_d$ from equation (2.4)

$$\vec{J} = -\frac{n \cdot e^2 \tau}{m}\vec{E} \qquad (2.11)$$

But conventionally, we take the direction of (conventional) current density as the direction of electric field. So the above equation becomes

$$\vec{J} = \sigma \vec{E}$$

where $\sigma = \frac{ne^2\tau}{m}$ is called conductivity. The equation 2.12 is called microscopic form of ohm's law.

The inverse of conductivity is called resistivity (ρ) [Refer section 2.2.1].

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad (2.13)$$

E X A M P L E

A copper wire of cross-sectional area 0.5 mm^2 carries a current of 0.2 A. If the free electron density of copper is 8.4×10^{28} m^{-3} then compute the drift velocity of free electrons. ***Solution*** The relation between drift velocity of electrons and current in a wire of cross- sectional area A is

$$v_d = \frac{I}{neA} = \frac{0.2}{8.4\times10^{28}\times1.6\times10^{-19}\times0.5\times10^{-6}}$$

$$v_d = 0.03 \times 10^{-3} \text{ m s}^{-1}$$

E X A M P L E

Determine the number of electrons flowing per second through a conductor, when a current of 32 A flows through it. ***Solution*** I = 32 A , t = 1 s

Charge of an electron, $e = 1.6 \times 10^{-19}$ C

The number of electrons flowing per second, n =?

$$I = \frac{q}{t} = \frac{ne}{t}$$

$$n = \frac{It}{e}$$

$$n = \frac{32\times1}{1.6\times10^{-19}C}$$

$$n = 20 \times 10^{19} = 2 \times 10^{20} \text{ electrons}$$

OHM'S LAW

The ohm's law can be derived from the equation J = σE. Consider a segment of wire of length *l* and cross sectional area A as shown in Figure 2.7.

When a potential difference V is applied across the wire, a net electric field is created in the wire which constitutes the current. For simplicity, we assume that the electric field is uniform in the entire length of the wire, the potential difference (voltage V) can be written as

$$V = El$$

As we know, the magnitude of current density

$$J = \sigma E = \sigma \frac{V}{l}$$

But $J = \frac{I}{A}$, so we write the equation (2.14) as

$$\frac{I}{A} = \sigma \frac{V}{l}.$$

By rearranging the above equation, we get

$$V = I\left(\frac{l}{\sigma A}\right) \quad (2.15)$$

The quantity $\frac{l}{\sigma A}$ is called resistance of the conductor and it is denoted as R. Note that the resistance is directly proportional to the length of the conductor and inversely proportional to area of cross section.

Therefore, the macroscopic form of ohm's law can be stated as

$$V = IR$$

From the above equation, **the resistance is the ratio of potential difference across the given conductor to the current passing through the conductor.**

$$R = \frac{V}{I}$$

The SI unit of resistance is ohm (Ω). From the equation (2.16), we infer that the graph between current versus voltage is straight line with a slope equal to the inverse of resistance R of the conductor. It is shown in the Figure 2.8 (a).

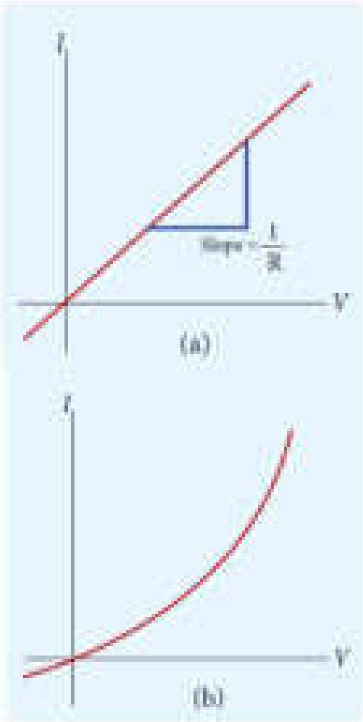


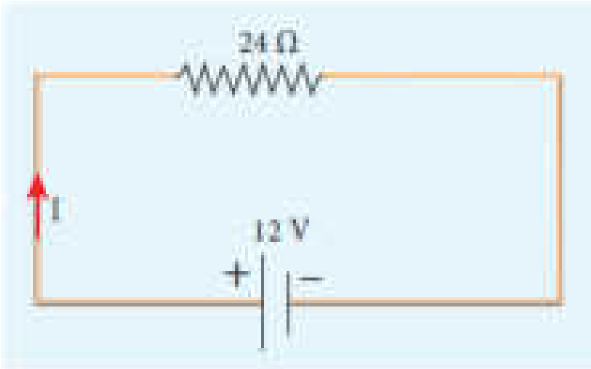
Figure 2.8 Current against voltage for (a) a conductor which obey Ohm's law and (b) for a non-ohmic device (Diode given in XII physics, unit 9 is an example of a non-ohmic device)

Materials for which the current against voltage graph is a straight line through the origin, are said to obey Ohm's law and their behaviour is said to be ohmic as shown in Figure 2.8(a). Materials or devices that do not follow Ohm's law are said to be nonohmic. These materials have more complex relationships between voltage and current. A plot of I against V for a non-ohmic material is non-linear and they do not have a constant resistance (Figure 2.8(b)).

E X A M P L E 2.5

A potential difference across 24 Ω resistor is 12 V. What is the current through the resistor?

Solution



V = 12 V and R = 24 Ω

Current, I = ?

From Ohm's law, $I = \frac{V}{R} = \frac{12}{24} = 0.5\,A$

Resistivity

In the previous section, we have seen that the resistance R of any conductor is given by

$$R = \frac{l}{\sigma A}$$

where σ is called the conductivity of the material and it depends only on the type of the material used and not on its dimension.

The resistivity of a material is equal to the reciprocal of its conductivity

$$\rho = \frac{1}{\sigma}$$

Now we can rewrite equation (2.18) using equation (2.19)

$$R = \rho \frac{l}{A}$$

The resistance of a material is directly proportional to the length of the conductor and inversely proportional to the area of cross section of the conductor. The proportionality constant ρ is called the resistivity of the material. If l = 1 m and A = 1 m^2 then the resistance R = ρ. In other words, the electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section. The SI unit of ρ is ohm-metre (Ω m). Based on the resistivity, materials are classified as conductors, insulators and semiconductors. The conductors have lowest resistivity, insulators have highest resistivity and semiconductors have resistivity greater than conductors but less than insulators. The typical resistivity values of some conductors, insulators and semiconductors are given in the Table 2.1

Table 2.1 Resistivity for various materials

Material	Resistivity, ρ (Ω m) at 200C
Insulators	
Pure Water	2.5×10^{5}
Glass	$10^{10} - 10^{14}$
Hard Rubber	$10^{14} - 10^{16}$
NaCl	10^{14}
Fused Quartz	10^{16}
Semiconductors	
Germanium	**0.46**
Silicon	**640**
Conductors	
Silver	1.6×10^{-8}
Copper	17×10^{-8}
Aluminium	2.7×10^{-8}
Tungsten	5.6×10^{-8}
Iron	10×10^{-8}

E X A M P L E 2.6

The resistance of a wire is 20 Ω. What will be new resistance, if it is stretched uniformly 8 times its original length?

EXAMPLE 2.6

The resistance of a wire is 20 Ω. What will be new resistance, if it is stretched uniformly 8 times its original length?

Solution

$R_1 = 20\ \Omega$, $R_2 = ?$

Let the original length (l_1) be l.

The new length, $l_2 = 8l_1$ (i.e) $l_2 = 8l$

The original resistance, $R_1 = \rho\frac{l_1}{A_1}$

The new resistance $R_2 = \rho\frac{l_2}{A_2} = \frac{\rho(8l)}{A_2}$

Though the wire is stretched, its volume is unchanged.

Initial volume = Final volume

$A_1l_1 = A_2l_2$, $\quad A_1l = A_2 8l$

$\frac{A_1}{A_2} = \frac{8l}{l} = 8$

By dividing equation R_2 by equation R_1, we get

$\frac{R_2}{R_1} = \frac{\rho(8l)}{A_2} \times \frac{A_1}{\rho l}$

$\frac{R_2}{R_1} = \frac{A_1}{A_2} \times 8$

Substituting the value of $\frac{A_1}{A_2}$, we get

$\frac{R_2}{R_1} = 8 \times 8 = 64$

$R_2 = 64 \times 20 = 1280\ \Omega$

Hence, stretching the length of the wire has increased its resistance.

EXAMPLE 2.7

Consider a rectangular block of metal of height A, width B and length C as shown in the figure.

If a potential difference of V is applied between the two faces A and B of the block (figure (a)), the current I_{AB} is observed. Find the current that flows if the same potential difference V is applied between the two faces B and C of the block (figure (b)). Give your answers in terms of I_{AB}.

Solution

In the first case, the resistance of the block

$$R_{AB} = \rho\frac{\text{length}}{\text{Area}} = \rho\frac{C}{AB}$$

The current $I_{AB} = \frac{V}{R_{AB}} = \frac{V}{\rho}\frac{AB}{C}$ (1)

In the second case, the resistance of the block $R_{BC} = \rho\frac{A}{BC}$

The current $I_{BC} = \frac{V}{R_{BC}} = \frac{V}{\rho}\frac{BC}{A}$ (2)

To express I_{BC} interms of I_{AB}, we multiply and divide equation (2) by AC, we get

$$I_{BC} = \frac{V}{\rho}\frac{BC\,AC}{A\,AC} = \left(\frac{V}{\rho}\frac{AB}{C}\right)\frac{C^2}{A^2} = \frac{C^2}{A^2}I_{AB}$$

Since C > A, the current $I_{BC} > I_{AB}$

The human body contains a large amount of water which has low resistance of around 200 Ω and the dry skin has high resistance of around 500 k Ω. But when the skin is wet, the resistance is reduced to around 1000 Ω. This is the reason, repairing the electrical connection with the wet skin is always dangerous.

Resistors in series and parallel An electric circuit may contain a number of resistors which can be connected in different ways. For each type of circuit, we can calculate the equivalent resistance produced by a group of individual resistors. Resistors in series When two or more resistors are connected end to end, they are said to be in series. The resistors could be simple resistors or bulbs or heating elements or other devices. Figure 2.9 (a) shows three resistors R1, R2 and R3 connected in series. The amount of charge passing through resistor R1 must also pass through resistors R2and R3 since the charges cannot accumulate anywhere in the circuit. Due to this reason, the current I passing through all the three resistors is the same. According to Ohm's law, if same current pass through different resistors of different values, then the potential difference across each resistor must be different. Let V1, V2 and V3 be the potential difference (voltage) across each of the resistors R1, R2 and R3 respectively, then we can write V1 = IR1, V2 = IR2 and V3 = IR3. But the total voltage V is equal to the sum of voltages across each resistor

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \quad (2.21)$$

$$V = I(R_1 + R_2 + R_3)$$

$$V = IR_S \quad (2.22)$$

where R_S is the equivalent resistance,

$$R_S = R_1 + R_2 + R_3 \quad (2.23)$$

When several resistances are connected in series, the total or equivalent resistance is the sum of the individual resistances as shown in the Figure 2.9 (b).

Note: The value of equivalent resistance in series connection will be greater than each individual resistance.

E X A M P L E 2.8

Calculate the equivalent resistance for the circuit which is connected to 24 V battery and also find the potential difference across 4 Ω and 6 Ω resistors in the circuit.

Solution

Since the resistors are connected in series, the effective resistance in the circuit

$= 4\,\Omega + 6\,\Omega = 10\,\Omega$

The Current I in the circuit $= \frac{V}{R_{eq}} = \frac{24}{10} = 2.4\,A$

Voltage across 4Ω resistor

$V_1 = IR_1 = 2.4\,A \times 4\Omega = 9.6\,V$

Voltage across $6\,\Omega$ resistor

$V_2 = IR_1 = 2.4\,A \times 6\Omega = 14.4\,V$

Resistors in parallel

Resistors are in parallel when they are connected across the same potential difference as shown in Figure 2.10 (a). In this case, the total current I that leaves the battery is split into three separate paths. Let I1, I2 and I3 be the current through the resistors R1, R2 and R3 respectively. Due to the conservation of charge, total current in the circuit I is equal to sum of the currents through each of the three resistors.

$$I = I_1 + I_2 + I_3 \qquad (2.24)$$

Since the voltage across each resistor is the same, applying Ohm's law to each resistor, we have

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \qquad (2.25)$$

Substituting these values in equation (2.24), we get

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$I = \frac{V}{R_p}$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (2.26)$$

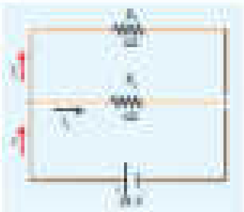
Here RP is the equivalent resistance of the parallel combination of the resistors. Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the values of resistance of the individual resistor is equal to the reciprocal of the effective resistance of the combination as shown in the Figure 2.10 (b)

Note: The value of equivalent resistance in parallel connection will be lesser than each individual resistance.

House hold appliances are always connected in parallel so that even if one is switched off, the other devices could function properly.

EXAMPLE 2.9

Calculate the equivalent resistance in the following circuit and also find the current I, I_1 and I_2 in the given circuit.



Solution

Since the resistances are connected in parallel, therefore, the equivalent resistance in the circuit is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{R_P} = \frac{5}{12}\Omega \quad \text{or } R_P = \frac{12}{5}\Omega$$

The resistors are connected in parallel, the potential (voltage) across each resistor is the same.

$$I_1 = \frac{V}{R_1} = \frac{24V}{4\Omega} = 6A$$

$$I_2 = \frac{V}{R_2} = \frac{24}{6} = 4A$$

The current I is the total of the currents in the two branches. Then,

$I = I_1 + I_2 = 6\text{ A} + 4\text{ A} = 10\text{ A}$

EXAMPLE 2.10

When two resistances connected in series and parallel their equivalent resistances are 15 Ω and $\frac{56}{15}$Ω respectively. Find the individual resistances.

Solution

$R_S = R_1 + R_2 = 15\,\Omega$ (1)

$R_P = \frac{R_1R_2}{R_1 + R_2} = \frac{56}{15}\Omega$ (2)

From equation (1) substituting for $R_1 + R_2$ in equation (2)

$\frac{R_1R_2}{15} = \frac{56}{15}\Omega$

$\therefore R_1R_2 = 56$

$R_2 = \frac{56}{R_1}\Omega$ (3)

Substituting for R_2 in equation (1) from equation (3)

$R_1 + \frac{56}{R_1} = 15$

Then, $\frac{R_1^2 + 56}{R_1} = 15$

$R_1^2 + 56 = 15\,R_1$

$R_1^2 - 15\,R_1 + 56 = 0$

The above equation can be solved using factorisation.

$R_1^2 - 8\,R_1 - 7\,R_1 + 56 = 0$

$R_1\,(R_1 - 8) - 7\,(R_1 - 8) = 0$

$(R_1 - 8)\,(R_1 - 7) = 0$

If $(R_1 = 8\,\Omega)$

using in equation (1)

$8 + R_2 = 15$

$R_2 = 15 - 8 = 7\,\Omega$,

$R_2 = 7\,\Omega$ i.e., (when $R_1 = 8\,\Omega$; $R_2 = 7\,\Omega$)

If $(R_1 = 7\,\Omega)$

Substituting in equation (1)

$7 + R_2 = 15$

$R_2 = 8\,\Omega$, i.e., (when $R_1 = 8\,\Omega$; $R_2 = 7\,\Omega$)

E X A M P L E

Calculate the equivalent resistance between A and B in the given circuit.

Solution

Parallel connection

Part 1

$$\frac{1}{R_{P_1}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{P_1}} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \quad R_{P_1} = 1\Omega$$

Part II

$$\frac{1}{R_{P_2}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}, \ \frac{1}{R_{P_2}} = \frac{1}{2}, \ R_{P_2} = 2\Omega$$

Part III

$$\frac{1}{R_{P_3}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$\frac{1}{R_{P_3}} = \frac{1}{3}, \quad R_{P_3} = 3\Omega$$

$$R = R_{P_1} + R_{P_2} + R_{P_3}$$

R = 1 + 2 + 3 R = 6 Ω

The circuit became:

E X A M P L E

Five resistors are connected in the configuration as shown in the figure. Calculate the equivalent resistance between the points a and b.

Solution

Case (a)

To find the equivalent resistance between the points a and b, we assume that current is entering the junction a. Since all the resistances in the outside loop are the same (1Ω), the current in the branches ac and ad must be equal. So the electric potential at the point c and d is the same hence no current flows into 5 Ω resistance. It implies that the 5 Ω has no role in determining the equivalent resistance and it can be removed. So the circuit is simplified as shown in the figure.
equ valent res stance of he cir uit bet een a a d b i *eq* = 1 **2 .**

Color code for Carbon resistors

Carbon resistors consists of a ceramic core, on which a thin layer of crystalline carbon is deposited as shown in Figure 2.11. These resistors are inexpensive, stable and compact in size. Color rings are used to indicate the value of the resistance according to the rules given in the Table 2.2.Three coloured rings are used to indicate the values of a resistor: the first two rings are significant figures of resistances, the third ring indicates the decimal multiplier after them. The fourth color, silver or gold,

Color	**Number**	**Multiplier**	**Tolerance**
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

shows the tolerance of the resistor at 10% or 5% as shown in the Figure 2.12 .If there is no fourth ring, the tolerance is 20%. For the resistor shown in Figure 2.12, the first digit = 5 (green), the second digit = 6 (blue), decimal multiplier = 103 (orange) and tolerance = 5% (gold). The value of resistance = 56 × 103 Ω or 56 kΩ with the tolerance value 5%.

A multimeter is a very useful electronic instrument used to measure voltage, current, resistance and capacitance. In fact, it can also measure AC voltage and AC current. The circular slider has to be kept in appropriate position to measure each electrical quantity.

Temperature dependence of resistivity

The resistivity of a material is dependent on temperature. It is experimentally found that for a wide range of temperatures, the resistivity of a conductor increases with
increase in temperature according to the expression,

$$\rho_T = \rho_o[1 + \alpha(T - T_o)]$$

where ρ*T* is the resistivity of a conductor at *T* oC, ρ*o* is the resistivity of the conductor at some reference temperature *To* (usually at 20oC) and *a* is the temperature coefficient of resistivity. **It is defined as the ratio of increase in resistivity per degree rise in temperature to its resistivity at T_o.**

From the equation (2.27), we can write

$$\rho_T - \rho_o = \alpha\rho_o(T - T_o)$$

$$\therefore \alpha = \frac{\rho_T - \rho_o}{\rho_o(T - T_o)} = \frac{\Delta\rho}{\rho_o \Delta T}$$

where *Δ*ρ = ρ*T* – ρ*o* is change in resistivity for a change in temperature *ΔT* = *T* – *To.* Its unit is per oC.

α of conductors

For conductors α *is positive.* If the temperature of a conductor increases, the average kinetic energy of electrons in the conductor increases. This results in more frequent collisions and hence the resistivity increases. The graph of the equation (2.27) is shown in Figure 2.13

Even though, the resistivity of conductors like metals varies linearly for wide range of temperatures, there also exists a non-linear region at very low temperatures. The resistivity approaches some finite value as the temperature approaches absolute zero as shown in Figure 2.13(b).

As the resistance is directly proportional to resistivity of the material, we can also write the resistance of a conductor at temperature T oC as

$$R_T = R_o\left[1+\alpha\left(T-T_o\right)\right]$$

The temperature coefficient can be also be obtained from the equation (2.28),

$$R_T - R_o = \alpha R_o\left(T-T_o\right)$$

$$\therefore \alpha = \frac{R_T - R_o}{R_o\left(T-T_o\right)} = \frac{1}{R_o}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{R_o}\frac{\Delta R}{\Delta T}$$

where $\Delta R = R_T - R_o$ is change in resistance during the change in temperature $\Delta T = T - T_o$

***a* of semiconductors**

For semiconductors, the resistivity decreases with increase in temperature. As the temperature increases, more electrons will be liberated from their atoms (Refer unit 9 for conduction in semi conductors). Hence the current increases and therefore the resistivity decreases as shown in Figure 2.14. A semiconductor with a negative temperature coefficient of resistance is called a thermistor.

The typical values of temperature coefficients of various materials are given in table 2.3.

Color	Temperature Coefficient α [(°C)
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.92×10^{-3}
Lead	3.9×10^{-3}
Nichrome	0.4×10^{-3}
Carbon	-0.5×10^{-3}
Germanium	-48×10^{-3}
Silicon	-75×10^{-3}

We can understand the temperature dependence of resistivity in the following way. In section 2.1.3, we have shown that the electrical conductivity,

the electrical conductivity, $\sigma = \frac{ne^2\tau}{m}$. As the resistivity is inverse of σ, it can be written as,

$$\rho = \frac{m}{ne^2\tau} \qquad (2.30)$$

The resistivity of materials is

i) inversely proportional to the number density (n) of the electrons

ii) inversely proportional to the average time between the collisions (τ).
In metals, if the temperature increases, the average time between the collision (τ) decreases and n is independent of temperature. In

semiconductors when temperature increases, n increases and τ decreases, but increase in n is dominant than decreasing τ, so that overall resistivity decreases.

The resistance of certain materials become zero below certain temperature T_c. This temperature is known as critical temperature or transition temperature. The materials which exhibit this property are known as superconductors. This phenomenon was first observed by Kammerlingh Onnes in 1911. He found that mercury exhibits superconductor behaviour at 4.2 K. Since R = 0, current once induced in a superconductor persists without any potential difference.

EXAMPLE 2.14

Resistance of a material at 10^0C and 40^0C are 45 Ω and 85 Ω respectively. Find its temperature co-efficient of resistance.

Solution

$T_0 = 10\ ^0C$, $T = 40\ ^0C$, $R_0 = 45\ \Omega$, $R = 85\ \Omega$

$$\alpha = \frac{1}{R_o}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{45}\left(\frac{85-45}{40-10}\right) = \frac{1}{45}\left(\frac{40}{30}\right)$$

$$\alpha = 0.0296\ per\ ^\circ C$$

EXAMPLE 2.13

If the resistance of coil is 3 Ω at 20 °C and $\alpha = 0.004/^0C$ then determine its resistance at 100 °C.

Solution

$R_0 = 3\ \Omega, \quad T = 100^0C, \quad T_0 = 20^0C$

$\alpha = 0.004/^0C, \quad R_T = ?$

$R_T = R_0(1 + \alpha(T - T_0))$

$R_{100} = 3(1 + 0.004 \times 80)$

$R_{100} = 3(1 + 0.32)$

$R_{100} = 3(1.32)$

$R_{100} = 3.96\ \Omega$

ENERGY AND POWER IN ELECTRICAL CIRCUITS

When a battery is connected between the ends of a conductor, a current is established. The battery is transporting energy to the device which is connected in the circuit. Consider a circuit in which a battery of voltage V is connected to the resistor as shown in Figure 2.15.

Assume that a positive charge of *dQ* moves from point *a* to *b* through the battery and moves from point *c* to *d* through the resistor and back to point a. When the moves from point a to b, it gains potential energy dU = V.dQ and the chemical potential energy of the battery decreases by the same amount. When this charge dQ passes through resistor it loses the potential energy dU = V.dQ due to collision with atoms in the resistor and again reaches the point a. This process occurs continuously till the battery is connected in the circuit. The rate at which the charge loses its electrical potential energy in the resistor can be calculated.

The electrical power P is the rate at which the electrical potential energy is delivered,

$$P = \frac{dV}{dt} = \frac{d}{dt}(V.dQ) = V\frac{dQ}{dt} \qquad (2.31)$$

Since the electric current $I = \frac{dQ}{dt}$.

So the equation (2.31) can be rewritten as

$$P = VI \qquad (2.32)$$

This expression gives the power delivered by the battery to any electrical system, where *I* is the current passing through it and *V* is the potential difference across it. The SI unit of electrical power is watt (1*W* = 1 *J s*-1). Commercially, the electrical bulbs used in houses come with the power and voltage rating of 5W-220V, 30W-220V, 60W-220V etc.

Usually these voltage rating refers AC RMS voltages. For a given bulb, if the voltage drop across the bulb is greater than voltage rating, the bulb will fuse.
Using Ohm's law, power delivered to the resistance R is expressed in other forms

$$P = IV = I\,(IR) = I^2R$$

$$P = IV = \frac{V}{R}V = \frac{V^2}{R}$$

The electrical power produced (dissipated) by a resistor is *I*2*R*. It depends on the square of the current. Hence, if current is doubled, the power will increase by four times. Similar explanation holds true for voltage also.

The total energy used by any device is obtained by multiplying the power and duration of the time when it is ON. If the power is in watts and the time is in seconds, the energy will be in joules. In practice, electrical energy is measured in kilowatt hour (*kWh*). 1 *kWh* is known as 1 unit of electrical energy.

(1 kWh = 1000 Wh = (1000 W) (3600 s) = 3.6 x 10^6 J)

The Tamilnadu Electricity Board is charging for the amount of energy you use and not for the power. A current of 1A flowing through a potential difference of 1V produces a power of 1W.

E X A M P L E

A battery of voltage V is connected to 30 W bulb and 60 W bulb as shown in the figure. (a) Identify brightest bulb (b) which bulb has greater resistance? (c) Suppose the two bulbs are connected in series, which bulb will glow brighter?

(a) The power delivered by the battery P = VI. Since the bulbs are connected in parallel, the voltage drop across each bulb is the same. If the voltage is kept fixed, then the power is directly proportional to current ($P \propto I$). So 60 W bulb draws twice as much as current as 30 W and it will glow brighter then others.

(b) To calculate the resistance of the bulbs, we use the relation $P = \frac{V2}{R}$
In both the bulbs, the voltage drop is the same, so the power is inversely proportional to the resistance or resistance is inversely proportional to the power $\left(R \propto \frac{1}{P}\right)$. It implies that, the 30W has twice as much as resistance as 60 W bulb.

(c) When these two bulbs are connected in series, the current passing through each bulb is the same. It is equivalent to two resistors connected in series. The bulb which has higher resistance has higher voltage drop. So 30W bulb will glow brighter than 60W bulb. So the higher power rating does not always imply more brightness and it depends whether bulbs are connected in series or parallel.

E X A M P L E

Two electric bulbs marked 20 W – 220 V and 100 W – 220 V are connected in series to 440 V supply. Which bulb will be fused?

Solution

To check which bulb will be fused, the voltage drop across each bulb has to be calculated.

The resistance of a bulb,

$$R = \frac{V^2}{P} = \frac{(\text{Rated voltage})^2}{\text{Rated power}}$$

For 20W-220V bulb,

$$R_1 = \frac{(220)^2}{20}\Omega = 2420\Omega$$

For 100W-220V bulb,

$$R_2 = \frac{(220)^2}{100}\Omega = 484\Omega$$

Both the bulbs are connected in series. So the current which passes through both the bulbs are same. The current that passes through the circuit ,I=$\frac{V}{R}$

$$R_{net} = (R_1 + R_2)$$

$$R_{net} = (484 + 2420)\Omega = 2904\Omega$$

$$I = \frac{440V}{2904\Omega} \approx 0.151A$$

The voltage drop across the 20W bulb is

$$V_1 = IR_1 = \frac{440}{2904} \times 2420 \approx 366.6\ V$$

The voltage drop across the 100W bulb is

$$V_2 = IR_2 = \frac{440}{2904} \times 484 \approx 73.3\ V$$

The 20 W bulb will be fused because its voltage rating is only 220 V and 366.6 V is dropped across it.

An electric cell converts chemical energy into electrical energy to produce electricity. It contains two electrodes immersed in an electrolyte

ELECTRIC CELLS AND BATTERIES

Several electric cells connected together form a battery. When a cell or battery is connected to a circuit, electrons flow from the negative terminal to the positive terminal through the circuit. By using chemical reactions, a battery produces potential difference across its terminals. This potential difference provides the energy to move the electrons through the circuit. Commercially available electric cells and batteries are

If we connect copper and zinc rod in a lemon, it acts as an electric cell. The citric acid in the lemon acts as an electrolyte. The potential can be measured using a multimeter.

Electromotive force and internal resistance

A battery or cell is called a source of electromotive force (emf). The term 'electromotive force' is a misnomer since it does not really refer to a force but describes a potential difference in volts. The emf of a battery or cell is the voltage provided by the battery when no current flows in the external circuit.

Electromotive force determines the amount of work a battery or cell does to move a certain amount of charge around the circuit. It is denoted by the symbol ξ and to be pronounced as 'xi'. An ideal battery has zero internal resistance and the potential difference (terminal voltage) across the battery equals to its emf. But a real battery is made of electrodes and electrolyte, there is resistance to the flow of charges within the battery. This resistance is called internal resistance *r*. For a real battery, the terminal voltage is not equal to the emf of the battery. A freshly prepared cell has low internal resistance and it increases with ageing.

Determination of internal resistance

The circuit connections are made as shown in Figure 2.20.

The emf of cell ξ is measured by connecting a high resistance voltmeter across it without connecting the external resistance R as shown in Figure 2.20(a). Since the voltmeter draws very little current for deflection, the circuit may be considered as open. Hence the voltmeter reading gives the emf of the cell. Then, external resistance R is included in the circuit and current I is established in the circuit.

The potential difference across R is equal to the potential difference across the cell (V) as shown in Figure 2.20(b).

The potential drop across the resistor R is

$$V = IR$$

Due to internal resistance r of the cell, the voltmeter reads a value V, which is less than the emf of cell ξ. It is because, certain amount of voltage (Ir) has dropped across the internal resistance *r*.

Then $V = \xi - Ir$

$$Ir = \xi - V \quad (2.36)$$

Dividing equation (2.36) by equation (2.35), we get

$$\frac{Ir}{IR} = \frac{\xi - V}{V}$$

$$r = \left[\frac{\xi - V}{V}\right] R \quad (2.37)$$

Since ξ, *V* and *R* are known, internal resistance *r* can be determined. We can also find the total current that flows in the circuit.

Due to this internal resistance, the power delivered to the circuit is not equal to power rating mentioned in the battery. For a battery of emf ξ, with an internal resistance *r*, the power delivered to the circuit of resistance *R* is given by

$$P = I\xi = I\,(V + Ir) \text{ (from equation 2.36)}$$

Here *V* is the voltage drop across the resistance R and it is equal to IR.
Therefore, P = I (IR +Ir)

$$P = I2\,R + I2\,r \quad \textbf{(2.38)}$$

Here I2 r is the power delivered to the internal resistance and I2 R is the power delivered to the electrical device (here it is the resistance R). For a good battery, the internal resistance r is very small, then *I*2 *r* << *I*2 R and almost entire power is delivered to the resistance.

E X A M P L E

A battery has an emf of 12 V and connected to a resistor of 3 Ω. The current in the circuit is 3.93 A. Calculate (a) terminal voltage and the internal resistance of the battery (b) power delivered by the battery and power delivered to the resistor

Solution

The given values *I* = 3.93 A, ξ = 12 V, *R* = 3 Ω

(a) The terminal voltage of the battery is equal to voltage drop across the resistor

$$\mathbf{V = IR = 3.93 \times 3 = 11.79\ V}$$

The internal resistance of the battery,

$$r = \left[\frac{\xi - V}{V}\right] R = \left[\frac{12 - 11.79}{11.79}\right] \times 3 = 0.05\ \Omega$$

(b) The power delivered by the battery P = Iξ = 3.93 × 12 = 47.1 W
The power delivered to the resistor = I2 R = 46.3 W
The remaining power = (47.1 – 46.3) *P* = 0.772 W is delivered to the internal resistance and cannot be used to do useful work. (it is equal to *I*2 *r*).

Cells in series

Several cells can be connected to form a battery. In series connection, the negative terminal of one cell is connected to the positive terminal of the second cell, the negative terminal of second cell is connected to the positive terminal of the third cell and so on. The free positive terminal of the first cell and the free negative terminal of the last cell become the terminals of the battery.

Suppose n cells, each of emf ξ volts and internal resistance *r* ohms are connected in series with an external resistance R as shown in Figure 2.21

The total emf of the battery = $n\xi$
The total resistance in the circuit = $nr + R$
By Ohm's law, the current in the circuit is

$$I = \frac{\text{total emf}}{\text{total resistance}} = \frac{n\xi}{nr + R} \qquad (2.39)$$

Case (a) If $r << R$, then,

$$I = \frac{n\xi}{R} \approx nI_1 \qquad (2.40)$$

where, I_1 is the current due to a single cell

$$\left(I_1 = \frac{\xi}{R}\right)$$

Thus, if r is negligible when compared to R the current supplied by the battery is n times that supplied by a single cell.

Case (b) If $r >> R$, $I = \frac{n\xi}{nr} \approx \frac{\xi}{r}$

It is the current due to a single cell. That is, current due to the whole battery is the same as that due to a single cell and hence there is no advantage in connecting several cells.

Thus series connection of cells is advantageous only when the effective internal resistance of the cells is negligibly small compared with R.

E X A M P L E 2.18

From the given circuit,
Find

i) Equivalent emf of the combination
ii) Equivalent internal resistance
iii) Total current
iv) Potential difference across external resistance
v) Potential difference across each cell

Solution

i) Equivalent emf of the combination $\xi_{eq} = n\xi = 4\ 9 = 36$ V

ii) Equivalent internal resistance $r_{eq} = nr$
$= 4 \times 0.1 = 0.4\ \Omega$

iii) Total current $I = \frac{n\xi}{R+nr}$

$$= \frac{4\times 9}{10+(4\times 0.1)}$$

$$= \frac{4\times 9}{10+0.4} = \frac{36}{10.4}$$

$I = 3.46$ A

iv) Potential difference across external resistance V = IR = 3.46 × 10 = 34.6 V. The remaining 1.4 V is dropped across the internal resistance of cells.

v) Potential difference across each cell
$$\frac{V}{n} = \frac{34.6}{4} = 8.65V$$

Cells in parallel

In parallel connection all the positive terminals of the cells are connected to one point and all the negative terminals to a second point. These two points form the positive and negative terminals of the battery. Let n cells be connected in parallel between the points A and B and a resistance R is connected between the points A and B as shown in Figure 2.22. Let ξ be the emf and r the internal resistance of each cell.

The equivalent internal resistance of the

$$I = \frac{n\xi}{r + nR} \qquad (2.42)$$

Case (a) If r >> R, $I = \frac{n\xi}{r} = nI_1$ (2.43)

where I_1 is the current due to a single cel and is equal to $\frac{\xi}{r}$ when R is negligible. Thus the current through the external resistanc due to the whole battery is n times th current due to a single cell.

Case (b) If r<<R, $I = \frac{\xi}{R}$ (2.44)

battery is $\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + ... \frac{1}{r}(n terms) = \frac{n}{r}$. So $r_{eq} = \frac{r}{n}$ and the total resistance in the circuit $= R + \frac{r}{n}$. The total emf is the potential difference between the points A and B, which is equal to ξ. The current in the circuit is given by

$$I = \frac{\xi}{\frac{r}{n} + R}$$

When the car engine is started with headlights turned on, they sometimes become dim. This is due to the internal resistance of the car battery.

The above equation implies that current due to the whole battery is the same as that due to a single cell. Hence it is advantageous to connect cells in parallel when the external resistance is very small compared to the internal resistance of the cells.

E X A M P L E 2.19

From the given circuit
Find
i) Equivalent emf
ii) Equivalent internal resistance
iii) Total current (I)
iv) Potential difference across each cell
v) Current from each cell

Solution

i) Equivalent emf ξ_{eq} = 5 V
ii) Equivalent internal resistance,

$$R_{eq} = \frac{r}{n} = \frac{0.5}{4} = 0.125\Omega$$

iii) total current, $I = \frac{\xi}{R + r/n}$

$$I = \frac{5}{10 + 0.125} = \frac{5}{10.125}$$

$$I = 0.5 \text{ A}$$

iv) Potential difference across each cell

$$V = IR = 0.5 \times 10 = 5 \text{ V}$$

v) Current from each cell, $I' = \frac{I}{n}$

$$I' = \frac{0.5}{4} = 0.125 A$$

KIRCHHOFF'S RULES

Ohm's law is useful only for simple circuits. For more complex circuits, Kirchhoff 's rules can be used to find current and voltage. There are two generalized rules: i) Kirchhoff 's current rule ii) Kirchhoff 's voltage rule.

Kirchhoff's first rule (Current rule or Junction rule)

It states that the algebraic sum of the currents at any junction of a circuit is zero. It is a statement of conservation of electric charge. All charges that enter a given junction in a circuit must leave that junction since charge cannot build up or disappear at a junction. Current entering the junction is taken as positive and current leaving the junction is taken as negative.

$I1 + I2 - I3 - I4 - I5 = 0$
(or)
$I1 + I2 = I3 + I4 + I5$

E X A M P L E 2.20

From the given circuit find the value of I.

Solution

Applying Kirchoff 's rule to the point P in the circuit,
The arrows pointing towards P are positive and away from P are negative.

Therefore, $0.2A - 0.4A + 0.6A - 0.5A + 0.7A - I = 0$
$1.5A - 0.9A - I = 0$
$0.6A - I = 0$
$I = 0.6\ A$

Kirchhoff's Second rule (Voltage rule or Loop rule)

It states that in a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. This rule follows from the law of conservation of energy for an isolated system (The energy supplied by the emf sources is equal to the sum of the energy delivered to all resistors).

The product of current and resistance is taken as positive when the direction of the current is followed. Suppose if the direction of current is

opposite to the direction of the loop, then product of current and voltage across the resistor is negative. It is shown in Figure 2.24 (a) and (b). The emf is considered positive when proceeding from the negative to the positive terminal of the cell. It is shown in Figure 2.24 (c) and (d).
Kirchhoff voltage rule has to be applied only when all currents in the circuit reach a steady state condition (the current in various branches are constant).

E X A M P L E 2.21

The following figure shows a complex network of conductors which can be divided into two closed loops like ACE and ABC. Apply Kirchoff 's voltage rule.

Solution

Thus applying Kirchoff 's second law to the closed loop EACE

$I1R1 + I2R2 + I3R3 = \xi$ and for the closed loop ABCA
$I4R4 + I5R5 - I2R2 = 0$

E X A M P L E 2.22

Calculate the current that flows in the 1 Ω resistor in the following circuit.
We can denote the current that flows from 9V battery as I1 and it splits into I2 and I1 – I2 in the junction according Kirchoff 's current rule (KCR). It is shown below.
Now consider the loop EFCBE and apply KVR, we get

1I2 + 3I1 + 2I1 = 9
5I1 + I2 = 9 (1)

Applying KVR to the loop EADFE, we get

3 (I1 – I2) – 1I2 = 6
3I1 – 4I2 = 6 (2)

Solving equation (1) and (2), we get

I1 = 1.83 A and I2 = -0.13 A

It implies that the current in the 1 ohm resistor flows from F to E.

Wheatstone's bridge

An important application of Kirchhoff 's rules is the Wheatstone's bridge. It is used to compare resistances and also helps in determining the unknown resistance in electrical network. The bridge consists of four resistances P, Q, R and S connected as shown in Figure 2.25. A galvanometer G is connected between the points B and D. The battery is connected between the points A and C. The current through the galvanometer is I_G and its resistance is G.

Applying Kirchhoff 's current rule to junction B

$$I_1 - I_G - I_3 = 0$$

Applying Kirchhoff 's current rule to junction D,

$$I_2 + I_G - I_4 = 0$$

Applying Kirchhoff 's voltage rule to loop ABDA,

I1P + IGG – I2R = 0 (2.47)

Applying Kirchhoff 's voltage rule to loop ABCDA,

I1P + I3Q – I4S – I2R = 0

When the points B and D are at the same potential, the bridge is said to be balanced. As there is no potential difference between B and D, no current flows through galvanometer (IG = 0). Substituting IG = 0 in equation (2.45), (2.46) and (2.47), we get

I1= I3
I2 =I4

I1P =I2R

Substituting the equation (2.49) and (2.50) in equation (2.48)

I1P + I1Q – I2S – I2R = 0

I1 (P + Q) = I2 (R +S)

Dividing equation (2.52) by equation (2.51), we get

$$\frac{P+Q}{P}=\frac{R+S}{R}$$

$$1+\frac{Q}{P}=1+\frac{S}{R}$$

$$\frac{Q}{P}=\frac{S}{R}$$

$$\frac{P}{Q}=\frac{R}{S}$$

This is the bridge balance condition. Only under this condition, galvanometer shows null deflection. Suppose we know the values of two adjacent resistances, the other two resistances can be compared. If three of the resistances are known, the value of unknown resistance (fourth one) can be determined.

A galvanometer is an instrument used for detecting and measuring even very small electric currents. It is extensively useful to compare the potential difference between various parts of the circuit.

E X A M P L E 2.23

In a Wheatstone's bridge P = 100 Ω, Q = 1000 Ω and R = 40 Ω. If the galvanometer shows zero deflection, determine the value of S.

Solution

$$\frac{P}{Q}=\frac{R}{S}$$

$$S=\frac{Q}{P}\times R$$

$$S=\frac{1000}{100}\times 40 \quad S = 400\ \Omega$$

E X A M P L E 2.24

What is the value of x when the Wheatstone's network is balanced?

P = 500 Ω, Q = 800 Ω, R = x + 400,
S = 1000 Ω

Solution

$$\frac{P}{Q} = \frac{R}{S}$$

$$\frac{500}{800} = \frac{x+400}{1000}$$

$$\frac{x+400}{1000} = \frac{500}{800}$$

$$x + 400 = \frac{500}{800} \times 1000$$

$$x + 400 = \frac{5}{8} \times 1000$$

$$x + 400 = 0.625 \times 1000$$

$$x + 400 = 625$$

$$x = 625 - 400$$

$$x = 225\ \Omega$$

Meter bridge

The meter bridge is another form of Wheatstone's bridge. It consists of a uniform manganin wire AB of one meter length. This wire is stretched along a meter scale on a wooden board between two copper strips C and D. Between these two copper strips another copper strip E is mounted to enclose two gaps G1 and G2 as shown in Figure 2.26. An unknown resistance P is connected in G1 and a standard resistance Q is connected in G2. A jockey (conducting wire) is connected to the terminal E on the central copper strip through a galvanometer (G) and a high resistance (HR). The exact position of jockey on the wire can be read on the scale. A Lechlanche cell and a key (K) are connected across the ends of the bridge wire.

The position of the jockey on the wire is adjusted so that the galvanometer shows zero deflection. Let the point be J. The lengths AJ and JB of the bridge wire now replace the resistance R and S of the Wheatstone's bridge. Then

$$\frac{P}{Q}=\frac{R}{S}=\frac{R'.AJ}{R'.JB}$$

where R' is the resistance per unit length of wire

$$\frac{P}{Q}=\frac{AJ}{JB}=\frac{l_1}{l_2} \quad (2.55)$$

$$P=Q\frac{l_1}{l_2} \quad (2.56)$$

The bridge wire is soldered at the ends of the copper strips. Due to imperfect contact, some resistance might be introduced at the contact. These are called end resistances. This error can be eliminated, if another set of readings are taken with P and Q interchanged and the average value of P is found.

To find the specific resistance of the material of the wire in the coil P, the radius r and length l of the wire is measured. The specific resistance or resistivity ρ can be calculated using the relation

$$\text{Resistance}=\rho\frac{l}{A}$$

By rearranging the above equation, we get

$$\rho=\text{Resistance}\times\frac{A}{l} \quad (2.57)$$

If P is the unknown resistance equation (2.57) becomes,

$$\rho=P\frac{\pi r^2}{l}$$

E X A M P L E

In a meter bridge with a standard resistance of 15 Ω in the right gap, the ratio of balancing length is 3:2. Find the value of the other resistance.

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

$$P = Q\frac{l_1}{l_2}$$

$$P = 15\frac{3}{2} = 22.5\ \Omega$$

$$Q = 15\ \Omega, \quad l_1 : l_2 = 3:2$$

$$\frac{l_1}{l_2} = \frac{3}{2}$$

E X A M P L E 2.26

In a meter bridge, the value of resistance in the resistance box is 10 Ω. The balancing length is $l1$ = 55 cm. Find the value of unknown resistance.

Solution

Q = 10 Ω

$$\frac{P}{Q} = \frac{l_1}{100 - l_1} = \frac{l_1}{l_2}$$

$$P = Q \times \frac{l_1}{100 - l_1}$$

$$P = \frac{10 \times 55}{100 - 55}$$

$$P = \frac{550}{45} = 12.2\ \Omega$$

Potentiometer

Potentiometer is used for the accurate measurement of potential differences, current and resistances. It consists of ten meter long uniform wire of manganin or constantan stretched in parallel rows each of 1 meter length, on a wooden board. The two free ends A and B are brought to the same side and fixed to copper strips with binding screws. A meter scale is fixed parallel to the wire. A jockey is provided for making contact.

The principle of the potentiometer is illustrated in Fig *Bt*. The battery, key and the potentiometer wire are connected in series forms the primary circuit. The positive terminal of a primary cell of emf ξ is connected to the point C and negative terminal is connected to the jockey through a galvanometer G and a high resistance HR. This forms the secondary circuit. ure 2.27. A steady current is maintained across the wire CD by a battery

Bt. The battery, key and the potentiometer wire are connected in series forms the primary circuit. The positive terminal of a primary cell of emf ξ is connected to the point C and negative terminal is connected to the jockey through a galvanometer G and a high resistance HR. This forms the secondary circuit.

Let contact be made at any point J on the wire by jockey. If the potential difference across CJ is equal to the emf of the cell ξ then no current will flow through the galvanometer and it will show zero deflection. CJ is the balancing length l. The potential difference across CJ is equal to Irl where I is the current flowing through the wire and r is the resistance per unit length of the wire.

Hence $\xi = Irl$

Since I and r are constants, $\xi \propto l$. The emf of the cell is directly proportional to the balancing length.

Comparison of emf of two cells with a potentiometer

To compare the emf of two cells, the circuit connections are made as shown in Figure 2.28. Potentiometer wire CD is connected to a battery *Bt* and a key K in series. This is the primary circuit. The end C of the wire is connected to the terminal M of a DPDT (Double Pole Double Throw) switch and the other terminal N is connected to a jockey through a galvanometer G and a high resistance HR. The cells whose emf ξ_1 and ξ_2 to be compared are connected to the terminals M_1,N_1 and M_2,N_2 of the DPDT switch. The positive terminals of *Bt*, ξ_1 and ξ_2 should be connected to the same end C.

The DPDT switch is pressed towards M_1, N_1 so that cell ξ_1 is included in the secondary circuit and the balancing length l_1 is found by adjusting the jockey for zero deflection. Then the second cell ξ_2 is included in the circuit and the balancing length l_2 is determined. Let r be the resistance per unit length of the potentiometer wire and I be the current flowing through the wire.

we have $\xi_1 = Irl_1$

$\xi_2 = Irl_2$

By dividing equation

$$\frac{\xi_1}{\xi_2} = \frac{l_1}{l_2}$$

By including a rheostat (Rh) in the primary circuit, the experiment can be repeated several times by changing the current flowing through it.

Measurement of internal resistance of a cell by potentiometer

To measure the internal resistance of a cell, the circuit connections are made as shown in Figure 2.29. The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and the negative terminal of the battery is connected to the end D through a key K1. This forms the primary circuit.

The positive terminal of the cell ξ whose internal resistance is to be determined is also connected to the end C of the wire. The negative terminal of the cell ξ is connected to a jockey through a galvanometer and a high resistance. A resistance box R and key K2 are connected across the cell ξ. With K2 open, the balancing point J is obtained and the balancing length CJ = *l*1 is measured. Since the cell is in open circuit, its emf is

$$\xi \propto l_1$$

A suitable resistance (say, 10 Ω) is included in the resistance box and key K2 is closed. Let r be the internal resistance of the cell. The current passing through the cell and the resistance R is given by

$$I = \frac{\xi}{R+r}$$

The potential difference across R is

$$V = \frac{\xi R}{R+r}$$

Then $\frac{\xi R}{R+r} \propto l_2$

From equations (2.62) an

$$\frac{R+r}{R} = \frac{l_1}{l_2}$$

$$1 + \frac{r}{R} = \frac{l_1}{l_2};$$

$$r = R\left[\frac{l_1}{l_2} - 1\right]$$

$$\therefore r = R\left(\frac{l_1 - l_2}{l_2}\right)$$

Substituting the values of the *R*, l_1 and l_2 , the internal resistance of the cell is determined. The experiment can be repeated for different values of *R*. It is found that the internal resistance of the cell is not constant but increases with increase of external resistance connected across its terminals.

HEATING EFFECT OF ELECTRIC CURRENT

When current flows through a resistor, some of the electrical energy delivered to the resistor is converted into heat energy and it is dissipated. This heating effect of current is known as Joule's heating effect**.** Just as current produces thermal energy, thermal energy may also be suitably used to produce an electromotive force. This is known as thermoelectric effect.

Joule's law

If a current I flows through a conductor kept across a potential difference V for a time t, the work done or the electric potential energy spent is

W = VIt (2.66)

In the absence of any other external effect, this energy is spent in heating the conductor. The amount of heat(H) produced is

H = VIt (2.67)

For a resistance R,

H = $I2Rt$ (2.68)

This relation was experimentally verified by Joule and is known as Joule's law of heating. It states that **the heat developed in an electrical**

circuit due to the flow of current varies directly as

(i) the square of the current
(ii) the resistance of the circuit and
(iii) the time of flow.

E X A M P L E 2.27

Find the heat energy produced in a resistance of 10 Ω when 5 A current flows through it for 5 minutes.

Solution

R = 10 Ω, I = 5 A, t = 5 minutes = 5 × 60 s
H = I2 R t
= 52 × 10 × 5 × 60
= 25 × 10 × 300
= 25 × 3000
= 75000 J (or) 75 Kj

Application of Joule's heating effect

1. Electric heaters

Electric iron, electric heater, electric toaster shown in Figure 2.30 are some of the home appliances that utilize the heating effect of current. In these appliances, the heating elements are made of nichrome, an alloy of nickel and chromium. Nichrome has a high specific resistance and can be heated to very high temperatures without oxidation.

E X A M P L E

An electric heater of resistance 10 Ω connected to 220 V power supply is immersed in the water of 1 kg. How long the electrical heater has to be switched on to increase its temperature from 30°C to 60°C. (The specific heat of water is s = 4200 J kg-1)

Solution

According to Joule's heating law H = I2 Rt
The current passed through the electrical

$$\text{heater} = \frac{220V}{10\Omega} = 22\,A$$

The heat produced in one second by the electrical heater H = I2 R

The heat produced in one second H = (22)2 × 10 = 4840 J = 4.84 k J. In fact the power rating of this electrical heater is 4.84 k W.

The amount of energy to increase the temperature of 1kg water from 30°C to 60°C is

Q = ms ΔT (Refer XI physics vol 2, unit 8)

Here m = 1 kg,

s = 4200 J kg-1,
ΔT = 30,
so Q = 1 × 4200 x 30 = 126 kJ

The time required to produce this heat

energy $t = \frac{Q}{I^2R} = \frac{126 \times 10^3}{4840} \approx 26.03\ s$

Electric fuses

Fuses as shown in Figure 2.31, are connected in series in a circuit to protect the electric devices from the heat developed by the passage of excessive current. It is a short length of a wire made of a low melting point material. It melts and breaks the circuit if current exceeds a certain value. Lead and copper wire melts and burns out when the current increases above 5 A and 35 A respectively.

The only disadvantage with the above fuses is that once fuse wire is burnt due to excessive current, they need to be replaced. Nowadays in houses, circuit breakers (trippers) are also used instead of fuses.

Whenever there is an excessive current produced due to faulty wire connection, the circuit breaker switch opens. After repairing the faulty connection, we can close the circuit breaker switch. It is shown in the Figure 2.32.

3. Electric furnace

Furnaces as shown in Figure 2.33 are used to manufacture a large number of technologically important materials such as steel, silicon carbide, quartz, gallium arsenide, etc). To produce temperatures up to 1500°C, molybdenum-nichrome wire wound on a silica tube is used. Carbon arc furnaces produce temperatures up to 3000 °C.

4. Electrical lamp

It consists of a tungsten filament (melting point 3380 0C) kept inside a glass bulb and heated to incandescence by current. In incandescent electric lamps only about 5% of electrical energy is converted into light and the rest is wasted as heat. Electric discharge lamps, electric welding and electric arc also utilize the heating effect of current as shown in Figure 2.34.

THERMOELECTRIC EFFECT

Conversion of temperature differences into electrical voltage and vice versa is known as thermoelectric effect. A thermoelectric device generates voltage when there is a temperature difference on each side. If a voltage is applied, it generates a temperature difference.

Seebeck effect

Seebeck discovered that in a closed circuit consisting of two dissimilar metals, when the junctions are maintained at different temperatures an emf (potential difference) is developed. The current that flows due to the emf developed is called thermoelectric current. The two dissimilar metals connected to form two junctions is known as thermocouple
If the hot and cold junctions are interchanged, the direction of current also reverses. Hence the effect is reversible.

The magnitude of the emf developed in a thermocouple depends on (i) the nature of the metals forming the couple and (ii) the temperature difference between the junctions.

Applications of Seebeck effect

1. Seebeck effect is used in thermoelectric generators (Seebeck generators).These thermoelectric generators are used in power plants to convert waste heat into electricity.

2.This effect is utilized in automobiles as automotive thermoelectric generators for increasing fuel efficiency.

3. Seebeck effect is used in thermocouples and thermopiles to measure the temperature difference between the two objects.

Peltier effect

In 1834, Peltier discovered that when an electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and absorbed at the other junction. This is known as Peltier effect.

In the Cu-Fe thermocouple the junctions A and B are maintained at the same temperature. Let a current from a battery flow through the thermocouple (Figure 2.36 (a)). At the junction A, where the current flows from Cu to Fe, heat is absorbed and the junction A becomes cold. At the junction B, where the current flows from Fe to Cu heat is liberated and it becomes hot. When the direction of current is reversed, junction A gets heated and junction B gets cooled as shown in the Figure 2.36(b). Hence Peltier effect is reversible.

Thomson effect

Thomson showed that if two points in a conductor are at different temperatures, the density of electrons at these points will differ and as a result the potential difference is created between these points. Thomson effect is also reversible.

If current is passed through a copper bar AB which is heated at the middle point C, the point C will be at higher potential. This indicates that the heat is absorbed along AC and evolved along CB of the conductor as shown in Figure 2.37(a). Thus heat is transferred due to the current flow in the direction of the current. It is called positive Thomson effect. Similar effect is observed in metals like silver, zinc, and cadmium. When the copper bar is replaced by an iron bar, heat is evolved along CA and absorbed along BC. Thus heat is transferred due to the current flow in the direction opposite to the direction of current. It is called negative Thomson effect as shown in the Figure 2.37(b). Similar effect is observed in metals like platinum, nickel, cobalt, and mercury.