# MONTHLY TEST – IV
# PART – I

| LIGHT | | |
|---|---|---|
| 8th term 1 | Unit-3 | Light |
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**8th term 1 Unit-3. Light**

## Introduction

Lofty mountains covered with greenish vegetation, magnificent trees reaching up to the clouds, beautiful streams drifting down the valleys, bluish sea water roaring towards the coast and the radiant sky in the morning being filled with golden red color, all give delight to our eyes and peace to our mind. But, can we see them all without light? No, because, we can see things around us only when the light reflected by them reaches our eyes.

Light is a form of energy and it travels in a straight line. You have studied in your lower classes, how it is reflected by the polished surfaces such as plane mirrors. In this lesson, you will study about other types of mirrors like the spherical mirrors and parabolic mirrors and their applications in our daily life. You will also study about the laws of reflection and the laws of refraction and some of the optical instruments, such as periscope and kaleidoscope, which work on these principles.

## Types of Mirrors

We use mirrors in our daily life for various purposes. We use them for decoration. In vehicles, they are used as rear view mirrors. They are also used in scientific apparatus, like telescope. The mirror is an optical device with a polished surface that reflects the light falling on it. A typical mirror is a glass sheet coated with aluminium or silver on one of its sides to produce an image. Mirrors have a plane or curved surface. Curved mirrors have surfaces that are spherical, cylindrical, parabolic and ellipsoid. The shape of a mirror determines the type of image it forms. Plane mirrors form the perfect image of an object. Whereas, curved mirrors produce images that are either enlarged or diminished. You would have studied about plane mirrors in your lower classes. In this section, you will study about spherical and parabolic mirrors.

**Do You Know?**

Method of coating a glass plate with a thin layer of reflecting metals was in practice during the 16th century in Venice, Italy. They used an amalgam of tin and mercury for this purpose. Nowadays, a thin layer of molten aluminium or silver is used for coating glass plates that will then become mirrors.

## Spherical mirrors

Spherical mirrors are one form of curved mirrors. If the curved mirror is a part of a sphere, then it is called a 'spherical mirror'. It resembles the shape of a piece cut out from a spherical surface. One side of this mirror is silvered and the reflection of light occurs at the other side.

**Concave mirrors**

A spherical mirror, in which the reflection of light occurs at its concave surface, is called a concave mirror. These mirrors magnify theobject placed close to them. The most common example of a concave mirror is the make-up mirror.

**Convex mirror**

A spherical mirror, in which the reflection of light occurs at its convex surface, is called a convex mirror. The image formed by these mirrors is smaller than the object. Most common convex mirrors are rear viewing mirrors used in vehicles.

**Do You Know?**

Convex mirrors used in vehicles as rear-view mirrors are labeled with the safety warning: 'Objects in the mirror are closer than they appear' to warn the drivers. This is because inside the mirrors, vehicles will appear to be coming at a long distance.

**Parabolic mirrors**

A parabolic mirror is one type of curved mirror, which is in the shape of a parabola. It has a concave reflecting surface and this surface directs the entire incident beam of light to converge at its focal point.

In the same way, light rays generated by the source placed at this focal point will fall on this surface and they will be diverged in a direction, which is parallel to the principal axis of the parabolic mirror. Hence, the light rays will be reflected to travel a long distance, without getting diminished.

Parabolic mirrors, also known as parabolic reflectors, are used to collect or project energy such as light, heat, sound and radio waves. They are used in reflecting telescopes, radio telescopes and parabolic microphones. They are also used in solar cookers and solar water heaters.

**Do You Know?**

The principle behind the working of a parabolic mirror has been known since the Greco-Roman times. The first mention of these structures was found in the book, 'On Burning Mirrors', written by the mathematician Diocles. They were also studied in the 10th century, by a physicist called IbnSahl. The first parabolic mirrors were constructed by Heinrich Hertz, a German physicist, in the form of reflector antennae in the year 1888.

**TERMS RELATED TO SPHERICAL MIRRORS**

In order to understand the image formation in spherical mirrors, you need to know about some of the terms related to them.

**Center of Curvature:** It is the center of the sphere from which the mirror is made. It is denoted by the letter C in the ray diagrams. (A ray diagram represents the formation of an image by the spherical mirror. You will study about them in your next class).

**Pole:** It is the geometric centre of the spherical mirror. It is denoted by the letter P. Radius of Curvature: It is the distance between the center of the sphere and the vertex. It is shown by the letter R in ray diagrams. (The vertex is the point on the mirror's surface where the principal axis meets the mirror. It is also called as 'pole'.)

**Principal Axis:** The line joining the pole of the mirror and its center of curvature is called principal axis.

**Focus:** When a beam of light is incident on a spherical mirror, the reflected rays converge (concave mirror) at or appear to diverge from (convex mirror) a point on the principal axis. This point is called the 'focus' or 'principal focus'. It is also known as the focal point. It is denoted by the letter F in ray diagrams.

**Focal length:** The distance between the pole and the principal focus is called focal length (f) of a spherical mirror. There is a relation between the focal length of a spherical mirror and its radius of curvature. The focal length is half of the radius of curvature.

That is, focal length= $\frac{Radiusofcurvature}{2}$

**PROBLEM 1**

The radius of curvature of a spherical mirror is 20cm. Find its focal length

**Solution:**

Radius of curvature =20cm

Focal length (f0 = $\frac{Radiusofcurvature}{2}$

= $\frac{R}{2}$ = $\frac{20}{2}$ =10cm

**PROBLEM 2**

Focal length of a spherical mirror is 7 cm. What is its radius of curvature?

**Solution:**

Focal length = 7 cm

Radius of curvature (R) = 2 × focal length = 2 × 7 = 14 cm

**IMAGES FORMED BY SPHERICAL MIRRORS**

Images formed by spherical mirrors are of two types: i) real image and ii) virtual image. Real images can be formed on a screen, while virtual images cannot be formed on a screen.

Image formed by a convex mirror is always erect, virtual and diminished in size. As a result, images formed by these mirrors cannot be projected on a screen.

The characteristics of an image are determined by the location of the object. As the object gets closer to a concave mirror, the image gets larger, until attaining approximately the size of the object, when it reaches the centre of curvature of the mirror. As the object moves away, the image diminishes in size and gets gradually closer to the focus, until it is reduced to a point at the focus when the object is at an infinite distance from the mirror.

The size and nature of the image formed by a convex mirror is given in Table 3.1.

Concave mirrors form a real image and it can be caught on a screen. Unlike convex mirrors, concave mirrors show different image types. Depending on the position of the object in front of the mirror, the position, size and nature of the image will vary. Table 3.2 provides a summary of images formed by a concave mirror.

**Table 3.1 Image formed by a convex mirror**

| POSITION OF THE OBJECT | POSITION OF THE IMAGE | IMAGE SIZE | NATURE OF THE IMAGE |
|---|---|---|---|
| At infinity | At F | Highly diminished, point sized | Virtual and erect |
| Between infinity the pole (P) | Between P and F | Diminished | Virtual erect |

**Table 3.2 Image formed by a concave mirror**

| POSITION OF THE OBJECT | POSITIOIN OF THE IMAGE | IMAGE SIZE | NATURE OF THE IMAGE |
|---|---|---|---|
| At infinity | At F | Highly diminished | Real and inverted |
| Beyond C | Between C and F | Diminished | Real and inverted |
| At C | At C | Same size as the object | Real and inverted |
| Between C and F | Beyond C | Magnified | Real and inverted |
| At F | At infinity | Highly magnified | Real and inverted |
| Between F and P | Behind the mirror | Magnified | Virtual and erect |

You can observe from the table that a concave mirror always forms a real and inverted image except when the object is placed between the focus and the pole of the mirror. In this position, it forms a virtual and erect image.

**Application of curved Mirrors**
**Concave mirrors**

1. Concave mirrors are used while applying make-up or shaving, as they provide a magnified image.
2. They are used in torches, search lights and head lights as they direct the light to a long distance.
3. They can collect the light from a larger area and focus it into a small spot. Hence, they are used in solar cookers.
4. They are used as head mirrors by doctors to examine the eye, ear and throat as they provide a shadow-free illumination of the organ.
5. They are also used in reflecting telescopes. Figure 3.3 Concave mirrors

**Convex mirrors**

1. Convex mirrors are used in vehicles as rear view mirrors because they give an upright image and provide a wider field of view as they are curved outwards.

2. They are found in the hallways of various buildings including hospitals, hotels, schools and stores. They are usually mounted on a wall or ceiling where hallways make sharp turns.

3. They are also used on roads where there are sharp curves and turns.

Not all the objects can produce the same effect as produced by the plane mirror. A ray oflight, falling on a body having a shiny, polished and smooth surface alone is bounced back. This bouncing back of the light rays as they fall on the smooth, shiny and polished surface is called reflection.

Reflection involves two rays: i) incident ray and ii) reflected ray. The incident ray is the light ray in a medium falling on the shiny surface of a reflecting body. After falling on the surface, this ray returns into the same medium. This ray is called the reflected ray. An imaginary line perpendicular to the reflecting surface, at the point of incidence of the light ray, is called the normal.

The relation between the incident ray, the reflected ray and the normal is given as the law of reflection. The laws of reflection are as follows:

- The incident ray, the reflected ray and the normal at the point of incidence, all lie in the same plane.
- The angle of incidence and the angle of reflection are always equal.

**Do You Know?**

Silver metal is the best reflector of light. That's why a thin layer of silver is deposited on the side of materials like plane glass sheets, to make mirrors.

**TYPES OF REFLECTION**

You have learnt that not all bodies can reflect light rays. Th e amount of reflection depends on the nature of the reflecting surface of a body. Based on the nature of the surface, reflection can be classified into two types namely, i) regular reflection and ii) irregular reflection.

**Regular reflection**

When a beam of light (collection of parallel rays) falls on a smooth surface, it gets reflected. After reflection, the reflected rays will be parallel to each other. Here, the angle of incidence and the angle of reflection of each ray will be equal. Hence, the law of reflection is obeyed in this case and thus a clear image is formed. This reflection is called 'regular reflection' or 'specular reflection'. Example: Reflection of light by a plane mirror and reflection of light from the surface of still water.

**Irregular reflection**

In the case of a body having a rough or irregular surface, each region of the surface is inclined at different angles. When light falls on such a surface, the light rays are reflected at different angles. In this case, the angle of incidence and the angle of reflection of each ray are not equal. Hence, the law of reflection is not obeyed in this case and thus the image is not clear. Such a reflection is called 'irregular reflection' or 'diff used reflection'. Example: Reflection of light from a wall.

**MULTIPLE REFLECTIONS**

You can see three images. How is it possible to have three images with two mirrors? In the activity given above, you observed that for a body kept in between two plane mirrors, which were inclined to each other, you could see many images. Th is is because, the 'image' formed by one mirror acts as an 'object' for the other mirror. Th e image formed by the first mirror acts as an object for the second mirror and the image formed by the second mirror acts as an object for the first mirror. Th us, we have three images of a single body. Th is is known as multiple reflection. This type of reflections can be seen in show rooms and saloons.

The number of images formed, depends on the angle of inclination of the mirrors. If the angle between the two mirrors is a factor of 360°, then the total number of reflections is finite. If θ (Theta) is the angle of inclination of the plane mirrors, the number of images formed = $\frac{360}{0}$-1. As you decrease this angle, the number of images formed increases. When they are parallel to each other, the number of images formed becomes infinite.

**Problem.3**
If two plane mirrors are inclined to each other at an angle of 90°, find the number of images formed.
**Solution:**
Angle of inclination = 90°
Number of images formed =
$\frac{360^0}{\theta} - \frac{360^o}{90^o} - 1 = - 1 = 4 - 1 = 3$

**Kaleidoscope**

It is a device, which functions on the principle of multiple reflection of light, to produce numerous patterns of images. It has two or more mirrors inclined with each other. It can be designed from inexpensive materials and the colourful image patterns formed by this will be pleasing to you. This instrument is used as a toy for children.

## Periscope

It is an instrument used for viewing bodies or ships, which are over and around another body or a submarine. It is based on the principle of the law of reflection of light. It consists of a long outer case and inside this case mirrors or prisms are kept at each end, inclined at an angle of 45°. Light coming from the distant body, falls on the mirror at the top end of the periscope and gets reflected vertically downward. This light is reflected again by the second mirror kept at the bottom, so as to travel horizontally and reach the eye of the observer. In some complex periscopes, opticfibre is used instead of mirrors for obtaining a higher resolution. Th e distance between the mirrors also varies depending on the purpose of using the periscope.

**Uses**

• It is used in warfare and navigation of the submarine.
• In military it is used for pointing and firing guns from a 'bunker'. • Photographs of important places can be taken through periscopes without trespassing restricted military regions.
• Fibre optic periscopes are used by doctors as endoscopes to view internal organs of the body.

## REFRACTION OF LIGHT

We know that when a light ray falls on a polished surface placed in air, it is reflected into the air itself. When it falls on a transparent material, it is not reflected completely, but a part of it is reflected and a part of it is absorbed and most of the light passes through it. Th rough air, light travels with a speed of $3 \times 10^8$ m s-1, but it cannot travel with the same speed in water or glass, because, optically denser medium such as water and glass offer some resistance to the light rays.

So, light rays travelling from a rarer medium like air into a denser medium like glass or water are deviated from their straight line path. Th is bending of light about the normal, at the point of incidence; as it passes from one transparent medium to another is called refraction of light.

When a light ray travels from the rarer medium into the denser medium, it bends towards the normal and when it travels from the denser medium into the rarer medium, it bends away from the normal. You can observe this phenomenon with the help of the activity given below.

In this activity, the light rays actually travel from the water (a denser medium) into the air (a rarer medium). As you saw earlier, when a light ray travels from a denser medium to a rarer medium, it is deviated from its straight line path. So, the pencil appears to be bent when you see it through the glass of water.

**Refractive Index**

Refraction of light in a medium depends on the speed of light in that medium. When the speed of light in a medium is more, the bending is less andwhen the speed of light is less, the bending is more.

The amount of refraction of light in a medium is denoted by a term known as refractive index of the medium, which is the ratio of the speed of light in the air to the speed of light in that particular medium. It is also known as the absolute refractive index and it is denoted by the Greek letter 'μ' (pronounced as 'mew').

$$\mu = \frac{Speedoflighinair(c)}{Speedoflight\ \mathrm{int}\ hemedium(v)}$$

Refractive index is a ratio of two similar quantities (speed) and so, it has no unit. Since, the speed of light in any medium is less than its speed in air, refractive index of any transparent medium is always greater than 1.
Refractive indices of some common substances are given in Table 3.3.

| **Substances** | **Refractive Index** |
|---|---|
| Air | 1.0 |
| Water | 1.33 |
| Ether | 1.36 |
| Kerosene | 1.41 |
| Ordinary Glass | 1.5 |
| Quartz | 1.56 |
| Diamond | 2.41 |

In general, the refractive index of one medium with respect to another medium is given by the ratio of their absolute refractive indices.

$$\mu_2 \frac{Absoluterefactiveindexofthe\ \mathrm{sec}\ ondmedium}{Absoluterefactiveindexofthefivemedum}$$

$_1\mu_2$ = [illegible] or $_1\mu_2$ = [illegible]

Thus, the refractive index of one medium with respect to another medium is also given by the ratio of the speed of light in first medium to its speed in the second medium.

**PROBLEM 4**
Speed of light in air is 3 × 108 m s-1 and the speed of light in a medium is 2 ×108 ms-1. Find the refractive index of the medium with respect to air.
**Solution:**
Refractive index (μ)= $\frac{Speedoflightinair(c)}{Speedoflight\ \mathrm{int}\ hemedum(v)}$

$$\mu = \frac{3X10^8}{2X10^8} = 1.5$$

**PROBLEM 5**

Refractive index of water is 4/3 and the refractive index of glass is 3/2. Find the refractive index of glass with respect to the refractive index of water.

**Solution:**

$${}_w\mu_g = \frac{\text{Refractive index of glass}}{\text{Refractive index of water}} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8} = 1.125$$

**Snell's Law of Refraction**

Refraction of light rays, as they travel from one medium to another medium, obeys two laws, which are known as Snell's laws of refraction. They are:

I) The incident ray, the refracted ray and the normal at the point of intersection, all lie in the same plane.

II) The ratio of the sine of the angle of incidence (i) to the sine of the angle of refraction (r) is equal to the refractive index of the medium, which is a constant.

**Figure 3.12 Snell's Law**

In the above activity, you can see that the first prism splits the white light into seven coloured light rays and the second prism recombines them into white light, again. Thus, it is clear that white light consists of seven colours. You can also recall the Newton's disc experiment, which you studied in VII standard.

Splitting of white light into its seven constituent colours (wavelength), on passing through a transparent medium is known as dispersion of light.

Why does dispersion occur? It is because, light of different colours present in white light have different wavelength and they travel at different speeds in a medium. You know that refraction of a light ray in a medium depends on its speed. As each coloured light has a different speed, the constituent coloured lights are refracted at different extents, inside the prism. Moreover, refraction of a light ray is inversely proportional to its wavelength.

Thus, the red coloured light, which has a large wavelength, is deviated less while the violet coloured light, which has a short wavelength, is deviated more.

**9 th book**

**UNIT- 6 –LIGHT**

**Introduction**

Light is a form of energy which travels as electromagnetic waves. The branch of physics that deals with the properties and applications of light is called optics. In our day to day life we use number of optical instruments. Microscopes are inevitable in science laboratories. Telescopes, binoculars, cameras and projectors are used in educational, scientific and entertainment fields. In this lesson, you will learn about spherical mirrors (concave and convex). Also, you will learn about the properties of light, namely reflection and refraction and their applications.

**Reflection of Light**

Light falling on any polished surface such as a mirror, is reflected. This reflection of light on polished surfaces follows certain laws and you have studied about them in your lower classes. Let us study about them little elaborately here.

**Laws of reflection**

Consider a plane mirror MM′ as shown in Figure 6.1. Let AO be the light ray incident on the plane mirror at O. The ray AO is called incident ray. The plane mirror reflects the incident ray along OB. The ray OB is called reflected ray. Draw a line ON at O perpendicular to MM′. This line ON is called normal.

The angle made by the incident ray with the normal (i = angle AON) is called angle of incidence. The reflected ray OB makes an angle (r = angle NOB) with the normal and this is called angle of reflection. From the figure you can observe that the angle of incidence is equal to the angle of reflection. i.e., i = r. Also, the incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane. These are called the laws of reflection. Laws of reflection are given as: The incident ray, the reflected ray and the normal at the point of incidence, all lie in the same plane.

The angle of incidence is equal to angle of reflection.

| |
|---|
| The most common usage of mirror writing can be found on the front of ambulances, where the word "AMBULANCE" is often written in very large mirrored text. |

**Lateral inversion**

You might have heard about inversion. But what is lateral inversion? The word lateral comes from the Latin word latus which means side. Lateral inversion means sidewise inversion. It is the apparent inversion of left and right that occurs in a plane mirror. Why do plane mirrors reverse left and right, but they do not reverse up and down? Well, the answer is surprising. Mirrors do not actually reverse left and right and they do not reverse up and down also. What actually mirrors do is reverse inside out. Look at the image below (Figure

6.2) and observe the arrows, which indicate the light ray from the object falling on the mirror. The arrow from the object's head is directed towards the top of the mirror and the arrow from the feet is directed towards the bottom. The arrow from left hand goes to the left side of the mirror and the arrow from the right hand goes to the right side of the mirror. Here, you can see that there is no switching. It is an optical illusion. Thus, the apparent lateral inversion we observe is not caused by the mirror but the result of our perception.

**Real and Virtual Image**

If the light rays coming from an object actually meet, after reflection, the image formed will be a real image and it is always inverted. A real image can be produced on a screen. When the light rays coming from an object do not actually meet, but appear to meet when produced backwards, that image will be virtual image. The virtual image is always erect and cannot be caught on a screen (Figure
binoculars, cameras and projectors are used in educational, scientifi c and entertainment fields.

In this lesson, you will learn about spherical mirrors (concave and convex). Also, you will learn about the properties of light, namely reflection and refraction and their applications.

**Curved Mirrors**

We studied about laws of reflection. These laws are applicable to all types of reflecting surfaces including curved surfaces. Let us learn about image formation in curved surfaces in this part.

In your earlier classes, you have studied that there are many types of curved mirrors, such as spherical and parabolic mirrors. The most commonly used type of curved mirror is spherical mirror. The curved surfaces of a shining spoon could also be considered as a curved mirror.
Take a hemispherical spoon. It has an inner and outer surface like the inside and outside of the ball. See your face on these surfaces? How do they look?

Move the spoon slowly away from your face. Observe the image. How does it change? Reverse the spoon and repeat the activity. How does the image look like now?

**Spherical mirrors**

In curved mirrors, the reflecting surface can be considered to form a part of the surface of a sphere. Such mirrors whose reflecting surfaces are spherical are called spherical mirrors.

In some spherical mirrors the reflecting surface is curved inwards, that is, it faces towards the centre of the sphere. It is called concave mirror. In some other mirrors, the reflecting surface is curved outward. It is called convex mirror and are shown in Figure 2.

In order to understand reflection of light at curved surfaces, we need to know the following.

**Centre of curvature (C):** The centre of the hollow sphere of which the spherical mirror forms a part.

**Pole (P):** The geometrical centre of the spherical mirror.

**Principal axis (PC):** The perpendicular line joining the pole and the centre of curvature of the mirror.

**Radius of curvature(R):** The distance between the pole and the centre of curvature of the spherical mirror.

**Principal focus (F):** The point on the principal axis of the spherical mirror where the rays of light parallel to the principal axis meet or appear to meet after reflection from the spherical mirror.

**Focal length(f):** The distance between the pole and the principal focus.
Radius of curvature and focal length are related to each other by the formula: R=2f. All these are depicted in Figure 3.

Figure 3 Concave mirror

**Image Formed by Curved Mirrors**

Figure 4 Sunlight focused on a concave mirror

We have seen that the parallel rays of sun light (Figure 4) could be focused at a point using a concave mirror. Now let us place a lighted candle and a white screen in front of the concave mirror. Adjust the position of the screen. Move the screen front and back. Note the size of the image and its shape. Is it inverted? Is it small?

Next, slowly bring the candle closer to the mirror. What do you observe? As you bring the object closer to the mirror the image becomes bigger. Try to locate the image when you bring the candle very close to the mirror. Are you able to see an image on the screen? Now look inside the mirror. What do you see? An erect magnified image of the candle is seen. In some positions of the object an image is obtained on the screen. However at some position of the object no image is obtained. It is clear that the behaviour of the concave mirror is much more complicated than the plane mirror.

However, with the use of geometrical technique we can simplify and understand the behaviour of the image formed by a concave mirror. In the earlier case of plane mirror, we used only two rays to understand how to get full image of a person. But for understanding the nature of image formedby a concave mirror we need to look at four specific rules.

**Rules for the construction of image formed by spherical mirrors**

From each point of an object, number of rays travel in all directions. To find the position and nature of the image formed by a concave mirror, we need to know the following rules.

Rule 1: A ray passing through the centre of curvature is reflected back along its own path (Figure 5).

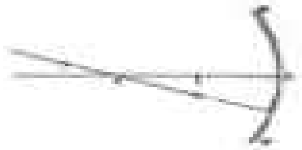


Figure 5 Ray passing centre of curvature

Rule 2: A ray parallel to the principal axis passes through the principal focus after reflection (Figure 6).

Figure 6 Ray parallel to principal axis

Rule 3: A ray passing through the focus gets reflected and travels parallel to the principal axis (Figure 7).

Figure 7 Ray travelling through the principal focus

Rule 4: A ray incident at the pole of the mirror gets reflected along a path such that the angle of incidence (APC) is equal to the angle of reflection (BPC) (Figure 8).

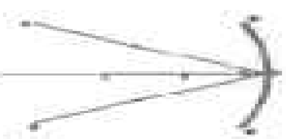


Figure 8 Angle of incidence equal to angle of reflection

**Concave Mirror**

## Ray diagrams for the formation of images

We shall now find the position, size and nature of image by drawing the ray diagram for a small linear object placed on the principal axis of a concave mirror at different positions.

**Case-I:** When the object is far away (at infinity), the rays of light reaching the concave mirror are parallel to each other (Figure 10).

Figure 10 Object at infinity

Position of the Image: The image is at the principal focus F.
Nature of the Image: It is (i) real, (ii) inverted and (iii) highly diminished in size.

**Case-II:** When the object is beyond the centre of curvature (Figure 11).
**Position of the image:** Between the principal focus F and centre of curvature C.

Figure 11 Object beyond the centre of curvature

**Nature of the image:** Real, inverted and smaller than object.
**Case – III:** When the object is at the centre of curvature (Figure 12).
**Position of the image:** The image is at the centre of curvature itself.
**Nature of the image:** It is i) Real, ii) inverted and iii) same size as the object.

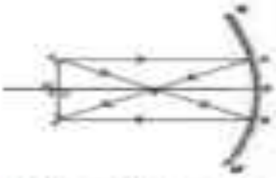


Figure 12 Object at the centre of curvature

**Case – IV:** When the object is in between the centre of curvature C and principal focus F (Figure 13).
**Position of the image:** The image is beyond C

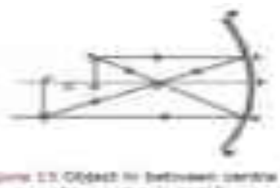


Figure 13 Object in between centre of curvature and principal focus

**Nature of the image:** It is i)Real ii) inverted and iii) magnified.

**Case – V:**When the object is at the principal focus F (Figure 14).
**Position of the image:** Theoretically, the image is at infinity.
**Nature of the image:** No image can be captured on a screen nor any virtual image can be seen.

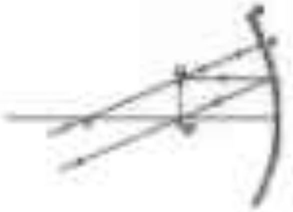
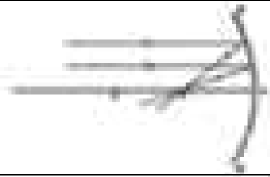
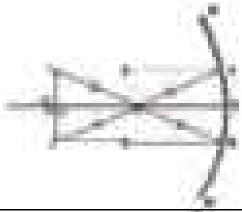
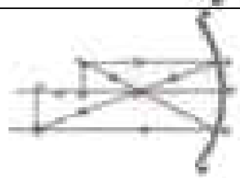
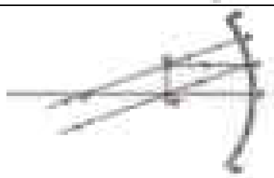
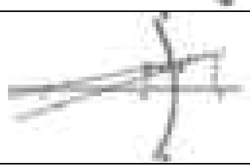


Figure 14 Object at principal focus

**Case – VI:** When the object is in between the focus F and the pole P (Figure 15). Position of the image: The image is behind the mirror.
Nature of the image: It is virtual,erect and magnified.

| S. No | Position of Object | Ray Diagram | Position of Image | Size of Image | Nature of Image |
|---|---|---|---|---|---|
| 1. | At infinity |  | At the principal focus | Point size | Real and Inverted |
| 2. | Beyond the Centre of Curvature C |  | Between F and C | Smaller than the object | Real and Inverted |
| 3. | At the Centre of Curvature C |  | A to C | Same size | Real and Inverted |
| 4. | Between C and F |  | Beyond C | Magnified | Real and inverted |
| 5. | At the principal focus F |  | At infinity | Infinitely large | Real and Inverted |
| 6. | Between the principal focus F and |  | Behind the mirror | Magnified | Virtual and Erect |

| | | | | | |
|---|---|---|---|---|---|
| | the pole P of the mirror | | | | |

**Sign convention for measurement of distances**

We follow a set of sign conventions called the cartesian sign convention. In this convention the pole (P) of the mirror is taken as the origin. The principal axis is taken as the x axis of the coordinate system (Figure 16).The object is always placed on the left side of the mirror. All distances are measured from the pole of the mirror.

Figure 16 Sign convention for spherical mirrors

- Distances measured in the direction of incident light are taken as positive and those measured in the opposite direction are taken as negative.
- All distances measured perpendicular to and above the principal axis are considered to be positive.
- All distances measured perpendicular to and below the principal axis are considered to be negative.

| Type of mirror | u | v | | f | R | Height of the subject | Height of the image | |
|---|---|---|---|---|---|---|---|---|
| | | real | virtual | | | | real | virtual |
| Concave mirror | - | - | + | - | - | | - | + |
| Convex mirror | - | No real image | + | + | + | | No real image | + |

**Mirror equation**

The expression relating the distance of the object u, distance of image v and focal length f of a spherical mirror is called the mirror equation. It is given as:

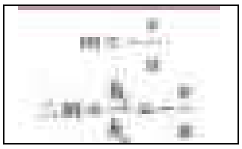
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

**Linear magnification (m)**

Magnification produced by a spherical mirror gives the how many times the image of an object is magnified with respect to the object size.
It can be defined as the ratio of the height of the image (//.) to the height of the object (h).

$$m = \frac{h_i}{h_o}$$

The magnification can be related to object distance (u) and the image distance (v)



Note: A negative sign in the value of magnification indicates that the image is real. A positive sign in the value of magnification indicates that the virtual image.

**Uses of concave mirror**

**Dentist's head mirror:**

In dentist's head mirror, a parallel beam of light is made to fall on the concave mirror. This mirror focuses the light beam on a small area of the body (such as teeth, throat etc.).

**Make-up mirror**:

When a concave mirror is held near the face, an upright and magnified image is seen. Here, our face will be seen magnified.

**Other applications**:

Concave mirrors are also used as reflectors in torches, head lights in vehicles and search lights to get powerful beams of light. Large concave mirrors are used in solar heaters.

Stellar objects are at an infinite distance. Therefore, the image formed by a concave mirror would be diminished, and inverted. Yet, astronomical telescopes use concave mirrors

**Convex Mirror**
**Image Formation**

Any two rays can be chosen to draw the position of the image in a convex mirror (Figure 6.10): a ray that is parallel to the principal axis (rule 1) and a ray that appears to pass through the centre of curvature (rule 2).

**Note**: All rays behind the convex mirror shall be shown with dotted lines.

The ray OA parallel to the principal axis is reflected along AD. The ray OB retraces its path. The two reflected rays diverge but they appear to intersect at I when produced backwards. Thus II' is the image of the object OO'. It is virtual, erect and smaller than the object.

**Uses of convex mirrors**

Convex mirrors are used as rear-view mirrors in vehicles. It always forms a virtual, erect, small-sized image of the object. As the vehicles approach the driver from behind, the size of the image increases. When the vehicles are moving away from the driver, then image size decreases. A convex mirror provides a much wider field of view (it is the observable area as seen through eye / any optical device such as mirror) compared to plane mirror. Convex mirrors are installed on public roads as traffic safety device. They are used in acute bends of narrow roads such as hairpin bends in mountain passes where direct view ofoncoming vehicles is restricted. It is also used in blind spots in shops.

In the rear view mirror, the following sentence is written. “Objects in the mirror are closer than they appear”. Why?

**Speed of light**

In early seventeenth century the Italian scientist Galileo Galilee (1564–1642) tried to measure the speed of light as it travelled from a lantern on a hill top about a mile (1.6 km) away from where he stood. His attempt was bound to fail, because he had no accurate clocks or timing instruments.

In 1665 the Danish astronomer Ole Roemer first estimated the speed of light by observing one of the twelve moons of the planet Jupiter. As these moons travel around the planet, at a set speed, it would take 42 hours to revolve around Jupiter. Roemer made a time schedule of the eclipses for the whole year. He made first observation in June and second observation in December. Roemer estimated the speed of light to be about 220,000 km per second.

In 1849 the first land based estimate was made by Armand Fizeau. Today the speed of light in vacuum is known to be almost exactly 300,000 km per second.

**Refraction of light**

This activity explains the refraction of light. The bending of light rays when they pass obliquely from one medium to another medium is called refraction of light.

**Cause of refraction**

Light rays get deviated from their original path while entering from one transparentmedium to another medium of different optical density. This deviation (change in direction) in the path of light is due to the change in velocity of light in the different medium. The velocity of light depends on the nature of the medium in which it travels. Velocity of light in a rarer medium (low optical density) is more than in a denser medium (high optical density).

**Refraction of light from a plane transparent surface**

When a ray of light travels from optically rarer medium to optically denser medium, it bends towards the normal. (Figure 22)

When a ray of light travels from an optically denser medium to an optically rarer medium it bends away from the normal. (Figure 23)

A ray of light incident normally on a denser medium goes without any deviation

**The laws of refraction of light**

The incident ray, the refracted ray and the normal to the interface of two transparent media at the point of incidence, all lie in the same plane.

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a light of a given colour and for the given pair of media. This law is also known as Snell's law of refraction.

If i is the angle of incidence and r is the angle of refraction, then

$$\frac{\sin i}{\sin r} = \text{constant}$$

This constant is called the refractive index of the second medium with respect to the first medium. It is generally represented by the Greek letter, $_1\mu_2$ (mew)

Note: The refractive index has no unit as it is the ratio of two similar quantities

**Verification of laws of refraction**

Figure 23: Verification of laws of refraction

**Speed of light in different media**

Light has the maximum speed in vacuum and it travels with different speeds in different media. The speed of light in some media is given below.

Note: The refractive index of a medium is also defined in terms of speed of light in different media

$$\mu = \frac{\text{speed of light in vacuum in air}(c)}{\text{speed of light in the vacuum (v)}}$$

in general, $1\mu2 = \frac{\text{speed of light in medium 1}}{\text{speed of light in medium 2}}$

**Total internal reflection**

When light travels from denser medium into a rarer medium, it gets refracted away from the normal. While the angle of incidence in the denser medium increases the angle of refraction also increases and it reaches a maximum value of r = 90° for a particular value.

This angle of incidence is called critical angle (Figure 6.12). The angle of incidence at which the angle of refraction is 90º is called the critical angle. At this angle, the refracted ray grazes the surface of separation between the two media.

When the angle of incidence exceeds the value of critical angle, the refracted ray is not possible. Since r > 90º the ray is totally reflected back to the same medium. This is called as total internal reflection.

**Conditions to achieve total internal reflection**

In order to achieve total internal refelection the following conditions must be met.

- Light must travel from denser medium to rarer medium. (Example: From water to air).
- The angle of incidence inside the denser medium must be greater than that of the critical angle.

**Total internal reflection in nature**

**Mirage**:

On hot summer days, patch of water may be on the road. This is an illusion. In summer, the air near the ground becomes hotter than the air at higher levels. Hotter air is less dense, and has smaller refractive index than the cooler air. Thus, a ray of light bends away from the normal and undergoes total internal reflection. Total internal reflection is the main cause for the spectacular brilliance of diamonds and twinkling of stars.

**Optical fibres**:

Optical fibres are bundles of high-quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding. Optical fibres work on the phenomenon of total internal reflection. When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflection along the length of the fibre and finally comes out at the other end. Optical fibres are extensively used for transmitting audio and video signals through long distances. Moreover, due to their flexible nature, optical fibers enable physicians to look and work inside the body through tiny incisions without having to perform surgery.

| An Indian-born physicist NarinderKapany is regarded as the Father of Fibre Optics. |
|---|

**10th Standard**
**Unit 2: Optics**

## INTRODUCTION

Light is a form of energy which travels in the form of waves. Th e path of light is called ray of light and group of these rays are called as beam of light. Any object which gives out light are termed as source of light. Some of the sources emit their own light and they are called as luminous objects. All the stars, including the Sun, are examples for luminous objects. We all know that we are able to see objects with the help of our eyes. But, we cannot see any object in a dark room. Can you explain why? If your answer is 'we need light to see objects', the next question is 'if you make the light from a torch to fall on your eyes, will you be able to see the objects?'Definitely, 'NO'. We can see the objects only when the light is made to fall on the objects and the light reflected from the objects is viewed by our eyes. You would have studied about the reflection and refraction of light elaborately in your previous classes. In this chapter, we shall discuss about the scattering of light, images formed by convex and concave lenses, human eye and optical instruments such as telescopes and microscopes.

## PROPERTIES OF LIGHT

Let us recall the properties of light and the important aspects on refraction of light.

- Light is a form of energy.
- Light always travels along a straight line.
- Light does not need any medium for its propagation. It can even travel through vacuum.
- The speed of light in vacuum or air is, $c = 3 \times 10^8 ms^{-1}$.
- Since, light is in the form of waves, it is characterized by a wavelength (λ) and a frequency (ν), which are related by the following equation: $c = \nu \lambda$ (c - velocity of light).
- Different coloured light has different wavelength and frequency.
- Among the visible light, violet light has the lowest wavelength and red light has the highest wavelength.
- When light is incident on the interface between two media, it is partly reflected and partly refracted.

## REFRACTION OF LIGHT

When a ray of light travels from one transparent medium into another obliquely, the path of the light undergoes deviation. This deviation of ray of light is called refraction. Refraction takes place due to the difference in the velocity of light in different media. The velocity of light is more in a rarer medium and less in a denser medium. Refraction of light obeys two laws of refraction.

**First law of refraction:**

The incident ray, the refracted ray of light and the normal to the refracting surface all lie in the same plane.

**Second law of refraction:**

The ratio of the sine of the angle of incidence and sine of the angle of refraction is equal to the ratio of refractive indices of the two media. This law is also known as Snell's law.

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{------------} \quad (2.1)$$

- Refractive index gives us an idea of how fast or how slow light travels in a medium. The ratio of speed of light in vacuum to the speed of light in a medium is defined as refractive index 'μ' of that medium.

- The speed of light in a medium is low if the refractive index of the medium is high and vice versa.

- When light travels from a denser medium into a rarer medium, the refracted ray is bent away from the normal drawn to the interface.

- When light travels from a rarer medium into a denser medium, the refracted ray is bent towards the normal drawn to the interface.

## REFRACTION OF A COMPOSITE LIGHT-DISPERSION OF LIGHT

We know that Sun is the fundamental and natural source of light. If a source of light produces a light of single colour, it is known as a monochromatic source. On the other hand, a composite source of light produces a white light which contains light of different colours. Sun light is a composite light which consists of light of various colours or wavelengths. Another example for a composite source is a mercury vapour lamp. What do you observe when a white light is refracted through a glass prism?

When a beam of white light or composite light is refracted through any transparent media such as glass or water, it is split into its component colours. This phenomenon is called as 'dispersion of light'.

The band of colours is termed as spectrum. This spectrum consists of following colours: Violet, Indigo, Blue, Green, Yellow, Orange, and Red. These colours are represented by the acronym "VIBGYOR". Why do we get the spectrum when white light is refracted by a transparent medium? This is because, different coloured lights are bent through different angles. That is the angle of refraction is different for different colours.

Angle of refraction is the smallest for red and the highest for violet. From Snell's law, we know that the angle of refraction is determined in terms of the refractive index of the medium. Hence, the refractive index of the medium is different for different coloured lights. This indicates that the refractive index of a medium is dependent on the wavelength of the light.

**SCATTERING OF LIGHT**

When sunlight enters the Earth's atmosphere, the atoms and molecules of different gases present in the atmosphere refract the light in all possible directions. This is called as 'Scattering of light'. In this phenomenon, the beam of light is redirected in all directions when it interacts with a particle of medium. The interacting particle of the medium is called as 'scatterer'.

**Types of scattering**

When a beam of light, interacts with a constituent particle of the medium, it undergoes many kinds of scattering. Based on initial and final energy of the light beam, scattering can be classified as,

**Elastic scattering**

- If the energy of the incident beam of light and the scattered beam of light are same, then it is called as 'elastic scattering'.

**Inelastic scattering**

- If the energy of the incident beam of light and the scattered beam of light are not same, then it is called as 'inelastic scattering'. The nature and size of the scatterer results in different types of scattering. They are

1) Rayleigh scattering
2) Mie scattering
3) Tyndall scattering
4) Raman scattering

**Rayleigh scattering**

The scattering of sunlight by the atoms or molecules of the gases in the earth's atmosphere is known as Rayleigh scattering.

**Rayleigh's scattering law**

Rayleigh's scattering law states that, "The amount of scattering of light is inversely proportional to the fourth power of its wavelength".

$$\text{Amount of scattering 'S'} \propto \frac{1}{\lambda^4}$$

According to this law, the shorter wavelength colours are scattered much more than the longer wavelength colours.

When sunlight passes through the atmosphere, the blue colour (shorter wavelength) is scattered to a greater extent than the red colour (longer wavelength). This scattering causes the sky to appear in blue colour.

At sunrise and sunset, the light rays from the Sun have to travel a larger distance in the atmosphere than at noon. Hence, most of the blue lights are scattered away and only the red light which gets least scattered reaches us. Therefore, the colour of the Sun is red at sunrise and sunset.

**Mie scattering**

Mie scattering takes place when the diameter of the scatterer is similar to or larger than the wavelength of the incident light. It is also an elastic scattering. The amount of scattering is independent of wave length. Mie scattering is caused by pollen, dust, smoke, water droplets, and other particles in the lower portion of the atmosphere.

Mie scattering is responsible for the white appearance of the clouds. When white light falls on the water drop, all the colours are equally scattered which together form the white light.

**Tyndall Scattering**

When a beam of sunlight, enters into a dusty room through a window, then its path becomes visible to us. This is because, the tiny dust particles present in the air of the room scatter the beam of light. This is an example of Tyndall Scattering
The scattering of light rays by the colloidal particles in the colloidal solution is called Tyndall Scattering or Tyndall Effect.

**Raman scattering**

When a parallel beam of monochromatic (single coloured ) light passes through a gas or liquid or transparent solid, a part of light rays are scattered.

The scattered light contains some additional frequencies (or wavelengths) other than that of incident frequency (or wavelength). This is known as Raman scattering or Raman Effect.

Raman Scattering is defined as **"The interaction of light ray with the particles of pure liquids or transparent solids, which leads to a change in wavelength or frequency."**

The spectral lines having frequency equal to the incident ray frequency is called 'Rayleigh line' and the spectral lines which are having frequencies other than the incident ray frequency are called 'Raman lines'. The lines having frequencies lower than the incident frequency is called stokes lines and the lines having frequencies higher than the incident frequency are called Antistokes lines.

You will study more about Raman Effect in higher classes.

**LENSES**

A lens is an optically transparent medium bounded by two spherical refracting surfaces or one plane and one spherical surface.

Lens is basically classified into two types. They are: (i) Convex Lens (ii) Concave Lens

- **Convex or bi-convex lens:** It is a lens bounded by two spherical surfaces such that it is thicker at the centre than at the edges. A beam of light passing through it, is converged to a point. So, a convex lens is also called as converging lens.

- **(ii) Concave or bi-concave Lens:** It is a lens bounded by two spherical surfaces such that it is thinner at the centre than at the edges. A parallel beam of light passing through it, is diverged or spread out. So, a concave lens is also called as diverging lens.

**Other types of Lenses**

- **Plano-convex lens:** If one of the faces of a bi-convex lens is plane, it is known as a plano-convex lens.
- **Plano-concave lens:** If one of the faces of a bi-concave lens is plane, it is known as a plano-concave lens.

All these lenses are shown in Figure 2.2 given below:

## IMAGES FORMED DUE TO REFRACTION THROUGH A CONVEX AND CONCAVE LENS

When an object is placed in front of a lens, the light rays from the object fall on the lens. Th e position, size and nature of the image formed can be understood only if we know certain basic rules.

**Rule-1:** When a ray of light strikes the convex or concave lens obliquely at its optical centre, it continues to follow its path without any deviation (Figure 2.3).

**Rule-2:** When rays parallel to the principal axis strikes a convex or concave lens, the refracted rays are converged to (convex lens) or appear to diverge from (concave lens) the principal focus (Figure 2.4).

**Rule-3:** When a ray passing through (convex lens) or directed towards (concave lens) the principal focus strikes a convex or concave lens, the refracted ray will be parallel to the principal axis (Figure 2.5).

## REFRACTION THROUGH A CONVEX LENS

Let us discuss the formation of images by a convex lens when the object is placed at various positions.

### Object at infinity

When an object is placed at infinity, a real image is formed at the principal focus. Th e size of the image is much smaller than that of the object (Figure 2.6).

### Object placed beyond C (>2F)

When an object is placed behind the center of curvature(beyond C), a real and inverted image is formed between the center of curvature and the principal focus. Th e size of the image is the same as that of the object (Figure 2.7).

### Object placed at C

When an object is placed at the center of curvature, a real and inverted image is formed at the other center of curvature. Th e size of the image is the same as that of the object (Figure 2.8).

### Object placed between F and C

When an object is placed in between the center of curvature and principal focus, a real and inverted image is formed behind the center of curvature. The size of the image is bigger than that of the

**Object placed at the principal focus F**

When an object is placed at the focus, a real image is formed at infinity. The size of the image is much larger than that of the object (Figure 2.10).

**Object placed between the principal focus F and optical centre O**

When an object is placed in between principal focus and optical centre, a virtual image is formed. The size of the image is larger than that of the object (Figure 2.11).

**APPLICATIONS OF CONVEX LENSES**

- Convex lenses are used as camera lenses
- They are used as magnifying lenses
- They are used in making microscope, telescope and slide projectors
- They are used to correct the defect of vision called hypermetropia

**REFRACTION THROUGH A CONCAVE LENS**

Let us discuss the formation of images by a concave lens when the object is placed at two possible positions.

**Object at Infinity**

When an object is placed at infinity, a virtual image is formed at the focus. The size of the image is much smaller than that of the object (Figure 2.12).

Figure 2.12 Concave lens-Object at infinity

**Object anywhere on the principal axis at a finite distance**

When an object is placed at a finite distance from the lens, a virtual image is formed between optical center and focus of the concave lens. The size of the image is smaller than that of the object (Figure 2.13).

But, as the distance between the object and the lens is decreased, the distance between the image and the lens also keeps decreasing. Further, the size of the image formed increases as the distance between the object and the lens is decreased. This is shown in (figure 2.14).

**APPLICATIONS OF CONCAVE LENSES**

- Concave lenses are used as eye lens of 'Galilean Telescope'
- They are used in wide angle spy hole in doors.

❖ They are are used to correct the defect of vision called 'myopia'

**LENS FORMULA**

Like spherical mirrors, we have lens formula for spherical lenses. The lens formula gives the relationship among distance of the object (u), distance of the image (v) and the focal length (f ) of the lens. It is expressed as

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \ldots\ldots\ldots\ldots 2.2$$

It is applicable to both convex and concave lenses. We need to give an at most care while solving numerical problems related to lenses in taking proper signs of different quantities.

**SIGN CONVENTION**

Cartesian sign conventions are used for measuring the various distances in the ray diagrams of spherical lenses. According to cartesian sign convention,

- ❖ The object is always placed on the left side of the lens.
- ❖ All the distances are measured from the optical centre of the lens.
- ❖ The distances measured in the same direction as that of incident light are taken as positive.
- ❖ The distances measured against the direction of incident light are taken as negative.
- ❖ The distances measured upward and perpendicular to the principal axis is taken as positive.
- ❖ The distances measured downward and perpendicular to the principal axis is taken as negative.

**MAGNIFICATION OF A LENS**

Like spherical mirrors, we have magnification for spherical lenses. Spherical lenses produce magnification and it is defined as the ratio of the height of the image to theheight of an object. Magnification is denoted by the letter 'm'. If height of the object is *h* and height of the image is *h ´*, the magnification produced by lens is,

$$m = \frac{\text{height of the image}}{\text{height of the object}} = \frac{h'}{h} \quad \ldots\ldots (2.3)$$

Also it is related to the distance of the object (*u*) and the distance of the image (*v*) as follows:

$$m = \frac{\text{Distance of the image}}{\text{Distance of the object}} = \frac{v}{u} \quad \ldots\ldots (2.4)$$

If the magnification is greater than 1, then we get an enlarged image. On the other hand, if the magnification is less than 1, then we get a diminished image.

**LENS MAKER'S FORMULA**

All lenses are made up of transparent materials. Any optically transparent material will have a refractive index. The lens formula relates the focal length of a lens with the distance of object and image. For a maker of any lens, knowledge of radii of curvature of the lens is required. This clearly indicates the need for an equation relating the radii of curvature of the lens, the refractive index of the given material of the lens and the required focal length of the lens. The lens maker's formula is one such equation. It is given as

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \cdots\cdots\cdots (2.5)$$

where$\mu$ is the refractive index of the material of the lens; $R$1 and $R$2 are the radii of curvature of the two faces of the lens; f is the focal length of the lens.

**POWER OF A LENS**

When a ray of light falls on a lens, the ability to converge or diverge these light rays depends on the focal length of the lens. This ability of a lens to converge (convex lens) or diverge (concave lens) is called as its power. Hence, the power of a lens can be defined as the degree of convergence or divergence of light rays. Power of a lens is numerically defined as the reciprocal of its focal length.

$$P = \frac{1}{f} \cdots\cdots\cdots\cdots\cdots (2.6)$$

The SI unit of power of a lens is dioptre. It is represented by the symbol D. If focal length is expressed in 'm', then the power of lens is expressed in 'D'. Thus 1D is the power of

**Table 2.1 Differences between a Convex Lens and a Concave Lens**

| S.No | Convex Lens | Concave Lens |
|---|---|---|
| 1 | A convex lens is thicker in the middle than at edges. | A concave lens is thinner in the middle than at edges. |
| 2 | It is a converging lens. | It is a diverging lens. |
| 3 | It produces mostly real images. | It produces virtual images. |
| 4 | It is used to treat hypermeteropia. | It is used to treat myopia. |

11th vol II

# WAVES

## INTRODUCTION

In the previous chapter, we have discussed the oscillation of a particle. Consider a medium which consists of a collection of particles. If the disturbance is created at one end, itpropagates and reaches the other end. That is, the disturbance produced at the first mass point is transmitted to the next neighbouring mass point, and so on. Notice that here, only the disturbance is transmitted, not the mass points. Similarly, the speech we deliver is due to the vibration of our vocalchord inside the throat. This leads to the vibration of the surrounding air molecules and hence, the effect of speech (information) is transmitted from one point in space to another point in space without the medium carrying the particles. Thus, the disturbance which carries energy and momentum from one point in space to another point in space without the transfer of the medium is known as a wave.

Standing near a beach, one can observe tides in the ocean reaching the seashore with a similar wave pattern; hence they are called ocean waves. A rubber band when plucked vibrates like a wave which is an example of a standing wave. These are shown in Figure 11.2. Other examples of waves are light waves (electromagnetic waves), through which we observe and enjoy the beauty of nature and sound waves using which we hear and enjoy pleasant melodious songs. Day to day applications of waves are numerous, as in mobile phone communication, laser surgery, etc.

### Ripples and wave formation on the water surface

Suppose we drop a stone in a trough of still water, we can see a disturbance produced at the place where the stone strikes the water surface as shown in Figure 11.3. We find that this disturbance spreads out (diverges out) in the form of concentric circles of ever increasing radii (ripples) and strike the boundary of the trough. This is because some of the kinetic energy of the stone is transmitted to the water molecules on the surface. Actually the particles of the water (medium) themselves do not move outward with the disturbance. This can be observed by keeping a paper strip on the water surface. The strip moves up and down when the disturbance (wave) passes on the water surface. This shows that the water molecules only undergo vibratory motion about their mean positions.

### Formation of waves on stretched string

Let us take a long string and tie one end of the string to the wall as shown in Figure 11.4 (a). If we give a quick jerk, a bump (like pulse) is produced in the string as shown in Figure 11.4 (b). Such a disturbance is sudden and it lasts for a short duration, hence it is known as a wave pulse. If jerks are given continuously then the waves produced are standing waves. Similar waves are produced by a plucked string in a guitar.

### Formation of waves in a tuning fork

When we strike a tuning fork on a rubber pad, the prongs of the tuning fork vibrate about their mean positions. The prong vibrating about a mean position means moving outward and inward, as indicated in the Figure 11.5. When a prong moves outward, it pushes the layer of air in its neighbourhood which means there is more accumulation of air molecules in this region. Hence, the density and also the pressure increase. These regions are known as compressed regions or compressions. This compressed air layer moves forward and compresses the next neighbouring layer in a similar manner. Thus a wave of compression advances or passes through air. When the prong moves inwards, the particles of the medium are moved to the right. In this region both density and pressure are low. It is known as a rarefaction or elongation.

### Characteristics of wave motion

- For the propagation of the waves, the medium must possess both inertia and elasticity, which decide the velocity of the wave in that medium.
- In a given medium, the velocity of a wave is a constant whereas the constituent particles in that medium move with different velocities at different positions. Velocity is maximum at their mean position and zero at extreme positions.
- Waves undergo reflections, refraction, interference, diffraction and polarization.

| Point to ponder | |
|---|---|
| 1. | The medium possesses both inertia and elasticity for propagation of waves. |
| 2. | Light is an electromagnetic wave. what is the medium for its transmission? |

### Mechanical wave motion and its types

Wave motion can be classified into two types

a. **Mechanical wave** – Waves which require a medium for propagation are known as mechanical waves.
   **Examples:** sound waves, ripples formed on the surface of water, etc.
b. **Non mechanical wave** – Waves which do not require any medium for propagation are known as non-mechanical waves.
   **Example:** light

Further, waves can be classified into two types

a. Transverse waves
b. Longitudinal waves

**Transverse wave motion**

In transverse wave motion, the constituents of the medium oscillate or vibrate about their mean positions in a direction perpendicular to the direction of propagation (direction of energy transfer) of waves.

**Example:** light (electromagnetic waves)

**Longitudinal wave motion**

In longitudinal wave motion, the constituent of the medium oscillate or vibrate about their mean positions in a direction parallel to the direction of propagation (direction of energy transfer) of waves as shown in Figure 11.7.

**Example:** Sound waves travelling in air.

**Discuss with your Teacher**

- Tsunami (pronounced soo-nah-mee in Japanese) means Harbour waves. A tsunami is a series of huge and giant waves which come with great speed and huge force. What happened on 26th December2004 in southern part of India? - Discuss
- Gravitational waves - LIGO (Laser lnterferometer Gravitational wave Observatory) experiment Nobel Prize winners in Physics 2017
  i. Prof. Rainer Weiss
  ii. Prof. Barry C. Barish
  iii. Prof. Kip S. Thorne

  "For decisive contributions to the LIGO detector and observation of gravitational forces"

| Comparison of transverse and longitudinal waves | | |
|---|---|---|
| S.No | Transverse waves | Longitudinal waves |
| 1. | The direction of vibration of particles of the medium is perpendicular to the direction of propagation of waves. | The direction of vibration of particles of the medium is parallel to the direction of propagation of waves. |
| 2. | The disturbances are in the form of crests and troughs. | The disturbances are in the form of compressions and rarefactions |
| 3. | Transverse waves are possible in elastic medium. | Longitudinal waves are possible in all types of media (solid, liquid and gas). |

NOTE:
1. Absence of medium is also known as vacuum. Only electromagnetic waves can travel through vacuum.
2. Rayleigh waves are considered to be mixture of transverse and longitudinal.

**TERMS AND DEFINITIONS USED IN WAVE MOTION**

Suppose we have two waves as shown in Figure 11.8. Are these two waves identical?. No. Though, the two waves are both sinusoidal, there are many difference between them. Therefore, we have to define some basic terminologies to distinguish one wave from another.

Consider a wave produced by a stretched string as shown in Figure 11.9.

If we are interested in counting the number of waves created, let us put a reference level (mean position) as shown in Figure 11.9. Here the mean position is the horizontal line shown. The highest point in the shaded portion is called crest. With respect to the reference level, the lowest point on the un-shaded portion is called trough. This wave contains repetition of a section O to B and hence we define the length of the smallest section without repetition as one wavelength as shown in Figure 11.10. In Figure 11.10 the length OB or length BD is one wave lengh. A Greek letter lambda λ is used to denote one wavelength.

For transverse waves (as shown in Figure 11.11), the distance between two neighbouring crests or troughs is known as the wavelength. For longitudinal waves, (as shown in Figure 11.12) the distance between two neighbouring compressions or rarefactions is known as the wavelength. The SI unit of wavelength is meter.

**E X A M P L E**

Which of the following has longer wavelength?

In order to understand frequency and time period, let us consider waves (made of three wavelengths) as shown in Figure 11.13 (a). At time t = 0 s, the wave reaches the point A from left. After time t = 1 s (shown in figure 11.13(b)), the number of waves which have crossed the point A is two. Therefore, the frequency is defined as “the number of waves crossing a point per second” It is measured in hertz whose symbol is Hz. In this example,

$$f = 2 \text{ Hz}$$

wave consisting of three wavelengths passing a point A at time (a) t = 0 s and (b) after time t = 1 sin

If two waves take one second (time) to cross the point A then the time taken by one wave to cross the point A is half a second. This defines the time period T as

$$T = \frac{1}{2} = 0.5\, s$$

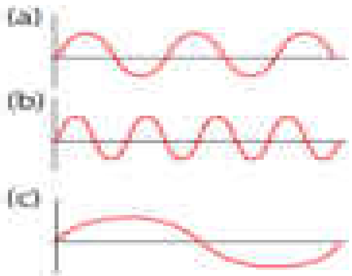
From equation (11.1) and equation (11.2), frequency and time period are inversely related i.e.

$$T = \frac{1}{f}$$

Time period is defined as the time taken by one wave to cross a point.in

**E X A M P L E**

Three waves are shown in the figure below




Write down

(a) the frequency in ascending order
(b) the wavelength in ascending order

**Solution**

$$f_c < f_a < f_b$$
$$\lambda_b < \lambda_a < \lambda_c$$

From the example 11.2, we observe that the frequency is inversely related to the wavelength, $f \alpha \frac{1}{\lambda}$

Then, fλ is equal to what?

$$[(i.e) \;\; f\lambda = ?]$$

A simple dimensional argument will help us to determine this unknown physical quantity. Dimension of wavelength is, [λ] = L

Frequency $f = \frac{1}{Time\ period}$ which implies that the dimension of frequency is,

$$[f] = \frac{1}{[T]} = T^{-1}$$
$$\Rightarrow [\lambda f] = [\lambda][f] = LT^{-1} = [velocity]$$

Therefore,

Velocity, $\lambda f = v$

where v is known as the wave velocity or phase velocity. This is the velocity with which the wave propagates. Wave velocity is the distance travelled by a wave in one second.

**Note:**

The number of cycles (or revolutions) per unit time is called angular frequency. Angular frequency, $\omega = \frac{2\pi}{T} = 2\pi f$ (unit is radians/second)

The number of cycles per unit distance or number of waves per unit distance is called wave number. wave number, $k = \frac{2\pi}{\lambda}$ (unit is radians/ meter In two, three or higher dimensional case, the wave number is the magnitude of a vector called \
wave vector. The points in space of wave vectors are called reciprocal vectors, $\dot{k}$

**Example**

The average range of frequencies at which human beings can hear sound waves varies from 20 Hz to 20 kHz. Calculate the wavelength of the sound wave in these limits. (Assume the speed of sound to be 340 m $s^{-1}$.

**Solution**

$$\lambda_1 = \frac{v}{f_1} = \frac{340}{20} = 17\text{m}$$

$$\lambda_2 = \frac{v}{f_2} = \frac{340}{20 \times 10^3} = 0.017\text{ m}$$

Therefore, the audible wavelength region is from 0.017 m to 17 m when the velocity of sound in that region is 340 m $s^{-1}$.

**Example**

A man saw a toy duck on a wave in an ocean. He noticed that the duck moved up and down 15 times per minute. He roughly measured the wavelength of the ocean wave as 1.2 m. Calculate the time taken by the toy duck for going one time up and down and also the velocity of the ocean wave.

**Solution**

Given that the number of times the toy duck moves up and down is 15 times per minute. This information gives us frequency (the number of times the toy duck moves up and down)

$$f = \frac{15 \text{ times toy duck moves up and down}}{\text{one minute}}$$

But one minute is 60 second, therefore, expressing time in terms of second

$$f = \frac{15}{60} = \frac{1}{4} = 0.25\,\text{Hz}$$

The time taken by the toy duck for going one time up and down is time period which is inverse of frequency

$$T = \frac{1}{f} = \frac{1}{0.25} = 4\,\text{s}$$

The velocity of ocean wave is

$$v = \lambda f = 1.2 \times 0.25 = 0.3\ \text{m s}^{-1}.$$

**Amplitude of a wave:**

The waves shown in thesame wavelength, same frequency andsame time period and also move with samevelocity. The only difference between twowaves is the height of either crest or trough.This means, the height of the crest or troughalso signifies a wave character. So we definea quantity called an amplitude of the wave, as the maximum displacement of the medium with respect to a reference axis (for example in this case x-axis). Here, it is denoted by A.

**Example**

Consider a string whose one end is attached to a wall. Then compute the following in both situations given in figure (assume waves crosses the distance in one second)

(a) Wavelength, (b) Frequency and (c) Velocity

**Solution**

| | **First Class** | **Second Class** |
|---|---|---|
| (a) Wavelength | $\lambda = 6$ m | $\lambda = 2$ m |

| | | |
|---|---|---|
| (b)Frequency | f = 2 Hz | f = 6 Hz |
| (c) Velocity | v = 6 × 2 = 12 m s-1 | v = 2 × 6 = 12 m s-1 |

This means that the speed of the wave along a string is a constant. Higher the frequency, shorter the wavelength and vice versa, and their product is velocity which remain the same.

**Velocity of Waves in different Media**

Suppose a hammer is stroked on long rails at a distance and when a person keeps his ear near the rails at the other end he/she will hear two sounds, at different instants. The sound that is heard through the rails (solid medium) is faster than the sound we hear through the air (gaseous medium). This implies the velocity of sound is different in different media.

In this section, we shall derive the velocity of waves in two different cases:

1. The velocity of a transverse waves along a stretched string.
2. The velocity of a longitudinal waves in an elastic medium.

**Velocity of transverse waves in a stretched string**

Let us compute the velocity of transverse travelling waves on a string. When a jerk is given at one end (left end) of the rope, the wave pulses move towards right end with a velocity v. This means that the pulses move with a velocity v with respect to an observer who is at rest frame. Suppose an observer also moves with same velocity v in the direction of motion of the wave pulse, then that observer will notice that the wave pulse is stationary and the rope is moving with pulse with the same velocity v.

Consider an elemental segment in the string as shown in the Figure. Let A andB be two points on the string at an instant of time. Let dl and dm be the length and mass of the elemental string, respectively. By definition, linear mass density, μ is

$$\mu = \frac{dm}{dl}$$
$$dm = \mu\, dl$$

The elemental string AB has a curvature which looks like an arc of a circle with centre at O, radius R and the arc subtending an angle θ at the origin O as shown in Figure. The angle θ can be written in terms of arc length and radius as $\theta = \frac{dl}{R}$. The centripetal acceleration supplied by the tension in the string is

$$a_{cp} = \frac{v^2}{R}$$

Then, centripetal force can be obtained when mass of the string (dm) is included in equation.

$$F_{cp} = \frac{(dm)v^2}{R}$$

The centripetal force experienced by elemental string can be calculated by substituting equation

$$\frac{(dm)v^2}{R} = \frac{\mu v^2 \, dl}{R}$$

The tension T acts along the tangent of the elemental segment of the string at A and B. Since the arc length is very small, variation in the tension force can be ignored. We can resolve T into horizontal component $T\cos\left(\frac{\theta}{2}\right)$ and vertical component $T\sin\left(\frac{\theta}{2}\right)$. The horizontal components at A and B are equal in magnitude but opposite in direction; therefore, they cancel each other. Since the elemental arc length AB is taken to be very small, the vertical components at A and B appears to acts vertical towards the centre of the arc and hence, they add up. The net radial force Fr is

$$F_r = 2T\sin\left(\frac{\theta}{2}\right)$$

Since the amplitude of the wave is very small when it is compared with the length of the string, the sine of small angle is approximated as $\sin\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$. Hence, equation can be written as

$$F_r = 2T \times \frac{\theta}{2} = T\theta$$

But $\theta = \frac{dl}{R}$ therefore substituting in equation (11.11), we get

$$F_r = T\frac{dl}{R}$$

Applying Newton's second law to the elemental string in the radial direction, under equilibrium, the radial component of the force is equal to the centripetal force. Hence equating equation (11.9) and equation (11.12), we have $T\frac{dl}{R} = \mu v^2 \frac{dl}{R}$

$$v = \sqrt{\frac{T}{\mu}} \text{ measured in m s}^{-1}$$

**Observations:**

• The velocity of the string is

a. directly proportional to the square root of the tension force

b. inversely proportional to the square root of linear mass density

c. independent of shape of the waves.

**Example**

Calculate the velocity of the travelling pulse as shown in the figure below. The linear mass density of pulse is 0.25 kg m-1. Further, compute the time taken by the travelling pulse to cover a distance of 30 cm on the string.

**Solution**

The tension in the string is T = m g = 1.2 × 9.8 = 11.76 N

The mass per unit length is μ = 0.25 kg m-1

Therefore, velocity of the wave pulse is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{11.76}{0.25}} = 6.858\ m\ s^{-1} = 6.8\ m\ s^{-1}$$

The time taken by the pulse to cover the distance of 30 cm is

$$t = \frac{d}{v} = \frac{30 \times 10^{-2}}{6.8} = 0.044\ s = 44\ ms \text{ where,}$$

ms = milli second

**Velocity of longitudinal waves in an elastic medium**

Consider an elastic medium (here we assume air) having a fixed mass contained in a long tube (cylinder) whose cross sectional area is A and maintained under a pressure P. One can generate longitudinal waves in the fluid either by displacing the fluid using a piston or by keeping a vibrating tuning fork at one end of the tube. Let us assume that the direction of propagation of waves coincides with theaxis of the cylinder. Let ρ be the density of the fluid which is initially at rest. At t = 0, the piston at left end of the tube is set in motion toward the right with a speed u.

Let u be thevelocity of the piston and v be the velocity of the elastic wave. In time interval Δt, the distance moved by the piston Δd = u Δt. Now, the distance moved by the elastic disturbance is $\Delta x = v\Delta t$. Let $\Delta m$ be the mass of the air that has attained a velocity v in a time $\Delta t$ . Therefore,

$$\Delta m = \rho\, A\, \Delta x = \rho\, A\, (v\, \Delta t)$$

Then, the momentum imparted due to motion of piston with velocity u is

$$\Delta p = [\rho\, A\, (v\, \Delta t)]u$$

But the change in momentum is impulse. The net impulse is

$$I = (\Delta P\, A)\Delta t$$

Or $$(\Delta P\, A)\Delta t = [\rho\, A\, (v\, \Delta t)]u$$

$$\Delta P = \rho\, v\, u \qquad (1$$

When the sound wave passes through air, the small volume element ($\Delta V$) of theair undergoes regular compressions and rarefactions. So, the change in pressure can also be written as

$$\Delta P = B\frac{\Delta V}{V}$$

where, V is original volume and B is known as bulk modulus of the elastic medium.
But $V = A\, \Delta x = A\, v\, \Delta t$ and
$\Delta V = A\, \Delta d = A\, u\, \Delta t$
Therefore,

$$\Delta P = B\frac{A\, u\, \Delta t}{A\, v\, \Delta t} = B\frac{u}{v}$$

$$\rho\, v\, u = B\frac{u}{v} \text{ or } v^2 = \frac{B}{\rho}$$

$$\Rightarrow v = \sqrt{\frac{B}{\rho}}$$

In general, the velocity of a longitudinal wave in elastic medium is $v = \sqrt{\frac{E}{\rho}}$ where E is the modulus of elasticity of the medium.

**Cases: For a solid:**

**(i) one dimension rod (1D)**

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y is the Young's modulus of the material of the rod and ρ is the density of the rod. The 1D rod will have only Young's modulus.

**(ii) Three dimension rod (3D)**The speed of longitudinal wave in a solid is

$$v = \sqrt{\frac{K + \frac{4}{3}\eta}{\rho}}$$

where η is the modulus of rigidity, K is the bulk modulus and ρ is the density of the rod.

**Cases: For liquids:**

$$v = \sqrt{\frac{K}{\rho}}$$

where, K is the bulk modulus and ρ is the density of the rod.

**E X A M P L E**

Calculate the speed of sound in a steel rod whose Young's modulus $Y = 2 \times 10^{11}$ N $m^{-2}$ and $\rho = 7800$ kg $m^{-3}$.

**Solution**

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{7800}} = \sqrt{0.2564 \times 10^{8}}$$

$$= 0.506 \times 10^{4} m s^{-1} = 5 \times 10^{3} m s^{-1}$$

Therefore, longitudinal waves travel faster in a solid than in a liquid or a gas. Now you may understand why a shepherd checks before crossing railway track by keeping his ears on the rails to safeguard his cattle.

**E X A M P L E**

An increase in pressure of 100 kPa causes a certain volume of water to decrease by

0.005% of its original volume.
(a) Calculate the bulk modulus of water?.
(b) Compute the speed of sound (compressional waves) in water?.

**Solutions**

**a) Bulk modules**

$$B = V\left|\frac{\Delta P}{\Delta V}\right| = \frac{100\times10^{3}}{0.005\times10^{-2}} =$$

$$= \frac{100\times10^{3}}{5\times10^{-5}} = 2000\ MPa, \text{ where } MP_a$$

Mega Pascal

(b) Speed of sound in water is

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2000\times10^{6}}{1000}} = 1414\,ms^{-1}$$

The velocities of both transverse waves and longitudinal waves depend on elastic property (like string tension T or bulk modulus B) and inertial property (like density or mass per unit length) i.e., $v = \sqrt{\frac{\text{Elastic property}}{\text{Inertial property}}}$

**Speed of Sound in Various media**

| S.No | Medium | Speed in $ms^{-1}$ |
|---|---|---|
| 1 | Rubber | 1600 |
| 2 | Gold | 3240 |
| 3 | Brass | 4700 |
| 4 | Copper | 5010 |
| 5 | Iron | 5950 |
| 6 | Aluminium | 6420 |
| | | Liquids at $25^0$C |
| 1 | Kerosene | 1324 |
| 2 | Mercury | 1450 |
| 3 | Water | 1493 |

| | | |
|---|---|---|
| 4 | Sea water | 1533 |
| Gas (at $0^0$C) | | |
| 1 | Oxygen | 317 |
| 2 | Air | 337 |
| 3 | Helium | 972 |
| 4 | Hydrogen | 1286 |
| Gas (at 20°C) | | |
| 1 | Air | 343 |

**PROPAGATION OF SOUND WAVES**

We know that sound waves are longitudinal waves, and when they propagate compressions and rarefactions are formed. In the following section, we compute the speed of sound in air by Newton's method and also discuss the Laplace correction and the factors affecting sound in air.

**Newton's formula for speed of sound waves in air**

Sir Isaac Newton assumed that when sound propagates in air, the formation of compression and rarefaction takes placein a very slow manner so that the process is isothermal in nature. That is, the heat produced during compression (pressure increases, volume decreases), and heat lost during rarefaction (pressure decreases, volume increases) occur over a period of time such that the temperature of the medium remains constant. Therefore, by treating the air molecules to form an ideal gas, the changes in pressure and volume obey Boyle's law, Mathematically

$$PV = \text{Constant} \qquad (11.20)$$

Differentiating equation (11.20), we get

$$PdV + VdP = 0$$

$$\text{or,} \quad P = -V\frac{dP}{dV} = B_T \qquad (11.21)$$

where, $B_T$ is an isothermal bulk modulus of air. Substituting equation (11.21) in equation (11.16), the speed of sound in air is

$$v_T = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{P}{\rho}} \qquad (11.22)$$

Since P is the pressure of air whose value at NTP (Normal Temperature and Pressure) is 76 cm of mercury, we have

$$P = (0.76 \times 13.6 \times 10^3 \times 9.8) \text{ N m}^{-2}$$

$\rho$ = 1.293 kg m$^{-3}$. here $\rho$ is density of air

Then the speed of sound in air at Normal Temperature and Pressure (NTP) is

$$v_T = \sqrt{\frac{(0.76 \times 13.6 \times 10^3 \times 9.8)}{1.293}}$$

= 279.80 m s$^{-1}$ ≈ 280 ms$^{-1}$ (theoretical value)

But the speed of sound in air at 0°C is experimentally observed as 332 m s$^{-1}$ which is close upto 16% more than theoretical value (Percentage error is $\frac{(332-280)}{332} \times 100\% + 15.6\%)$. This error is not small.

**Laplace's correction**

In 1816, Laplace satisfactorily corrected this discrepancy by assuming that when the sound propagates through a medium, the particles oscillate very rapidly such that the compression and rarefaction occur very fast. Hence the exchange of heat produced due to compression and cooling effect due to rarefaction do not take place, because, air (medium) is a bad conductor of heat. Since, temperature is no longer considered as a constant here, sound propagation is an adiabatic process. By adiabatic considerations, the gas obeys Poisson's law (not Boyle's law as Newton assumed), which is

$$PV^{\gamma} = \text{constant} \qquad (11.23)$$

where, $\gamma = \frac{C_P}{C_V}$, which is the ratio between specific heat at constant pressure and specific heat at constant volume.

Differentiating equation (11.23) on both the sides, we get

$$V^{\gamma} dP + P(\gamma V^{\gamma-1} dV) = 0$$

$$\text{or, } \gamma P = -V\frac{dp}{dV} = B_A$$

where, $B_A$ is the adiabatic bulk modulus of air. Now, substituting equation (11.24) in equation (11.16), the speed of sound in air is

$$v_A = \sqrt{\frac{B_A}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma} v_T$$

Since air contains mainly, nitrogen, oxygen, hydrogen etc, (diatomic gas), we take γ = 1.47. Hence, speed of sound in air is $v_A = \left(\sqrt{1.4}\right)\left(280ms^{-1}\right) = 331.30ms^{-1}$ which is very much closer to experimental data.

**Factors affecting speed of sound in gases**

Let us consider an ideal gas whose equation of state is

$$PV = n\,R\,T$$

where, P is pressure, V is volume, T is temperature, n is number of mole and R is universal gas constant. For a given mass of a molecule, equation (11.26) can be written as

$$\frac{PV}{T} = \text{constant}$$

For a fixed mass m, density of the gas inversely varies with volume. i.e.,

$$\rho \alpha \frac{1}{V}, V = \frac{m}{\rho}$$

Substituting equation (11.28) in equation (11.27), we get

$$\frac{P}{\rho} = cT$$

where c is constant.

The speed of sound in air given in equation (11.25) can be written as

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma c T}$$

From the above relation we observe the following

**(a) Effect of pressure:**

For a fixed temperature, when the pressure varies, correspondingly density also varies such that the ratio $\left(\frac{P}{\rho}\right)$ becomes constant. This means that the speed of sound is independent of pressure for a fixed temperature. If the temperature remains same at the top and the bottom of a mountain then the speed of sound will remain same at these two points.

But, in practice, the temperatures are not same at top and bottom of a mountain; hence, the speed of sound is different at different points.

**(b) Effect of temperature:**

Since, $v \alpha \sqrt{T}$, the speed of sound varies directly to the square root of temperature in kelvin.

Let $v_0$ be the speed of sound at temperature at 0° C or 273 K and v be the speed of sound at any arbitrary temperature T (in kelvin), then

$$\frac{v}{v_0} = \sqrt{\frac{T}{273}} = \sqrt{\frac{273+t}{273}}$$

$$v = v_0\sqrt{1+\frac{t}{273}} \doteq v_0\left(1+\frac{t}{546}\right)$$

(using binomial expansion)

Since $v_0 = 331 \text{m s}^{-1}$ at $0^0$C, v at any temperature in $t^0$C is

v = (331 + 0.60t) m s-1

Thus the speed of sound in air increases by 0.61 m $s^{-1}$ per degree celcius rise in temperature. Note that when the temperature is increased, the molecules will vibrate faster due to gain in thermal energy and hence, speed of sound increases.

(**c) Effect of density:**

Let us consider two gases with different densities having same temperature and pressure. Then the speed of sound in the two gases are

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}}$$

and

$$v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}}$$

Taking ratio of equation (11.31) and equation (11.32), we get

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho 1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

For gases having same value of γ,

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

**(e) Effect of wind:**

The speed of sound is also affected by blowing of wind. In the direction along the wind blowing, the speed of sound increases whereas in the direction opposite to wind blowing, the speed of sound decreases.

**Example**

The ratio of the densities of oxygen andnitrogen is 16:14. Calculate the temperaturewhen the speed of sound in nitrogen gasat 17°C is equal to the speed of sound inoxygen gas.

**Solution**
From equation (11.25), we have

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

But, $\rho = \frac{M}{V}$

Therefore, $v = \sqrt{\frac{\gamma PV}{M}}$

Using equation (11.26)

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Where, R is the universal gas constant and M is the molecular mass of the gas. The speed of sound in nitrogen gas at 17°C is

$$v_N = \sqrt{\frac{\gamma R(273K + 17K)}{M_N}}$$

$$= \sqrt{\frac{\gamma R(290K)}{M_N}}$$

Similarly, the speed of sound in oxygen gas at t in K is

$$v_0 = \sqrt{\frac{\gamma R(273K + t)}{M_0}}$$

Given that the value of γ is same for both the gases, the two speeds must be equal. Hence, equating equation (1) and (2), we get

$$v_o = v_N$$

$$\sqrt{\frac{\gamma R(273 + t)}{M_0}} = \sqrt{\frac{\gamma R(290)}{M_N}}$$

Squaring on both sides and cancelling γ R term and rearranging, we get

$$\frac{M_0}{M_N} = \frac{273 + t}{290}$$

Since the densities of oxygen and nitrogen is 16:14,

$$\frac{\rho_0}{\rho_N} = \frac{16}{14}$$

$$\frac{\rho_0}{\rho_N} = \frac{\frac{M_0}{V}}{\frac{M_N}{V}} = \frac{M_0}{M_N} \Rightarrow \frac{M_0}{M_N} = \frac{16}{14} \qquad (5)$$

Substituting equation (5) in equation (3), we get

$$\frac{273+t}{290}=\frac{16}{14}\Rightarrow 3822+14t=4640$$

$$\Rightarrow t=58.4\text{ K}$$

**REFLECTION OF SOUND WAVES**

When sound wave passes from one medium to another medium, the following things can happen

**(a) Reflection of sound:** If the medium is highly dense (highly rigid), the sound can be reflected completely (bounced back) to the original medium.

**(b) Refraction of sound:** When the sound waves propagate from one medium to another medium such that there can be some energy loss due to absorption by the second medium.

In this section, we will consider only the reflection of sound waves in a medium when it experiences a harder surface. Similar to light, sound can also obey the laws of reflection, which states that

(i) The angle of incidence of sound is equal to the angle of reflection.

(ii) When the sound wave is reflected by a surface then the incident wave, reflected wave and the normal at the point of incidence all lie in the same plane.

Similar to reflection of light from a mirror, sound also reflects from a harder flat surface, This is called as specular reflection.

Specular reflection is observed only when the wavelength of the source is smaller than dimensions of the reflecting surface, as well as smaller than surface irregularities.

**Reflection of sound through the plane surface**

When the sound waves hit the plane wall, they bounce off in a manner similar to that of light. Suppose a loudspeaker is kept at an angle with respect to a wall (plane surface), then the waves coming from the source (assumed to be a point source) can be treated as spherical wave fronts (say, compressions moving like a spherical wave front). Therefore, the reflected wave front on the plane surface is also spherical, such that its centre of curvature (which lies on the other side of plane surface) can be treated as the image of the sound source (virtual or imaginary loud speaker) which can be assumed to be at a position behind the plane surface.

**Reflection of sound through the curved surface**

The behaviour of sound is different when it is reflected from different surfaces-convex or concave or plane. The sound reflected from a convex surface is spread out and so it is easily attenuated and weakened. Whereas, if it is reflected from the concave surface it will converge at a point and this can be easilyamplified. The parabolic reflector (curved reflector) which is used to focus the sound precisely to a point is used in designing the parabolic mics which are known as high directional microphones.

We know that any surface (smooth or rough) can absorb sound. For example, the sound produced in a big hall or auditorium or theatre is absorbed by the walls, ceilings, floor, seats etc. To avoid such losses, a curved sound board (concave board) is kept in front of the speaker, so that the board reflects the sound waves of the speaker towards the audience. This method will minimize the spreading of sound waves in all possible direction in that hall and also enhances the uniform distribution of sound throughout the hall. That is why a person sitting at any position in that hall can hear the sound without any disturbance.

**Applications of reflection of sound waves**

**(a) Stethoscope:** It works on the principle of multiple reflections.

It consists of three main parts:
(i) Chest piece
(ii) Ear piece
(iii) Rubber tube

**(i) Chest piece:** It consists of a small disc-shaped resonator (diaphragm) which is very sensitive to sound and amplifies the sound it detects.

**(ii) Ear piece:** It is made up of metal tubes which are used to hear sounds detected by the chest piece.

**(iii) Rubber tube:** This tube connects both chest piece and ear piece. It is used to transmit the sound signal detected by the diaphragm, to the ear piece. The sound of heart beats (or lungs) or any sound produced by internal organs can be detected, and it reaches the ear piece through this tube by multiple reflections.

**(b) Echo:** An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at 20°C is 344 m $s^{-1}$. If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall. After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds.
Scientists have estimated that we can hear two sounds properly if the time gap or time interval between each sound is $\left(\frac{1}{10}\right)^{th}$ of a second (persistence of hearing) i.e., 0.1 s. Then,

$$\text{velocity}=\frac{\text{Distance travelled}}{\text{time taken}}=\frac{2d}{t}$$

$$2d = 344 \times 0.1 = 34.4 \text{ m}$$
$$d = 17.2 \text{ m}$$

The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meter.

**(c) SONAR:**SOundNAvigation and Ranging. Sonar systems make use of reflections of sound waves in water to locate the position or motion of an object. Similarly, dolphins and bats use the sonar principle to find their way in the darkness.

**(d) Reverberation:** In a closed room the sound is repeatedly reflected from the walls and it is even heard long after the sound source ceases to function. The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation. The duration for which the sound persists is called reverberation time. It should be noted that the reverberation time greatly affects the quality of sound heard in a hall. Therefore, halls are constructed with some optimum reverberation time.

**Example**

Suppose a man stands at a distance from a cliff and claps his hands. He receives an echo from the cliff after 4 second. Calculate the distance between the man and the cliff. Assume the speed of sound to be 343 m s-1.

**Solution**

The time taken by the sound to come back as echo is $2t = 4 \Rightarrow t = 2$ s
∴The distance is $d = vt = (343 \text{ m s}^{-1})(2 \text{ s}) = 686$ m.

Note: Classification of sound waves: Sound waves can be classified in three groups according to their range of frequencies:

**(1) Infrasonic waves:**
Sound waves having frequencies below 20 Hz are called infrasonic waves. These waves are produced during earthquakes. Human beings cannot hear these frequencies. Snakes can hear these frequencies.

**(2) Audible waves:**
Sound waves having frequencies between 20 Hz to 20,000 Hz (20kHz) are called audible waves. Human beings can hear these frequencies.

**(3) Ultrasonic waves:**
Sound waves having frequencies greater than 20 kHz are known as ultrasonic waves. Human beings cannot hear these frequencies. Bats can produce and hear these frequencies.

**(1.) Supersonic speed:**
An object moving with a speed greater than the speed of sound is said to move with a supersonic speed.

**(2.) Mach number:**
It is the ratio of the velocity of source to the velocity of sound.

$$\text{Mach number} = \frac{\text{velocity of source}}{\text{velocity of sound}}$$

**PROGRESSIVE WAVES (OR) TRAVELLING WAVES**

If a wave that propagates in a medium is continuous then it is known as progressive wave or travelling wave.

**Characteristics of progressive waves**

1. Particles in the medium vibrate about their mean positions with the same amplitude.
2. The phase of every particle ranges from 0 to 2π.
3. No particle remains at rest permanently. During wave propagation, particles come to the rest position only twice at the extreme points.
4. Transverse progressive waves are characterized by crests and troughs whereas longitudinal progressive waves are characterized by compressions and rarefactions.
5. When the particles pass through the mean position they always move with the same maximum velocity.
6. The displacement, velocity and acceleration of particles separated from each other by nλ are the same, where n is an integer, and λ is the wavelength.

**Equation of a plane progressive wave**

Suppose we give a jerk on a stretched string at time t = 0 s. Let us assume that the wave pulse created during this disturbance moves along positive x direction with constant speed v (a). We can represent the shape of the wave pulse, mathematically as y = y(x, 0) = f(x) at time t = 0 s. Assume that the shape of the wave pulse remains the same during the propagation. After some time t, the pulse moving towards the right and any point on it can be represented by x' (read it as x prime) (b). Then,

$$y(x, t) = f(x') = f(x - vt)$$

Similarly, if the wave pulse moves towards left with constant speed v, then y = f(x + vt). Both waves y = f(x + vt) and y = f(x − vt) will satisfy the following one dimensional differential equation known as the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

where the symbol ∂ represent partial derivative (read $\frac{\partial y}{\partial x}$ as partial y by partial x). Not all the solutions satisfying this differential equation can represent waves, because any physical acceptable wave must take finite values for all values of x and t. But if the function represents a wave then it must satisfy the differential equation. Since, in one dimension (one independent variable), the partial derivative with respect to x is the same as total derivative in coordinate x, we write

$$\frac{d^2y}{dx^2}=\frac{1}{v^2}\frac{d^2y}{dt^2}$$

This can be extended to more than one dimension (two, three, etc.). Here, for simplicity, we focus only on the one dimensional wave equation.

**Example**

Sketch y = x −a for different values of a.

**Solution**

This implies, when increasing the value of a, the line shifts towards right side. For a = vt, y = x − vt satisfies the differential equation. Though this function satisfies the differential equation, it is not finite for all values of x and t. Hence, it does not represent a wave.

**Example**

How does the wave y = sin(x − a) for a = 0, $a=\frac{\pi}{4}, a=\frac{\pi}{2}, a=\frac{3\pi}{2}$ and a = π look like? Sketch this wave.

**Solution**

From the above picture we observe that y = sin (x−a) for a = 0, $a=\frac{\pi}{4}, a=\frac{\pi}{2}, a=\frac{3\pi}{2}$ and a = π, the function y = sin (x−a) shifts towards right. Further, we can take a = vtand $v=\frac{\pi}{4}$, and sketching for different times t = 0s, t = 1s, t = 2s etc., we once again observe that y = sin(x−vt) moves towards the right. Hence, y = sin(x−vt) is a travelling (or progressive) wave moving towards the right. If y = sin(x+vt) then the travelling (or progressive) wave moves towards the left. Thus, any arbitrary function of type y = f(x−vt) characterising the wave must move towards right and similarly, any arbitrary function of type y = f(x+vt) characterizing the wave must move towards left.

**Example**

Check the dimensional of the wave y = sin(x−vt). If it is dimensionally wrong, write the above equation in the correct form.

**Solution**

Dimensionally it is not correct. we know that y = sin(x−vt) must be a dimensionless quantity but x−vt has dimension. The correct equation is y = sin (k x−ωt), where k and ω have the dimensions of inverse of length and inverse of time respectively. The sine functions and cosine functions are periodic functions with period 2π. Therefore, the correct expression is $y = \sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$ where λ and T are wavelength and time period, respectively. In general, y(x,t)=A sin(k x−ωt).

**Graphical representation of the wave**

Let us graphically represent the two forms of the wave variation
(a) Space (or Spatial) variation graph
(b) Time (or Temporal) variation graph

**(a) Space variation graph**

By keeping the time fixed, the change in displacement with respect to x is plotted. Let us consider a sinusoidal graph, y = A sin(kx), where k is a constant. Since the wavelength λ denotes the distance between any two points in the same state of motion, the displacement y is the same at both the ends y = x and y = x + λ, i.e.,

$$y = A\sin(kx) = A\sin(k(x+\lambda)) = A\sin(kx + k\lambda)$$

The sine function is a periodic function with period 2π. Hence,

$$y = A\sin(kx + 2\pi) = A\sin(kx)$$

Comparing equation, we get. kx + k λ = kx + 2π

That implies

$$k = \frac{2\pi}{\lambda} rad m^{-1}$$

where k is called wave number. This measures how many wavelengths are present in 2π radians.

The spatial periodicity of the wave is $\lambda = \frac{2\pi}{k}$ in m,

Then,
At t = 0 s y(x, 0) = y(x + λ, 0)
and
At any time t, y(x, t) = y(x + λ, t)

**Example**

The wavelength of two sine waves are λ1 = 1m and λ2 = 6m. Calculate the corresponding wave numbers.

**Solution**

$$k_1 = \frac{2\pi}{k} = 6.28 rad m^{-1}$$

$$k_1 = \frac{2\pi}{6} = 1.05 rad m^{-1}$$

**(b) Time variation graph**

By keeping the position fixed, the change in displacement with respect to time is plotted. Let us consider a sinusoidal graph, y =A sin(ωt), where ω is angular frequency of the wave which measures how quickly wave oscillates in time or number of cycles per second.

The temporal periodicity or time period is

$$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$$

The angular frequency is related to frequency f by the expression ω = 2 πf, where the frequency f is defined as the number of oscillations made by the medium particle per second. Since inverse of frequency is time period, we have,

$$T = \frac{1}{f} \text{ in seconds}$$

This is the time taken by a medium particle to complete one oscillation. Hence, we can define the speed of a wave (wave speed, v) as the distance traversed by the wave per second

$$v = \frac{\lambda}{T} = \lambda f \text{ in } ms^{-1}$$

which is the same relation as we obtained in equation (11.4).

**Particle velocity and wave velocity**

In a plane progressive harmonic wave, the constituent particles in the medium oscillate simple harmonically about their equilibrium positions. When a particle is in motion, the rate of change of displacement at any instant of time is defined as velocity of the particle at that instant of time. This is known as particle velocity.

$$v_P = \frac{dy}{dt} ms^{-1}$$

But y(x, t)= A sin(k x - ω t)

Therefore, $\frac{dy}{dt} = -\omega A \cos(kx - \omega t)$

Similarly, we can define velocity (here speed) for the travelling wave (or progressive wave). In order to determine the velocity of a progressive wave, let us consider a progressive wave moving towards right. This can be mathematically represented as a sinusoidal wave. Let P be any point on the phase of the wave and $y_P$ be its displacement with respect to the mean position. The displacement of the wave at an instant t is

$$y = y(x,t) = A \sin(kx - \omega t)$$

At the next instant of time $t' = t + \Delta t$ the position of the point P is $x' = x + \Delta x$. Hence, the displacement of the wave at this instant is

$$y = y(x', t') = y(x + \Delta x, t + \Delta t)$$
$$= A \sin[k(x + \Delta x) - \omega(t + \Delta t)]$$

Since the shape of the wave remains the same, this means that the phase of the wave remains constant (i.e., the y- displacement of the point is a constant). Therefore, equating equation (11.42) and equation (11.44), we get

y(x',t') = y(x,t), which implies A sin[k (x + Δx)− ω (t + Δt)]= A sin(k x− ω t) Or

$$k(x + \Delta x) - \omega(t + \Delta t) = kx - \omega t = \text{constant}$$

On simplification of equation (11.45), we get

$$v = \frac{\Delta x}{\Delta t} = \frac{\omega}{k} = v_p$$

where$v_p$ is called wave velocity or phase velocity.

By expressing the angular frequency and wave number in terms of frequency and wave length, we obtain

$$\omega = 2\pi f = \frac{2\pi}{T}$$
$$k = \frac{2\pi}{\lambda}$$
$$v = \frac{\omega}{k} = \lambda f$$

**Example**

A mobile phone tower transmits a wave signal of frequency 900MHz. Calculate the length of the waves transmitted from the mobile phone tower.
**Solution**

Frequency, f = 900MHz = 900 $\times 10^6$ Hz
The speed of wave is c = 3 $\times 10^8$m $s^{-1}$

$$\lambda = \frac{v}{f} = \frac{3\times10^8}{900\times10^6} = 0.33m$$

**SUPERPOSITION PRINCIPLE**

When a jerk is given to a stretched string which is tied at one end, a wave pulse is produced and the pulse travels along the string. Suppose two persons holding the stretched string on either side give a jerk simultaneously, then these two wave pulses move towards each other, meet at some point and move away from each other with their original identity. Their behaviour is very different only at the crossing/meeting points; this behaviour depends on whether the two pulses have the same or different shape. When the pulses have the same shape, at the crossing, the total displacement is the algebraic sum of their individual displacements and hence its net amplitude is higher than the amplitudes of the individual pulses. Whereas, if the two pulses have same amplitude but shapes are 180° out of phase at the crossing point, the net amplitude vanishes at that point and the pulses will recover their identities after crossing. Only waves can possess such a peculiar property and it is called superposition of waves. This means that the principle of superposition explains the net behaviour of the waves when they overlap. Generalizing to any number of wavesi.e, if two are more waves in a medium move simultaneously, when they overlap, their total displacement is the vector sum of the individual displacements. We know that the waves satisfy the wave equation which is a linear second order homogeneous partial differential equation in both space coordinates and time. Hence, their linear combination (often called as linear superposition of waves) will also satisfy the same differential equation.

To understand mathematically, let us consider two functions which characterize the displacement of the waves, for example,

$$y_1 = A_1\sin(kx - \omega t)$$

and

$$y_2 = A_2\cos(kx - \omega t)$$

Since, both y1 and y2 satisfy the wave equation (solutions of wave equation) then their algebraic sum

$$y = y_1 + y_2$$

also satisfies the wave equation. This means, the displacements are additive. Suppose we multiply $y_1$ and $y_2$ with some constant then their amplitude is scaled by that constant.

Further, if $C_1$ and $C_2$ are used to multiply the displacements $y_1$ and $y_2$, respectively, then, their net displacement y is

$$y = C_1 y_1 + C_2 y_2$$

This can be generalized to any number of waves. In the case of n such waves in more than one dimension the displacements are written using vector notation. Here, the net displacement $\dot{y}$ is

$$y = \sum_{i=1}^{n} C_i \dot{y}_i$$

The principle of superposition can explain the following :

(a) Space (or spatial) Interference (also known as Interference)

(b) Time (or Temporal) Interference (also known as Beats)

(c) Concept of stationary waves

Waves that obey principle of superposition are called linear waves (amplitude is much smaller than their wavelengths). In general, if the amplitude of the wave is not small then they are called non-linear waves. These violate the linear superposition principle, e.g. laser. In this chapter, we will focus our attention only on linear waves.

We will discuss the following in different subsections:

**Interference of waves**

Interference is a phenomenon in which two waves superimpose to form a resultant wave of greater, lower or the same amplitude.

Consider two harmonic waves having identical frequencies, constant phase difference φ and same wave form (can be treated as coherent source), but having amplitudes $A_1$ and $A_2$, then

$$y_1 = A_1 \sin(kx - \omega t)$$
$$y_2 = A_2 \sin(kx - \omega t + \varphi)$$

Suppose they move simultaneously in a particular direction, then interference occurs (i.e., overlap of these two waves). Mathematically

$$y = y1 + y2$$

Therefore, substituting equation (11.47) and equation (11.48) in equation (11.49), we get $y = A_1\sin(kx - \omega t) + A_2 \sin(kx - \omega t + \varphi)$

Using trigonometric identity $\sin(\alpha+\beta) = (\sin\alpha \cos\beta + \cos\alpha \sin\beta)$, we get
$y = A_1\sin(kx - \omega t)+A_2 [\sin(kx - \omega t) \cos\varphi + \cos(kx - \omega t) \sin\varphi]$

$$y = \sin(kx - \omega t)(A_1 + A_2\cos\varphi) + A_2\sin\varphi\cos(kx - \omega t)$$

Let us re-define

$$A \cos\theta =(A_1 + A_2\cos\varphi)$$
$$\text{and } A \sin\theta = A_2\sin\varphi$$

then equation (11.50) can be rewritten as $y = A \sin(kx-\omega t) \cos\theta + A \cos(kx-\omega t) \sin\theta$

$y = A (\sin(kx-\omega t) \cos\theta + \sin\theta\cos(kx-\omega t))$
$y = A \sin(kx-\omega t + \theta)$ (11.53)

By squaring and adding equation (11.51)
and equation (11.52), we get

$A_2 = A_1^2 + A_2^2 + 2A_1 A_2\cos\varphi$ (11.54)

Since, intensity is square of the amplitude ($I = A^2$), we have

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$$

This means the resultant intensity at any point depends on the phase difference at that point.

**(a) For constructive interference:**

When crests of one wave overlap with crests of another wave, their amplitudes will add up and we get constructive interference. The resultant wave has a larger amplitude than the individual waves.

The constructive interference at a point occurs if there is maximum intensity at that point, which means that

$$\cos\varphi = +1 \Rightarrow \phi = 0, 2\pi, 4\pi, ... = 2n\pi, \text{ where } n = 0,1,2,...$$

This is the phase difference in which two waves overlap to give constructive interference.

Therefore, for this resultant wave,

$$I_{\max imum} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(A_1 + A_2\right)^2$$

Hence, the resultant amplitude $A = A_1 + A_2$

**(b) For destructive interference:**

When the trough of one wave overlaps with the crest of another wave, their amplitudes "cancel" each other and we get destructive interference as shown in Figure 11.29 (b). The resultant amplitude is nearly zero. The destructive interference occurs if there is minimum intensity at that point, which means $\cos\varphi = -1 \Rightarrow \phi = \pi, 3\pi, 5\pi, ... = (2n-1)\pi$, where $n = 0,1,2,....$ i.e. This is the phase difference in which two waves overlap to give destructive interference. Therefore,

$$I_{\max imum} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(A_1 + A_2\right)^2$$

Hence, the resultant amplitude

$$A = |A1 - A2|$$

Let us consider a simple instrument to demonstrate the interference of sound waves as shown in Figure 11.30.

A sound wave from a loudspeaker S is sent through the tube P. This looks like a T-shaped junction. In this case, half of the sound energy is sent in one direction and the remaining half is sent in the opposite direction. Therefore, the sound waves that reach the receiver R can travel along either of two paths. The distance covered by the sound wave along any path from the speaker to receiver is called the path length . From the Figure 11.30, we notice that the lower path length is fixed but the upper path length can be varied by sliding the upper tube i.e., is varied. The difference in path length is known as path difference,

$$\Delta r = |r2 - r1|$$

Suppose the path difference is allowed to be either zero or some integer (or integral) multiple of wavelength λ. Mathematically, we have

$$\Delta r = n\lambda \text{ where, } n = 0, 1, 2, 3, ....$$

Then the two waves arriving from the paths $r_1$ and $r_2$ reach the receiver at any instant are in phase (the phase difference is 0° or 2π) and interfere constructively as shown in Figure 11.31.

Therefore, in this case, maximum sound intensity is detected by the receiver. If the path difference is some half-odd-integer (or half-integral) multiple of wavelength λ, mathematically, $\Delta r = n\frac{\lambda}{2}$

where, n = 1,3,... (n is odd) then the two waves arriving from the paths $r_1$ and $r_2$ and reaching the receiver at any instant are out of phase (phase difference of π or 180°). They interfere destructively as shown in Figure 11.32. They will cancel each other.

Therefore, the amplitude is minimum or zero amplitude which means no sound. No sound intensity is detected by the receiver in this case. The relation between path difference and phase difference is phase difference = $\frac{2\pi}{\lambda}$ (path difference) (11.56)

i.e., $\Delta\varphi = \frac{2\pi}{\lambda}\Delta r$ or $\Delta r = \frac{\lambda}{2\pi}\Delta\varphi$

**Example**

Consider two sources A and B as shown in the figure below. Let the two sources emit simple harmonic waves of same frequency but of different amplitudes, and both are in phase (same phase). Let O be any point equidistant from A and B as shown in the figure. Calculate the intensity at points O, Y and X. (X and Y are not equidistant from A & B)

**Solution**

The distance between OA and OB are the same and hence, the waves starting from A and B reach O after covering equal distances (equal path lengths). Thus, the path difference between two waves at O is zero.

$$OA - OB = 0$$

Since the waves are in the same phase, at the point O, the phase difference between two waves is also zero. Thus, the resultant intensity at the point O is maximum. Consider a point Y, such that the path difference between two waves is λ. Then the phase difference at Y is

$$\Delta\varphi = \frac{2\pi}{\lambda} \times \Delta r = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

Therefore, at the point Y, the two waves from A and B are in phase, hence, the intensity will be maximum.

Consider a point X, and let the path difference the between two waves be $\frac{\lambda}{2}$.

Then the phase difference at X is

$$\Delta\varphi = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

Therefore, at the point X, the waves meet and are in out of phase, Hence, due to destructive interference, the intensity will be minimum.

**Example**

Two speakers C and E are placed 5 m apart and are driven by the same source. Let a man stand at A which is 10 m away from the mid point O of C and E. The man walks towards the point O which is at 1 m (parallel to OC) as shown in the figure. He receives the first minimum in sound intensity at B. Then calculate the frequency of the source. (Assume speed of sound = 343 m $s^{-1}$)


**Solution**


The first minimum occurs when the two waves reaching the point B are 180° (out of phase). The path difference $\Delta x = \frac{\lambda}{2}$.

In order to calculate the path difference, we have to find the path lengths $x_1$ and $x_2$. In a right triangle BDC,

DB = 10m and OC = $\frac{1}{2}$(5) = 2.5m

CD = OC −1 = (2.5 m)−1 m = 1.5 m

$$x_1 = \sqrt{(10)^2 + (1.5)^2} = \sqrt{100 + 2.25}$$

$$= \sqrt{102.25} = 10.1m$$

In a right triangle EFB,

DB = 10m and OE = $\frac{1}{2}$ (5) = 2.5m = FA

FB = FA + AB = (2.5 m) + 1 m = 3.5 m

$$x_2 = \sqrt{(10)^2 + (3.5)^2} = \sqrt{100 + 12.25}$$

$$= \sqrt{112.25} = 10.6m$$

The path difference Δ x = x2 − x1 = 10.6 m−10.1 m = 0.5 m. Required that this path difference

$$\Delta x = \frac{\lambda}{2} = 0.5 \Rightarrow \lambda = 1.0m$$

To obtain the frequency of source, we use

$$v = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{343}{1} = 343\,Hz$$

$$= 0.3\,kHz$$

If the speakers were connected such that already the path difference is $\frac{\lambda}{2}$. Now, the path difference combines with a path difference of $\frac{\lambda}{2}$. This gives a total path difference of λ which means, the waves are in phase and there is a maximum intensity at point B.

**Formation of beats**

When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency.

Number of beats per second

n = | f1 - f2| per second

**Additional information (Not for examination): Mathematical treatment of beats**

For mathematical treatment, let us consider two sound waves having same amplitude and slightly different frequencies $f_1$ and $f_2$, superimposed on each other.

Since the sound wave (pressure wave) is a longitudinal wave, let us consider $y_1 = A \sin(\omega_1 t)$ and $y_2 = A \sin(\omega_2 t)$ to be displacements of the two waves at a point x = 0 with same amplitude (region having high pressures) and different angular frequencies $\omega_1$ and $\omega_2$, respectively. Then when they are allowed to superimpose we get the net displacement

$y = y_1 + y_2$

$y = A \sin(\omega_1 t) + A \sin(\omega_2 t)$

But

$\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$

Then

$y = A \sin(2\pi f_1 t) + A \sin(2\pi f_2 t)$

Using trigonometry formula

$$\sin C + \sin D = 2\cos\left(\frac{C-D}{2}\right)\sin\left(\frac{C+D}{2}\right)$$

$$y = 2A\cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right)\sin\left(2\pi\left(\frac{f_1+f_2}{2}\right)t\right)$$

Let, $y_p = 2A\cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right)$ (11.57)

and if f1 is slightly higher value than f2 then,

$\left(\frac{f_1-f_2}{2}\right)$ ∴∵ $\left(\frac{f_1+f_2}{2}\right)$ meansyp in equation (11.57) varies very slowly when compared to $\left(\frac{f_1+f_2}{2}\right)$. Therefore $y = y_P \sin(2\pi f_{avg} t)$ (11.58)

This represents a simple harmonic wave of frequency which is an arithmetic average of frequencies of the individual waves, $f_{avg} = \left(\frac{f_1+f_2}{2}\right)$ and amplitude yp varies with time t.

**Case (A):**

The resultant amplitude is maximum when $y_P$ is maximum. Since $y_p \alpha \cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right)$, this means maximum amplitude occurs only when cosine takes ±1,

$$\cos\left(2\pi\left(\frac{f_1-f_2}{2}\right)t\right) = \pm 1$$

$$\Rightarrow 2\pi\left(\frac{f_1-f_2}{2}\right)t = n\pi,$$

$$\text{or, } t = \frac{n}{(f_1-f_2)} \quad n = 0,1,2,3, ....$$

Hence, the time interval between two successive maxima is

$$t_2 - t_1 = t_3 - t_2 = ... = \frac{l}{(f_1-f_2)} n = |f_1 - f_2| = \frac{1}{|t_1-t_2|}$$

Therefore, the number of beats produced per second is equal to the reciprocal of the time interval between two consecutive maxima i.e., |f1 - f2|.

**Case (B):**

The resultant amplitude is minimum i.e., it is equal to zero when yp is minimum. Since $y_p \alpha \cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right)$, this means, minimum occurs only when cosine takes 0,

$$\cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right) = 0$$

$$\Rightarrow 2\pi\left(\frac{f_1 - f_2}{2}\right)t = (2n+1)\frac{\pi}{2},$$

$$\Rightarrow (f_1 - f_2)t = \frac{1}{2}(2n+1)$$

or, $t = \frac{1}{2}\left(\frac{2n+1}{f_1 - f_2}\right)$, where f1 ≠ f2 n = 0,1,2,3,.....

Hence, the time interval between two successive minima is $t_2 - t_1 = t_3 = ... = \frac{1}{(f_1 - f_2)}; n = |f_1 f_2| = \frac{1}{|t_1 - t_2|}$

Therefore, the number of beats produced per second is equal to the reciprocal of the time interval between two consecutive minima i.e., |f1 - f2|.

**Example**

Consider two sound waves with wavelengths 5 m and 6 m. If these two waves propagate in a gas with velocity 330 $ms^{-1}$. Calculate the number of beats per second.

**Solution**

Given $\lambda_1 = 5_m$ and $\lambda_2 = 6m$

Velocity of sound waves in a gas is v = 330 ms-1

The relation between wavelength and velocity is $v = \lambda f \Rightarrow f = \frac{v}{\lambda}$

The frequency corresponding to wavelength $\lambda_1$ *is* $f_1 = \frac{v}{\lambda 1} = \frac{330}{5} = 66Hz$

The frequency corresponding to wavelength

$\lambda_2$ *is* $f_2 = \frac{v}{\lambda 2} = \frac{330}{6} = 55Hz$

The number of beats per second is

| f1 − f2| = |66 − 55| = 11 beats per sec

**Example**

Two vibrating tuning forks produce waves whose equation is given by $y_1 = 5 \sin(240\pi t)$ and $y_2 = 4 \sin(244\pi t)$. Compute the number of beats per second.

**Solution**

Given $y_1 = 5 \sin(240\pi t)$ and $y_2 = 4 \sin(244\pi t)$

Comparing with $y = A \sin(2\pi f_1 t)$, we get

$$2\pi f_1 = 240\pi \Rightarrow f_1 = 120\text{Hz}$$

$$2\pi f_2 = 244\pi \Rightarrow f_2 = 122\text{Hz}$$

The number of beats produced is $|f1 - f2| = |120 - 122| = |-2| = 2$ beats per sec

**Standing Waves**

**Explanation of stationary waves**

When the wave hits the rigid boundary it bounces back to the original medium and can interfere with the original waves. A pattern is formed, which are known as standing waves or stationary waves. Consider two harmonic progressive waves (formed by strings) that have the same amplitude and same velocity but move in opposite directions. Then the displacement of the first wave (incident wave) is

$$y_1 = A \sin(kx - \omega t) \quad (11.59)$$

(waves move toward right)

and the displacement of the second wave (reflected wave) is

$$y2 = A \sin(kx + \omega t) \quad (11.60)$$

(waves move toward left)

both will interfere with each other by the principle of superposition, the net displacement is

$$= y1 + y2 \quad (11.61)$$

Substituting equation (11.59) and equation (11.60) in equation (11.61), we get

$$y = A \sin(kx - \omega t) + A \sin(kx + \omega t) \quad (11.62)$$

Using trigonometric identity, we rewrite equation (11.62) as

$$y(x, t) = 2A \cos(\omega t) \sin(kx) \quad (11.63)$$

This represents a stationary wave or standing wave, which means that this wave does not move either forward or backward, whereas progressive or travelling waves will move forward or backward. Further, the displacement of the particle in equation (11.63) can be written in more compact form,

$$y(x,t) = A'\cos(\omega t)$$

where, A′ = 2Asin(kx), implying that the particular element of the string executes simple harmonic motion with amplitude equals to A′. The maximum of this amplitude occurs at positions for which

$$\sin(kx) = 1 \Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots = m\pi$$

where m takes half integer or half integral values. The position of maximum amplitude is known as antinode. Expressing wave number in terms of wavelength, we can represent the anti-nodal positions as

$$x_m = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2}, \text{ where, m = 0,1,2... (11.64)}$$

For m = 0 we have maximum at $x_0 = \frac{\lambda}{2}$

For m = 1 we have maximum at $x_1 = \frac{3\lambda}{4}$

For m = 2 we have maximum at $x_2 = \frac{5\lambda}{4}$ and so on.

The distance between two successive antinodes can be computed by

$$x_m - x_{m-1} = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2} - \left(\frac{(2m+1)+1}{2}\right)\frac{\lambda}{2} = \frac{\lambda}{2}$$

Similarly, the minimum of the amplitude A' also occurs at some points in the space, and these points can be determined by setting

$$\sin(kx) = 0 \Rightarrow k\,x = 0, \pi, 2\pi, 3\pi, \ldots = n\,\pi$$

where n takes integer or integral values. Note that the elements at these points do not vibrate (not move), and the points are called nodes. The n<sup>th</sup>nodal positions is given by,

$$x_n = n\frac{\lambda}{2} \text{ where, n = 0,1,2,... (11.65)}$$

For n = 0 we have minimum at

$$x0 = 0$$

For n = 1 we have minimum at

$$x_1 = \frac{\lambda}{2}$$

For n = 2 we have maximum at

$$x2 = \lambda$$

and so on.

The distance between any two successive nodes can be calculated as

$$x_n - x_{n-1} = n\frac{\lambda}{2} - (n-1)\frac{\lambda}{2} = \frac{\lambda}{2}$$

**Example**

Compute the distance between anti-node and neighbouring node.

**Solution**

For nth mode, the distance between antinode and neighbouring node is

$$\Delta x_n = \left(\frac{2n+1}{2}\right)\frac{\lambda}{2} - n\frac{\lambda}{2} = \frac{\lambda}{4}$$

**Characteristics of stationary waves**

(1) Stationary waves are characterised by the confinement of a wave disturbance between two rigid boundaries. This means, the wave does not move forward or backward in a medium (does not advance), it remains steady at its place. Therefore, they are called "stationary waves or standing waves".

(2) Certain points in the region in which the wave exists have maximum amplitude, called as anti-nodes and at certain points the amplitude is minimum or zero, called as nodes.

(3) The distance between two consecutive nodes (or) anti-nodes is $\frac{\lambda}{2}$.

(4) The distance between a node and its neighbouring anti-node is $\frac{\lambda}{4}$.

(5) The transfer of energy along the standing wave is zero.

| **Comparison between progressive and stationary waves** | | |
|---|---|---|
| **S.No** | **Progressive waves** | **Stationary waves** |
| 1. | Crests and troughs are formed in transverse progressive waves, and compression and rarefaction are formed in longitudinal progressive waves. These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity. | Crests and troughs are formed in transverse stationary waves, and compression and rarefaction are formed in longitudinal stationary waves. These waves neither move forward nor backward in a medium i.e., they will not advance in a medium. |
| 2. | All the particles in the medium vibrate such that the amplitude of the vibration for all particles is same. | Except at nodes, all other particles of the medium vibrate such that amplitude of vibration is different for different particles. The amplitude is minimum or zero at nodes and maximum at antinodes. |
| 3. | These wave carry energy while propagating. | These waves do not transport energy. |

**Stationary waves in sonometer**

Sono means sound related, and sonometer implies sound-related measurements. It is a device for demonstrating the relationship between the frequency of the sound produced in the transverse standing wave in a string, and the tension, length and mass per unit length of the string. Therefore, using this device, we can determine the following quantities:

(a) the frequency of the tuning fork or frequency of alternating current
(b) the tension in the string
(c) the unknown hanging mass

**Construction:**

The sonometer is made up of a hollow box which is one meter long with a uniform metallic thin string attached to it. One end of the string is connected to a hook and the other end is connected to a weight hanger through a pulley. Since only one string is used, it is also known as monochord. The weights are added to the free end of the wire to increase the tension of the wire. Two adjustable wooden knives are put over the board, and their positions are adjusted to change the vibrating length of the stretched wire.

**Working :**

A transverse stationary or standing wave is produced and hence, at the knife edges P and Q, nodes are formed. In between the knife edges, anti-nodes are formed.

If the length of the vibrating element is l then

$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

Let f be the frequency of the vibrating element, T the tension of in the string and μ the mass per unit length of the string. Then using equation (11.13), we get

$$f = \frac{v}{\lambda} = \frac{1}{21}\sqrt{\frac{T}{\mu}} \text{ in Hertz (11.66)}$$

Let ρ be the density of the material of the string and d be the diameter of the string. Then the mass per unit length μ,

$$\mu = \text{Area} \times \text{density} = \pi r2\rho = \frac{\pi\rho d^2}{4}$$

Frequency $f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\frac{\pi d^2 \rho}{4}}}$

$$f = \frac{1}{ld}\sqrt{\frac{T}{\pi\rho}}$$

**Example**

Let f be the fundamental frequency of the string. If the string is divided into three segments $l_1$, $l_2$ and $l_3$ such that the fundamental frequencies of each segments be $f_1$, $f_2$ and $f_3$, respectively. Show that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

**Solution**

For a fixed tension T and mass density μ, frequency is inversely proportional to the string length i.e.

$$f \alpha \frac{1}{l} \Rightarrow f = \frac{v}{2l} \Rightarrow l = \frac{v}{2f}$$

For the first length segment

$$f_1 = \frac{v}{2l_1} \Rightarrow l_1 = \frac{v}{2f_1}$$

For the second length segment

$$f_2 = \frac{v}{2l_2} \Rightarrow l_2 = \frac{v}{2f_2}$$

Therefore, the total length

$$l = l_1 + l_2 + l_3$$

$$\frac{v}{2f} = \frac{v}{2f_1} + \frac{v}{2f_2} + \frac{v}{2f} \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

Fundamental frequency and overtones

Let us now keep the rigid boundaries at x = 0 and x = L and produce a standing waves by wiggling the string (as in plucking strings in a guitar). Standing waves with a specific wavelength are produced. Since, the amplitude must vanish at the boundaries, therefore, the displacement at the boundary must satisfy the following conditions y(x = 0, t) = 0 and y(x = L, t) = 0. Since the nodes formed are at a distance $\frac{\lambda_n}{2}$ apart, we have $n\left(\frac{\lambda_n}{2}\right) = L$, where n is an integer, L is the length between the two boundaries and λn is the specific wavelength that satisfy the specified boundary conditions. Hence,

$$\boxed{\lambda_n = \left(\frac{2L}{n}\right)}$$

Therefore, not all wavelengths are allowed. The (allowed) wavelengths should fit with the specified boundary conditions, i.e., for n = 1, the first mode of vibration has specific wavelength

$$\lambda_2 = \left(\frac{2L}{2}\right) = L$$

For n = 3, the third mode of vibration has specific wavelength

$$\lambda_3 = \left(\frac{2L}{3}\right)$$

and so on.

The frequency of each mode of vibration (called natural frequency) can be calculated.
We have,

$$f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right)$$

The lowest natural frequency is called the fundamental frequency.

$$f_1 = \frac{v}{\lambda_1} = \left(\frac{v}{2L}\right)$$

The second natural frequency is called the first over tone.

$$f_2 = 2\left(\frac{v}{2L}\right) = \frac{1}{L}\sqrt{\frac{T}{\mu}}$$

The third natural frequency is called the second over tone.

$$f_3 = 3\left(\frac{v}{2L}\right) = 3\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right)$$

and so on.

Therefore, the nth natural frequency can be computed as integral (or integer ) multiple of fundamental frequency, i.e.,

$f_n = nf_1$, where n is an integer. If natural frequencies are written as integral multiple of fundamental frequencies, then the frequencies are called harmonics. Thus, the first harmonic is $f_1 = f_1$ (the fundamental frequency is called first harmonic), the second harmonic is $f_2 = 2f_1$ , the third harmonic is $f_3 = 3f_1$ etc.

**Example**
Consider a string in a guitar whose length is 80 cm and a mass of 0.32 g with tension 80 N is plucked. Compute the first four lowest frequencies produced when it is plucked.

**Solution**
The velocity of the wave

$$v = \sqrt{\frac{T}{\mu}}$$

The length of the string, L = 80 cm=0.8 m The mass of the string, m = 0.32 g =0.32 × $10^{-3}$kg

Therefore, the linear mass density, $\mu = \frac{0.32\times10^{3}}{0.8} = 0.4\times10^{-3}\,kgm^{-1}$

The tension in the string, T = 80 N

$$v = \sqrt{\frac{80}{0.4\times10^{-3}}} = 447.2\ \mathrm{m\ s^{-1}}$$

The wavelength corresponding to the fundamental frequency $f_1$ is $\lambda_1$ = 2L = 2 × 0.8 = 1.6 m

The fundamental frequency $f_1$ corresponding to the wavelength $\lambda_1$

$$f_1 = \frac{v}{\lambda_1} = \frac{447.2}{1.6} = 279.5\,Hz$$

Similarly, the frequency corresponding to the second harmonics, third harmonics and fourth harmonics are

$$f_2 = 2f_1 = 559\ \text{Hz}$$
$$f_3 = 3f_1 = 838.5\ \text{Hz}$$
$$f_4 = 4f_1 = 1118\ \text{Hz}$$

**Laws of transverse vibrations in stretched strings**

There are three laws of transverse vibrations of stretched strings which are given as follows:

**(i) The law of length :**

For a given wire with tension T (which is fixed) and mass per unit length μ (fixed) the frequency varies inversely with the vibrating length. Therefore,

$$f \propto \frac{1}{l} \Rightarrow f = \frac{C}{l}$$

⇒l×f = C, where C is a constant

**(ii) The law of tension:**

For a given vibrating length l (fixed) and mass per unit length μ (fixed) the frequency varies directly with the square root of the tension T,

$$f \alpha \sqrt{T}$$

$\Rightarrow f \alpha \sqrt{T}$ .where A is a constant

**(iii) The law of mass:**

For a given vibrating length l (fixed) and tension T (fixed) the frequency varies inversely with the square root of the mass per unit length μ,

$$f \alpha \frac{1}{\sqrt{u}}$$

$\Rightarrow f = \frac{B}{\sqrt{\mu}}$, where B is a constant.

**INTENSITY AND LOUDNESS**

Consider a source and two observers (listeners). The source emits sound waves which carry energy. The sound energy emitted by the source is same regardless of whoever measures it, i.e., it is independent of any observer standing in that region. But the sound received by the two observers may be different; this is due to some factors like sensitivity of ears, etc. To quantify such thing, we define two different quantities known as intensity and loudness of sound.

**Intensity of sound**

When a sound wave is emitted by a source, the energy is carried to all possible surrounding points. The average sound energy emitted or transmitted per unit time or per second is called sound power. Therefore, the intensity of sound is defined as "the sound power transmitted per unit area taken normal to the propagation of the sound wave".

For a particular source (fixed source), the sound intensity is inversely proportional to the square of the distance from the source.

$$I = \frac{\text{power of the source}}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

This is known as inverse square law of sound intensity.

**Example**
A baby cries on seeing a dog and the cry is detected at a distance of 3.0 m such that the intensity of sound at this distance is $10^{-2}$ W $m^{-2}$. Calculate the intensity of the baby's cry at a distance 6.0 m.

**Solution**

$I_1$ is the intensity of sound detected at a distance 3.0 m and it is given as $10^{-2}$ W $m^{-2}$. Let $I_2$ be the intensity of sound detected at a distance 6.0 m. Then, $r_1$ = 3.0 m, $r_2$ = 6.0 m

and since $I \alpha \frac{1}{r^2}$

the power output does not depend on the observer and depends on the baby. Therefore,

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

$$I_2 = I_1 \frac{r_1^2}{r_2^2}$$

$$I_2 = 0.25 \times 10^{-2} \ W \ m^{-2}$$

**Loudness of sound**

Two sounds with same intensities need not have the same loudness. For example, the sound heard during the explosion of balloons in a silent closed room is very loud when compared to the same explosion happening in a noisy market. Though the intensity of the sound is the same, the loudness is not. If the intensity of sound is increased then loudness also increases. But additionally, not only does intensity matter, the internal and subjective experience of "how loud a sound is" i.e., the sensitivity of the listener also matters here. This is often called loudness. That is, loudness depends on both intensity of sound wave and sensitivity of the ear (It is purely observer dependent quantity which varies from person to person) whereas the intensity of sound does not depend on the observer. The loudness of sound is defined as "the degree of sensation of sound produced in the ear or the perception of sound by the listener".

**Intensity and loudness of sound**

Our ear can detect the sound with intensity level ranges from $10^{-2}$ $Wm^{-2}$to 20 W $m^{-2}$.

According to Weber-Fechner's law, "loudness (L) is proportional to the logarithm of the actual intensity (I) measured with an accurate non-human instrument". This means that

$$L \propto \ln I$$
$$L = k \ln I$$

where k is a constant, which depends on the unit of measurement. The difference between two loudnesses, L1 and L0 measures the relative loudness between two precisely measured intensities and is called as sound intensity level. Mathematically, sound intensity level is

$$\Delta L = L_1 - L_0 = k \ln I_1 - k \ln I_0 = k \ln \left[ \frac{I_1}{I_0} \right]$$

If k = 1, then sound intensity level is measured in bel, in honour of Alexander Graham Bell. Therefore,

$$\Delta L = \ln\left[\frac{I_1}{I_0}\right] bel$$

However, this is practically a bigger unit, so we use a convenient smaller unit, called decibel. Thus, decibel $= \frac{1}{10}$ bel. Therefore, by multiplying and dividing by 10, we get

$$\Delta L = 10\left(\ln\left[\frac{I_1}{I_0}\right]\right)\frac{1}{10} \text{ bel}$$

$$\Delta L = 10 \ln\left[\frac{I_1}{I_0}\right] \text{ decibel with } k = 10$$

For practical purposes, we use logarithm to base 10 instead of natural logarithm,

$$\Delta L = 10 \log_{10}\left[\frac{I_1}{I_0}\right] \text{ decibel}$$

**Example**

The sound level from a musical instrument playing is 50 dB. If three identical musical instruments are played together then compute the total intensity. The intensity of the sound from each instrument is $10^{-12}$ W m$^{-2}$

**Solution**

$$\Delta L = 10 \log_{10}\left[\frac{I_1}{I_0}\right] = 50\ dB$$

$$\log_{10}\left[\frac{I_1}{I_0}\right] = 5\ dB$$

$\frac{I_1}{I_0} = 10^5 \Rightarrow I_1 = 10^5 I_0 = 10^5 \times 10^{-12} Wm^{-2}$

$I_1 = 10^{-7} Wm^{-2}$

Since three musical instruments are played, therefore, $I_{total} = 3I_1 = 3 \times 10^{-7}$ Wm$^{-2}$.

**VIBRATIONS OF AIR COLUMN**

Musical instruments like flute, clarinet, nathaswaram, etc are known as wind instruments. They work on the principle of vibrations of air columns. The simplest form of a wind instrument is the organ pipe. It is made up of a wooden or metal pipe which produces the musical sound. For example, flute, clarinet and nathaswaram are organ pipe instruments. Organ pipe instruments are classified into two types:

**(a) Closed organ pipes:**

It is a pipe with one end closedand the other end open. If one end of a pipeis closed, the wave reflected at this closedend is 180° out of phase with the incomingwave. Thus there is no displacement of the particles at the closed end. Therefore, nodes are formed at the closed end and anti-nodes are formed at open end.

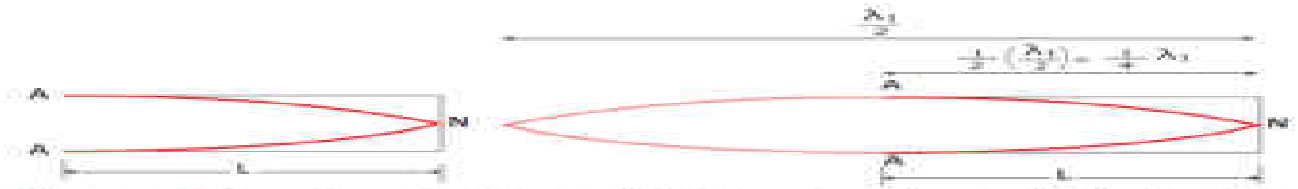


Figure 11.37 No motion of particles which leads to nodes at closed end and antinodes at open end (fundamental mode) (N-node, A-antinode)

Let us consider the simplest mode of vibration of the air column called the fundamental mode. Anti-node is formed at the open end and node at closed end. From the Figure, let L be the length of the tube and the wavelength of the wave produced. For the fundamental mode of vibration, we have,

$$L = \frac{\lambda_1}{4} \text{ or } \lambda_1 = 4L$$

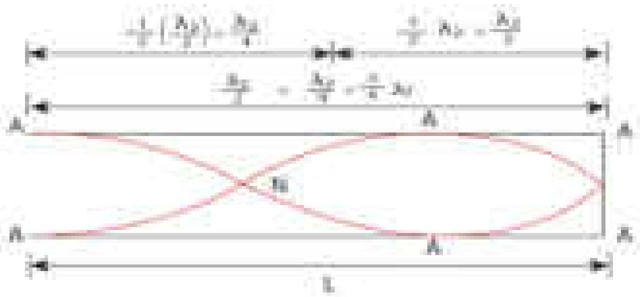
The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

which is called the fundamental note.

The frequencies higher than fundamental frequency can be produced by blowing air strongly at open end. Such frequencies are called overtones.

The Figure shows the second mode of vibration having two nodes and two antinodes, for which we have, from example.



second mode of vibration
having two nodes and two anti-nodes

$$4L = 3\lambda_2$$

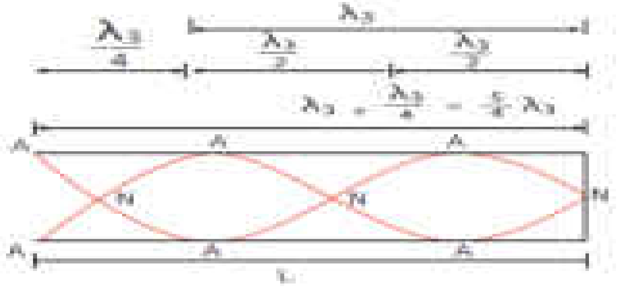
$$L = \frac{3\lambda_2}{4} \; or \; \lambda_2 = \frac{4L}{3}$$

The frequency for this,

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1$$

is called first over tone, since here, the frequency is three times the fundamental frequency it is called third harmonic.

The Figure shows third mode of vibration having three nodes and three anti-nodes.



Third mode of vibration
having three nodes and three anti-nodes

we have,

$$4L = 5\lambda_3$$

$$L = \frac{5\lambda_3}{4} \; or \; \lambda_3 = \frac{4L}{5}$$

The frequency

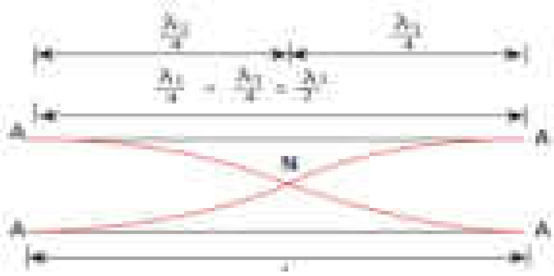
$$f^3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5f_1$$

is called second over tone, and since n = 5 here, this is called fifth harmonic. Hence, the closed organ pipe has only odd harmonics and frequency of the $n^{th}$ harmonic is $f_n = (2n+1)f_1$. Therefore, the frequencies of harmonics are in the ratio

$$f_1 : f_2 : f_3 : f_4 : \ldots = 1 : 3 : 5 : 7 : \ldots$$

**(b) Open organ pipes:**

It is a pipe with both the ends open. At both open ends, anti-nodes are formed. Let us consider the simplest mode of vibration of the air column called fundamental mode. Since anti-nodes are formed at the open end, a node is formed at the mid-point of the pipe.




Antinodes are formed at
the open end and a node is formed at
the middle of the pipe.

From Figure, if L be the length of the tube, the wavelength of the wave produced is given by

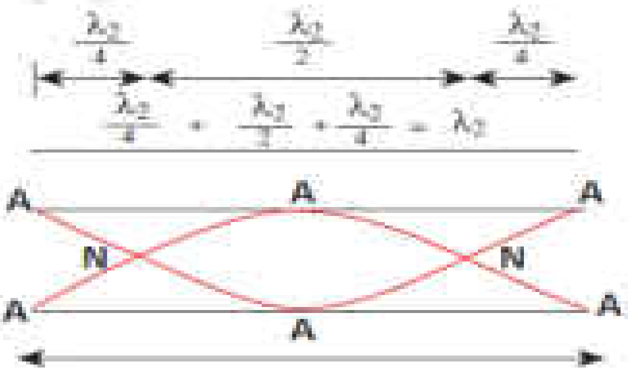
$$L = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2L$$

The frequency of the note emitted is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

which is called the fundamental note.

The frequencies higher than fundamental frequency can be produced by blowing air strongly at one of the open ends. Such frequencies are called overtones.




Second mode of
vibration in open pipes having two
nodes and three anti-nodes

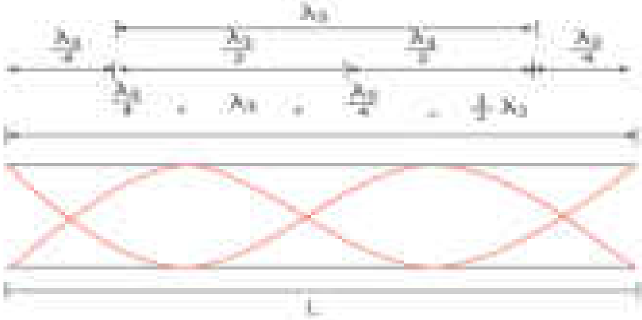
The Figure shows the second mode of vibration in open pipes. It has two nodes and three anti-nodes, and therefore,

$$L = \lambda_2 \text{ or } \lambda_2 = L$$

The frequency

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \times \frac{v}{2L} = 2\, f_1$$

is called first over tone. Since n = 2 here, it is called the second harmonic.



Third mode of vibration
having three nodes and four anti-nodes

The Figure above shows the third mode of vibration having three nodes and four anti-nodes

$$L = \frac{3}{2}\lambda_3 \; or \; \lambda_3 = \frac{2L}{3}$$

The frequency

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3\, f_1$$

is called second over tone. Since n = 3 here, it is called the third harmonic.

Hence, the open organ pipe has all the harmonics and frequency of nth harmonic is $f_n = nf_1$. Therefore, the frequencies of harmonics are in the ratio

$$f_1 : f_2 : f_3 : f_4 : \ldots = 1 : 2 : 3 : 4 : \ldots$$

**Example**

If a flute sounds a note with 450Hz, what are the frequencies of the second, third, and fourth harmonics of this pitch?. If the clarinet sounds with a same note as 450Hz, then what are the frequencies of the lowest three harmonics produced ?.

**Solution**

For a flute which is an open pipe, we have
Second harmonics f2 = 2 f1 = 900 Hz
Third harmonics f3 = 3 f1 = 1350 Hz
Fourth harmonics f4 = 4 f1 = 1800 Hz

For a clarinet which is a closed pipe, we have
Second harmonics f2 = 3 f1 = 1350 Hz
Third harmonics f3 = 5 f1 = 2250 Hz
Fourth harmonics f4 = 7 f1 = 3150 Hz

**Example**

If the third harmonics of a closed organ pipe is equal to the fundamental frequency of an open organ pipe, compute the length of the open organ pipe if the length of the closed organ pipe is 30 cm.

**Solution**

Let $l_2$ be the length of the open organ pipe, with $l_1$ =30 cm the length of the closedorgan pipe.It is given that the third harmonic of closed organ pipe is equal to the fundamental frequency of open organ pipe.
The third harmonic of a closed organ pipe is

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4l_1} = 3f_1$$

The fundamental frequency of open organ pipe is $f_1 = \frac{v}{\lambda_1} = \frac{v}{2l_2}$

Therefore,

$$\frac{v}{2l_2} = \frac{3v}{4l_1} \Rightarrow l_2 = \frac{2l_1}{3} = 20\,cm$$

**Resonance air column apparatus**

The resonance air column apparatus is one of the simplest techniques to measure the speed of sound in air at room temperature. It consists of a cylindrical glass tube of one meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube as shown in Figure. This cylindrical glass tube is mounted on a vertical stand with a scale attached to it. The tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R. The surface of the water will act as a closed end and other as the open end. Therefore, it behaves like a closed organ pipe, forming nodes at the surface of water and antinodes at the closed end. When a vibrating tuning fork is brought near the open end of the tube, longitudinal waves are formed inside the air column. These waves move downward as shown in Figure, and reach the surfaces of water and get reflected and produce standing waves. The length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork). At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the length of air column is proportional to $\left(\frac{1}{4}\right)^{th}$ of the wavelength of the sound waves produced.

Let the first resonance occur at length $L_1$, then

$$\frac{1}{4}\lambda = L_1$$

But since the antinodes are not exactly formed at the open end, we have to include a correction, called end correction e, by assuming that the antinode is formed at some small distance above the open end. Including this end correction, the first resonance is

$$\frac{1}{4}\lambda = L_1 + e$$

Now the length of the air column is increased to get the second resonance. Let $L_2$ be the length at which the second resonance occurs. Again taking end correction into account, we have

$$\frac{3}{4}\lambda = L_2 + e$$

In order to avoid end correction, let us take the difference of equation we get

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e) - (L_1 + e)$$

$$\Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L$$

$$\Rightarrow \lambda = 2\Delta L$$

The speed of the sound in air at room temperature can be computed by using the formula

$$v = f\lambda = 2f\,\Delta L$$

Further, to compute the end correction, we use equations, we get

$$e = \frac{L_2 - 3L_1}{2}$$

**Example**

A frequency generator with fixed frequency of 343 Hz is allowed to vibrate above a 1.0 m high tube. A pump is switched on to fill the water slowly in the tube. In order to get resonance, what must be the minimum height of the water?. (speed of sound in air is 343 m $s^{-1}$)

**Solution**

The wavelength, $\lambda = \frac{c}{f}$

$$\lambda = \frac{343ms^{1}}{343Hz} = 1.0m$$

Let the length of the resonant columns be $L_1$, $L_2$ and $L_3$. The first resonance occurs at length $L_1$

$$L_1 = \frac{\lambda}{4} = \frac{1}{4} = 0.25m$$

The second resonance occurs at length $L_2$

$$L_2 = \frac{3\lambda}{4} = \frac{3}{4} = 0.75m$$

The third resonance occurs at length

$$L_3 = \frac{5\lambda}{4} = \frac{5}{4} = 1.25m$$

and so on.

Since total length of the tube is 1.0 m the third and other higher resonances do not occur. Therefore, the minimum height of water $H_{min}$ for resonance is,

$H_{min}$ = 1.0 m − 0.75 m = 0.25 m

**Example**

A student performed an experiment to determine the speed of sound in air using the resonance column method. The length of the air column that resonates in the fundamental mode with a tuning fork is 0.2 m. If the length is varied such that the same tuning fork resonates with the first overtone at 0.7 m. Calculate the end correction.

**Solution**

End correction

$$e = \frac{L_2 - 3L_1}{2} = \frac{0.7 - 3(0.2)}{2} = 0.05m$$

**Example**

Consider a tuning fork which is used to produce resonance in an air column. A resonance air column is a glass tube whose length can be adjusted by a variable piston. At room temperature, the two successive resonances observed are at 20 cm and 85 cm of the column length. If the frequency of the length is 256 Hz, compute the velocity of the sound in air at room temperature.

**Solution**

Given two successive length (resonance) to be $L_1$ = 20 cm and $L_2$ = 85 cm
The frequency is f = 256 Hz

$$v = f\lambda = 2f\,\Delta L = 2f\,(L_2 - L_1)$$
$$= 2 \times 256 \times (85 - 20) \times 10^{-2}\ m\ s^{-1}$$
$$v = 332.8\ cm^{-1}$$

**DOPPLER EFFECT**

Often we have noticed that the siren sound coming from a police vehicle or ambulance increases when it comes closer to us and decreases when it moves away from us. When we stand near any passing train the train whistle initially increases and then it will decrease. This is known as Doppler Effect, named after Christian Doppler (1803 – 1853). Suppose a source produces sound with some frequency, we call it the as source frequency fs. If the source and an observer are at a fixed distance then the observer observes the sound with frequency $f_0$. This is the same as the sound frequency produced by the source $f_s$, i.e., $f_0 = f_s$. Hence, there is no difference in frequency, implying no Doppler effect is observed.

What happens if either source or an observer or both move?. Certainly, $fo \neq fs$. That is, when the source and the observer are in relative motion with respect to each other and to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect. The frequency perceived by the observer is known as apparent frequency. We can consider the following situations for the study of Doppler effect in sound waves

**(a) Source and Observer:** We can consider either the source or observer in motion or both are in motion. Further we can treat the motion to be along the line joining the source and the observer, or inclined at an angle θ to this line.

**(b) Medium:** We can treat the medium to be stationary or the direction of motion of the medium is along or opposite to the direction of propagation of sound.

**(c) Speed of Sound:** We can also consider the case where speed of the source or an observer is greater or lesser than the speed of sound.

In the following section, we make the following assumptions: the medium is stationary, and motion is along the line joining the source and the observer, and the speeds of the source and the observer are both less than the speed of sound in that medium.

We consider three cases:
(i) Source in motion and Observer is at rest.
(a) Source moves towards observer

(b) Source moves away from the observer

(ii) Observer in motion and Source is at rest.
(a) Observer moves towards Source
(b) Observer receding away from the Source

(iii) Both are in motion
(a) Source and Observer approach each other
(b) Source and Observer recede from each other
(c) Source chases Observer
(d) Observer chases Source

| Stationary observer and stationary source means the observer and source are both at rest with respect to medium respectively |
| --- |

**Source in motion and the observer at rest**

(a) Source moves towards the observer Suppose a source S moves to the right (as shown in Figure) with a velocity vs and let the frequency of the sound waves produced by the source be fs. We assume the velocity of sound in a medium is v. The compression (sound wave front) produced by the source S at three successive instants of time are shown in the Figure. When S is at position x1 the compression is at $C_1$. When S is at position x2, the compression is at $C_2$ and similarly for $x_3$ and $C_3$. Assume that if $C_1$ reaches the observer's position A then at that instant $C_2$ reaches the point B and $C_3$ reaches the point C as shown in the Figure 11.46. It is obvious to see that the distance between compressions $C_2$ and $C_3$ is shorter than distance between $C_1$ and $C_2$. This means the wavelength decreases when the source S moves towards the observer O (since sound travels longitudinally and wavelength is the distance between two consecutive compressions). But frequency is inversely related to wavelength and therefore, frequency increases.

Let λ be the wavelength of the source S as measured by the observer when S is at position $x_1$ and λ′ be wavelength of the source observed by the observer when S moves to position $x_2$. Then the change in wavelength is $\Delta\lambda = \lambda - \lambda' = v_s t$, where t is the time taken by the source to travel between $x_1$ and $x_2$. Therefore,

$$\lambda' = \lambda - v_s t$$

$$\text{But} \quad t = \frac{\lambda}{v}$$

On substituting equation (11.84) in equation (11.83), we get

$$\lambda' = \lambda\left(1 - \frac{v_s}{v}\right)$$

Since frequency is inversely proportional to wavelength, we have

$$f' = \frac{v}{\lambda'} \text{ and } f = \frac{v}{\lambda}$$

Hence, $$f' = \frac{f}{\left(1 - \frac{v_s}{v}\right)}$$

Since, $\frac{v_s}{v} \ll 1$, we use the binomial expansion and retaining only first order in $\frac{v_s}{v}$, we get

$$f' = f\left(1 + \frac{v_s}{v}\right)v$$

**(b) Source moves away from the observer:**

Since the velocity here of the source is opposite in direction when compared to case (a), therefore, changing the sign of the velocity of the source in the above case i.e, by substituting ($v_s \rightarrow -v_s$) in equation (11.83), we get

$$f' = \frac{f}{\left(1 + \frac{v_s}{v}\right)}$$

Using binomial expansion again, we get,

$$f' = f\left(1 - \frac{v_s}{v}\right)$$

**Observer in motion and source at rest**

(a) Observer moves towards Source

Let us assume that the observer O moves towards the source S with velocity $v_o$. The source S is at rest and the velocity of sound waves (with respect to the medium) produced by the source is v. From the Figure, we observe that both $v_o$ and v are in opposite direction. Then, their relative velocity is $v_r = v + v_0$. The wavelength of the sound wave is $\lambda = \frac{v}{f}$, which means the frequency observed by the observer O is $f_1 = \frac{v_r}{\lambda}$. Then

$$f' = \frac{v_r}{\lambda} = \left(\frac{v + v_o}{v}\right)f = f\left(1 + \frac{v_o}{v}\right)$$

**(b) Observer recedes away from the Source**

If the observer O is moving away (receding away) from the source S, then velocity $v_0$ and v moves in the same direction. Therefore, their relative velocity is $v_r = v - v_0$. Hence, the frequency observed by the observer O is

$$f' = \frac{v_r}{\lambda} = \left(\frac{v - v_0}{v}\right) f = f\left(1 - \frac{v_0}{v}\right)$$

**Both are in motion**
**(a) Source and observer approach each other**

Source and Observer approach towards each other.

Let vs and v0 be the respective velocities of source and observer approaching each other. In order to calculate the apparent frequency observed by the observer, as a simple calculation, let us have a dummy (behaving as observer or source) in between the source and observer. Since the dummy is at rest, the dummy (observer) observes the apparent frequency due to approaching source as given in equation as

$$f_d = \frac{f}{\left(1 - \frac{v_s}{v}\right)}$$

At that instant of time, the true observer approaches the dummy from the other side. Since the source (true source) comes in a direction opposite to true observer, the dummy (source) is treated as stationary source for the true observer at that instant. Hence, apparent frequency when the true observer approaches the stationary source (dummy source), from equation is

$$f' = f_d\left(1 + \frac{v_0}{v}\right) \Rightarrow f_d = \frac{f'}{\left(1 + \frac{v_0}{v}\right)}$$

Since this is true for any arbitrary time, therefore, comparing equation (11.91) and equation (11.92), we get

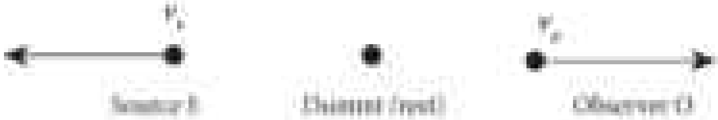
$$\frac{f}{\left(1 - \frac{v_s}{v}\right)} = \frac{f'}{\left(1 + \frac{v_0}{v}\right)}$$

$$\Rightarrow \frac{v f'}{(v + v_0)} = \frac{v f}{(v - v_s)}$$

Hence, the apparent frequency as seen by the observer is

$$f' = \left(\frac{v + v_0}{v - v_s}\right) f$$

**(b) Source and observer recede from each other**

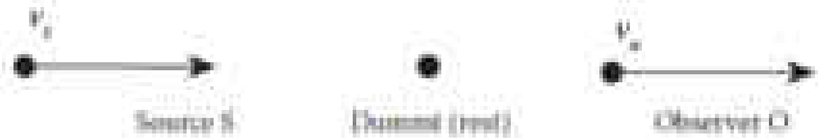


Source and Observer resides from each other

Here, we can derive the result as in the previous case. Instead of a detailed calculation, by inspection from Figure, we notice that the velocity of the source and the observer each point in opposite directions with respect to the case in (a) and hence, we substitute ($v_s \rightarrow -v_s$) and ($v_0 \rightarrow -v_0$) in equation, and therefore, the apparent frequency observed by the observer when the source and observer recede from each other is

$$f' = \left(\frac{v + v_0}{v - v_s}\right) f$$

**(c) Source chases the observer**

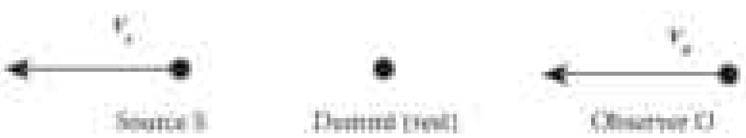



Source chases observer

Only the observer's velocity is oppositely directed when compared to case (a). Therefore, substituting ($v0 \rightarrow -v0$) in equation, we get

$$f' = \left(\frac{v - v_0}{v - v_s}\right) f$$

**(d) Observer chases the source**



Observer chases Source

Only the source velocity is oppositely directed when compared to case (a). Therefore, substituting $v_s \rightarrow -v_s$ in equation, we get

$$f' = \left(\frac{v \pm v_o}{v \mp v_s}\right) f$$

Discuss with your teacher
"Doppler effect in light"
"Doppler effect in sound is asymmetrical where as Doppler effect in light is symmetrical"

**Applications of Doppler effect**

Doppler effect has many applications. Specifically Doppler effect in light has many applications in astronomy. As an example, while observing the spectra from distant objects like stars or galaxies, it is possible to determine the velocities at which distant objects like stars or galaxies move towards or away from Earth. If the spectral lines of the star are found to shift towards red end of the spectrum (called as red shift) then the star is receding away from the Earth. Similarly, if the spectral lines of the star are found to shift towards the blue end of the spectrum (called as blue shift) then the star is approaching Earth. Let $\Delta\lambda$ be the Doppler shift. Then $\Delta\lambda = \frac{v}{c}\lambda$ where v is the velocity of the star. It may be noted that Doppler shift measures only the radial component (along the line of sight) of the relative velocity v.

**Example**

A sound of frequency 1500 Hz is emitted by a source which moves away from an observer and moves towards a cliff at a speed of 6 $ms_{-1}$.
(a) Calculate the frequency of the sound which is coming directly from the source.
(b) Compute the frequency of sound heard by the observer reflected off the cliff. Assume the speed of sound in air is 330 m s-1

**Solution**
(a) Source is moving away and observer is stationary, therefore, the frequency of sound heard directly from source is

$$f' = \frac{f}{\left(1 + \frac{v_s}{v}\right)} = \frac{1500}{\left(1 + \frac{6}{330}\right)} = 1473\,Hz$$

(b) Sound is reflected from the cliff and reaches observer, therefore,

$$f' = \frac{f}{\left(1 - \frac{v_s}{v}\right)} = \frac{1500}{\left(1 - \frac{6}{330}\right)} = 1528\,Hz$$

**Example**

An observer observes two moving trains, one reaching the station and other leaving the station with equal speeds of 8 m s$^{-1}$. If each train sounds its whistles with frequency 240 Hz, then calculate the number of beats heard by the observer.

**Solution:**

Observer is stationary

(i) Source (train) is moving towards an observer:

Apparent frequency due to train arriving station is

$$f_{in} = \frac{f}{\left(1 - \frac{v_s}{v}\right)} = \frac{240}{\left(1 - \frac{8}{330}\right)} = 246\,Hz$$

(ii) Source (train) is moving away form an observer:

Apparent frequency due to train leaving station is

$$f_{out} = \frac{f}{\left(1 - \frac{v_s}{v}\right)} = \frac{240}{\left(1 - \frac{8}{330}\right)} = 234\,Hz$$

So the number of beats = | $f_{in}$ -$f_{out}$ | = (246-234) = 12

**12th Volume II**
**Unit 6 – Ray Optics**

**Introduction**

Light is mystical. Yet, its behaviour is so fascinating. It is difficult to comprehend light as a single entity. The ray optics deals with light that is represented as a ray travelling in straight lines. Here, the geometrical constructs get the permanence to understand some of the characteristics of light and the phenomena associated with it. There are several other phenomena which can only be explained using wave optics, which we study in the next Unit. There is also a quantum aspect of light which we can study as quantum optics in graduate level courses.

**Ray optics**

Light travels in a straight line in a medium. Light may deviate in its path only when it encounters another medium or an obstacle. A ray of light gives information about only the direction of light. It does not give information about the other characteristics of light like intensity and colour. However, a ray is a sensible representation of light in ray optics. The path of the light is called a ray of light and a bundle of such rays is called a beam of light. In this chapter, we can explain the phenomena of reflection, refraction, dispersion and scattering of light, using the ray depiction of light.

**Reflection**

The bouncing back of light into the same medium when it encounters a reflecting surface is called reflectionof light. Polished surfaces can reflect light. Mirrors which are silver coated at their back can reflect almost 90% of the light falling on them. The angle of incidence $i$ and the angle **of reflection $r$ are measured with respect to the normal drawn to the surface at the point of incidence of light. According to law of reflection,**

1. The incident ray, reflected ray and normal to the reflecting surface all are coplanar (ie. lie in the same plane).
2. The angle of incidence $i$ is equal to the angle of reflection $r$.

$$i = r$$

The laws of reflection are valid at each point for any reflecting surface whether thesurface is flat (or) curved. If the reflecting surface is flat, then incident parallel rays after reflection come out as parallel rays as shown in. If the reflecting surface is irregular, then the incident parallel rays after reflection come out as irregular rays (not parallel rays). Still the laws of reflection are valid at every point of incidence in irregular reflection as shown in

**Angle of deviation due to reflection**

The angle between the incident and deviated light ray is called angle of deviation of the light ray.The incident light is AO. The reflected light is OB. The un-deviated light is OC which is the continuation of the incident light. The angle between OB and OC is the angle of deviation d. From the geometry, it is written as, d = 180 – (i+r). As, i = r in reflection, we can write angle of deviation in reflection at plane surface as,

$$d = 180 - 2i$$

The angle of deviation can also be measured in terms of the glancing angle $a$ which is measured between the incident ray AO and the reflecting plane surface XY as By geometry, the angles $\angle AOX = \alpha$, $\angle BOY = \alpha$ and $\angle YOC = \alpha$ (are all same). The angle of deviation (d) is the angle $\angle BOC$. Therefore,

$$d = 2\alpha$$

**Image formation in plane mirror**

Let us consider a point object A is placed in front of a plane mirror and the point of incidence is O on the mirror. A light ray AO from the point object is incident on the mirror and it is reflected along OB. The normal is ON.

The angle of incidence $\angle AON$ = angle of reflection $\angle BON$

Another ray $AD$ incident normally on the mirror at $D$ is reflected back along $DA$. When $BO$ and $AD$ are extended backwards, they meet at a point. Thus, the rays appear to come from a point which is behind the plane mirror. The object and its image in a plane mirror are at equal perpendicular distances from the plane mirror which can be shown by the following explanation.

Angle ∠AON = angle ∠DAO [Since they are alternate angles]

Angle ∠BON = angle ∠OA'D [Since they are corresponding angles]

Hence, it follows that angle, ∠DAO = ∠OA'D

The triangles ΔODA and ΔODA' are congruent

$$\therefore AD = A'D$$

This shows that the image distance $d_i$ inside the plane mirror is equal to the object distance $d_o$ in front of the plane mirror.

The image formed by the plane mirror for extended object is shown in

**Characteristics of the image formed by plane mirror**

1. The image formed by a plane mirror is virtual, erect, and laterally inverted.
2. The size of the image is equal to the size of the object.
3. The image distance far behind the mirror is equal to the object distance in front of it.
4. If an object is placed between two plane mirrors inclined at an angle θ, then the number of images n formed is as,

**Image by inclined mirrors**

| $\left(\frac{360}{\theta}\right)$ | **The position of object placed** | **Number of images n** |
|---|---|---|
| Even | Symmetrical | n = $\left(\frac{360}{\theta}-1\right)$ |
| | Unsymmetrical | n = $\left(\frac{360}{\theta}-1\right)$ |
| Odd | Symmetrical | n= $\left(\frac{360}{\theta}-1\right)$ |
| | Unsymmetrical | n = $\left(\frac{360}{\theta}\right)$ |

**Spherical Mirrors**

We shall now study about the reflections that take place in spherical surfaces.

A spherical surface is a part cut from a hollow sphere. Spherical mirrors are generally constructed from glass. One surface of the glass is silvered. The reflection takes place at the other polished surface. If the reflection takes place at the convex surface, it is called a convex mirror and if the reflection takes place at the concave surface, it is called a concave mirror.

We shall now become familiar with some of the terminologies pertaining to spherical mirrors. Centre of curvature: The centre of the sphere of which the mirror is a part is called the center of curvature (C) of the mirror.

**Centre of curvature:** The centre of the sphere of which the mirror is a part is called the centre of curvature C of the mirror.

**Radius of curvature:** The radius of the sphere of which the spherical mirror is a part is called the radius of curvature (R) of the mirror.

**Pole:** The middle point on the spherical surface of the mirror (or) the geometrical center of the mirror is called pole (P) of the mirror.

**Principal axis:** The line joining the pole and the centre of curvature is called the principal axis of the mirror. The light ray travelling along the principal axis towards the mirror after reflection travels back along the same principal axis. It is also called optical axis

**Focus (or) Focal point**: Light rays travelling parallel and close to the principal axis when incident on a spherical mirror, converge at a point for concave mirror or appear to diverge from a point for convex mirror on the principal axis. This point is called the focus or focal point (F) of the mirror.

**Focal length:** The distance between the pole and the focus is called the focal length (f) of the mirror.

**Focal plane:** The plane through the focus and perpendicular to the principal axis is called the focal plane of the mirror.

**Paraxial Rays and Marginal Rays**

The paraxial rays are the rays which travel very close to the principal axis and make small angles with it. They fall on the mirror very close to the pole. On the other hand, the marginal rays are the rays which travel far away from the principal axis and make large angles with it. They fall on the mirror far away from the pole. These two rays behave differently (get focused at different points) as shown in. In this chapter, we shall restrict our studies only to paraxial rays. As the angles made by the paraxial rays are very small, we can make good approximations.

**Relation between F and R**

Let C be the centre of curvature of the mirror. Consider a light ray parallel to the principal axis is incident on the mirror at M and passes through the principal focus F after reflection. The line CM is the normal to the mirror at M. Let i be the angle of incidence and the same will be the angle of reflection.

If MP is the perpendicular from M on the principal axis, then

The angles ∠MCP = i and ∠MFP = 2i
From right angle triangles ΔMCP and ΔMFP, we can write,

$$\tan i = \frac{PM}{PC} \text{ and } \tan 2i = \frac{PM}{PF}$$

As the angles are small, $\tan i \approx i$ and $\tan 2_i \approx 2_i$,

$$i = \frac{PM}{PC} \text{ and } 2i = \frac{PM}{PF}$$

simplifying further,

$$2\frac{PM}{PC} = \frac{PM}{PF}; 2PF = PC$$

PF is focal length f and PC is the radius of curvature R.

$$2f = R \text{ (or) } f = \frac{R}{2}$$

**Image formation in spherical mirrors**

The image can be located by graphical construction. To locate the point of an image, a minimum of two rays must meet at that point. We can use at least any two of the following four rays as shown in

1. A ray parallel to the principal axis after reflection will pass through or appear to pass through the principal focus.

2. A ray passing through or appear to pass through the principal focus, after reflection will travel parallel to the principal axis.
3. A ray passing through the centre of curvature retraces its path after reflection as it is a case of normal incidence.
4. A ray falling on the pole will get reflected as per law of reflection keeping principal axis as the normal.

**Cartesian sign convention**

While tracing the image, we would normally come across the object distance u, the image distance v, the object height h, the image height (h'), the focal length f and the radius of curvature R. A system of signs for these quantities must be followed so that the relations connecting them are consistent in all types of physical situations. We shall follow the Cartesian sign convention.

1. The Incident light is taken from left to right (i.e. object on the left of mirror).
2. All the distances are measured from the pole of the mirror (pole is taken as origin).
3. The distances measured to the right of pole along the principal axis are taken as positive.
4. The distances measured to the left of pole along the principal axis are taken as negative.
5. Heights measured in the upward perpendicular direction to the principal axis are taken as positive.
6. Heights measured in the downward perpendicular direction to the principal axis, are taken as negative.

**Mirror equation**

The mirror equation establishes a relation among object distance $u$, image distance $v$ and focal length $f$ for a spherical mirror.

An object AB is considered on the principal axis of a concave mirror beyond the center of curvature C. The first paraxial ray BD travelling parallel to principal axis is incident on the concave mirror at D, close to the pole P. After reflection the ray passes through the focus F. The second paraxial ray BP incident at the pole P is reflected along PB′. The third paraxial ray BC passing through centre of curvature C, falls normally on the mirror at E is reflected back along the same path. The three reflected rays intersect at the point . A perpendicular drawn as to the principal axis is the real, inverted image of the object AB.

As per law of reflection, the angle of incidence $\angle BPA$ is equal to the angle of reflection ∠B'PA'.

The triangles $\Delta BPA$ and Δare similar. Thus, from the rule of similar triangles.

$$\frac{A'B'}{AB} = \frac{PA'}{PA}$$

The other set of similar triangles are, $\Delta DPF$ and $\Delta B'A'F$. ($PD$ is almost a straight vertical line)

$$\frac{A'B}{PD} = \frac{A'F}{PF}$$

As, $PD = AB$ the above equation becomes,

$$\frac{A'B'}{AB} = \frac{A'F}{PF}$$

$$\frac{PA'}{PA} = \frac{A'F}{PF}$$

As, A′F = PA′ - PF, the above equation becomes,

$$\frac{PA'}{PA} = \frac{PA' - PF}{PF}$$

We can apply the sign conventions for the various distances in the above equation.

PA = -u,  PA′ = -v,  PF = -f

All the three distances are negative as per sign convention, because they are measured to the left of the pole.

$$\frac{-v}{-u} = \frac{-v - (-f)}{f}$$

On further Simplification,

$$\frac{v}{u} = \frac{v - f}{f}; \qquad \frac{v}{u} = \frac{v}{f} - 1$$

Dividing either side with v,

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

After rearranging,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

The above equation  is called mirror equation. Although this equation is derived for a special it is also valid for all other situations with any spherical mirror. This is because proper sign convention is followed for $u$, $v$ and $f$ in equation.

**Lateral magnification in spherical mirrors**

The lateral or transverse magnification is defined as the ratio of the height of the image to the height of the object. The height of the object and image are measured perpendicular to the principal axis.

magnification (m) = $\frac{height\ of\ the\ image\ (h')}{height\ of\ the\ object\ (h)}$

$$m = \frac{h'}{h}$$

Applying proper sign conventions for

$$\frac{A'B'}{AB} = \frac{PA'}{PA}$$

A′B′ = -h, AB = h, PA′ = -v, PA = -u

$$\frac{-h'}{h} = -\frac{v}{u}$$

On simplifying we get,

$$m = \frac{h'}{h} = -\frac{v}{u}$$

Using mirror equation, we can further write the magnification as,

$$m = \frac{h'}{h} = \frac{f - v}{f} = \frac{f}{f - u}$$

**Speed of Light**

Light travels with the highest speed in vacuum. The speed of light in vacuum is denoted as c and its value is, c = 3×108 m s-1. It is a very high value. Several attempts were made by scientists to determine the speed of light. The earliest attempt was made by a French scientist HippolyteFizeau (1819–1896). That paved way for the other scientists too to determine the speed of light.

**Fizeau's method to determine speed of light**

**Apparatus:** The apparatus used by Fizeau for determining speed of light in air. The light from the source S was first allowed to fall on a partially silvered glass plate G kept at an angle of 45o to the incident light from the source. The light then was allowed to pass through a rotating toothed-wheel with N teeth and N cuts of equal widths whose speed of rotation could be varied through an external mechanism. The light passing through one cut in the wheel will get reflected by a mirror M kept at a long distance d, about 8 km from the toothed wheel. If the toothed wheel was not rotating, the reflected light from the mirror would again pass through the same cut and reach the eyes of the observer through the partially silvered glass plate.

**Working:** The angular speed of rotation of the toothed wheel was increased from zero to a value ω until light passing through one cut would completely be blocked by the adjacent tooth. This is ensured by the disappearance of light while looking through the partially silvered glass plate.

**Expression for speed of light**: The speed of light in air v is equal to the ratio of the distance the light travelled from the toot hed wheel to the mirror and back 2d to the time taken t.

$$v = \frac{2d}{t}$$

The distance d is a known value from the arrangement. The time taken t for the light to travel the distance to and fro is calculated from the angular speed ω of the toothed wheel.

The angular speed ω of the toothed wheel when the light disappeared for the first time is,

$$\omega = \frac{\theta}{t}$$

Here, $\theta$ is the angle between the tooth and the slot which is rotated by the toothed wheel within that time $t$.

The angular speed ω (with unit rad $s^{-1}$) of the toothed-wheel when the light disappeared for the first time is,

$$\omega = \frac{\theta}{t}$$

Here, θ is the angle between one tooth and the next slot which is turned within that time t.

$$\theta = \frac{total\ angle\ of\ the\ circel\ in\ radian}{number\ of\ teeth + number\ of\ cuts}$$

$$\theta = \frac{2\pi}{2N} = \frac{\pi}{N}$$

Substituting for θ in the equation for ω

$$\omega = \frac{\pi / N}{t} = \frac{\pi}{Nt}$$

Rewriting the above equation for t,

$$t = \frac{\pi}{N\omega}$$

Substituting t from equation

$$v = \frac{2d}{\pi / N\omega}$$

After rearranging

$$v = \frac{2dN\omega}{\pi}$$

Fizeau had some difficulty to visually estimate the minimum intensity of the light when blocked by the adjacent tooth, and his value for speed of light was very close to the actual value. Later on, with the same idea of Fizeau and with much sophisticated instruments, the speed of light in air was determined as, $v = 2.99792 \times 10^8\ ms^{-1}$

After the disappearance of light for the first time while increasing the speed of rotation of the toothed-wheel from zero to ω, on further increase of speed of rotation of the wheel to 2ω, the light would appear again due to the passing of reflected light through the next slot. So, for every odd value of ω, light will disappear (stopped by tooth) and for every even value of ω light will appear (allowed by slot).

**Speed of light through different media**

Different transparent media like glass, water etc. were introduced in the path of light by scientists like Foucault (1819 - 1868) and Michelson (1852 - 1931) to find the speed of light in different media. Even evacuated glass tubes were also introduced in the path of light to find the speed of light in vacuum. It was found that light travels with lesser speed in any medium than its Speed in Vacuum. The speed of light in vacuum was determined as, C = 3 × $10^8$ $ms^{-1}$. We could notice that the speed of light in vacuum and in air are almost the same.

**Refractive index**

Refractive index of a transparent medium is defined as the ratio of speed of light in vacuum (or air) to the speed of light in that medium *v*.

refractive index n of a medium = $\frac{speed\ of\ ligth\ in\ vacuum\,(c)}{Speed\ of\ light\ in\ mdium\,(v)}$

$$n = \frac{c}{v}$$

Refractive index of a transparent medium gives an idea about the speed of light in that medium.

Refractive index does not have unit. The smallest value of refractive index is for vacuum, which is 1. For any other medium refractive index is greater than 1. Refractive index is also called as optical density of the medium. Higher the refractive index of a medium, greater is its optical density and speed of light through the medium is lesser and vice versa.

**Refraction index of different media**

| Media | Refraction index |
|---|---|
| Vacuum | 1.00 |
| Air | 1.0003 |
| Carbon dioxide gas | 1.0005 |
| Ice | 1.31 |
| Pure Water | 1.33 |
| Ethyl alcohol | 1.36 |
| Quartz | 1.46 |
| Vegetable oil | 1.47 |
| Olive oil | 1.48 |
| Acrylic | 1.49 |
| Table salt | 1.51 |
| Glass | 1.52 |
| Sapphire | 1.77 |
| Zircon | 1.92 |
| Qubic zirconia | 2.16 |
| Diamond | 2.42 |
| Gallium phosphide | 3.50 |

**Optical path**

Optical path of a medium is defined as the distance d' light travels in vacuum in the same time it travels a distance d in the medium.

Let us consider a medium of refractive index *n* and thickness *d*. Light travels with a speed *v* through the medium in a time t. Then we can write,

$$v = \frac{d}{t}\text{; rewritten t as, } t = \frac{d}{v}$$

In the same time, light can cover a greater distance *d'* in vacuum as it travels with greater speed *c* in vacuum.

$$c = \frac{d'}{t}\text{; rewritten as, } t = \frac{d}{c}$$

As the time taken in both the cases is the same, we can equate the time *t* as,

$$\frac{d'}{c} = \frac{d}{v}$$

Rewritten for the optical path d′ as, $d' = \frac{c}{v}d$

As, $\frac{c}{v} = n$; The optical path d′ is,

$$d' = nd$$

The value of n is always greater than 1, for a medium. Thus, the optical path *d'* of the medium is always greater than *d*.

**Refraction**

Refraction is passing through of light from one optical medium to another optical medium through a boundary. In refraction, the angle of incidence *i* in one medium and the angle of reflection *r* in the other medium are measured with respect to the normal drawn to the surface at the point of incidence of light. According to laws of refraction,

1. The incident ray, refracted ray and normal to the refracting surface are all coplanar (ie. lie in the same plane).
2. The ratio of sine of angle of incident *i* in the first medium to the angle of reflection *r* in the second medium is equal to the ratio of refractive index $n_2$ of the second medium to the refractive index $n_1$ of the first medium.

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$n_1 \sin i = n_2 \sin r$$

The above equation is in the ratio form. It can also be written in a product form as,

$$n_1 \sin i = n_2 \sin r$$

The law of refraction is also known as Snell's law.

| For normal incidence of light on a surface, the angle of incidence is zero. |
|---|

## Angle of Deviation due to refraction

The angle between the direction of incident ray and the refracted ray is called angle of deviation due to refraction. When light travels from rarer to denser medium, it deviates towards normal as shown in Figure 6.16. The angle of deviation in this case is,

$$d = i - r$$

We know that the angle between the incident and deviated light is called angle of deviation. When light travels from rarer to denser medium it deviates towards normal.

$$d = i - r$$

On the other hand, if light travels from denser to rarer medium, it deviates away from normal as shown in. The angle of deviation in this case is,

$$d = r - i$$

## simultaneous reflection (or) refraction

In any refracting surface there will also be some reflection taking place. Thus, the intensity of refracted light will be lesser than the incident light. The phenomenon in which a part of light from a source undergoing reflection and the other part of light from the same source undergoing refraction at the same surface is called simultaneous reflection (or) simultaneous refraction. This is shown in Figure 6.18. Such surfaces are available as partially silvered glasses.

> Production of optical surfaces capable of refracting as well as reflecting is possible by properly coating the surfaces with suitable materials. Thus, a glass can be made partially see through and partially reflecting. These glasses are commercially called as two-way mirror, half-silvered mirror, semi-silvered mirror etc. This gives a perception of regular mirror if the other side is made dark. But, still hidden cameras can be kept behind such mirrors. We need to be cautious when we stand in front of mirrors kept in unknown places. There is a method to test the two way mirror. Place the finger nail on the mirror surface. If there is a gap between nail and its image, then it is a regular mirror. If the fingernail directly touches its image, then it is a two way mirror.

## Principle of reversibility

The principle of reversibility states that light will follow exactly the same path if its direction of travel is reversed.

## Relative Refractive index

In the equation for Snell's law, the term $\left(\frac{n_2}{n_1}\right)$ is called relative refractive index of second medium with respect to the first medium which is denoted as ($n_{21}$).

$$n_{21} = \frac{n_2}{n_1}$$

The concept of relative refractive index gives rise to other useful relation such as,

**a) Inverse rule:**

$$n_{12} = \frac{1}{n_{21}} \text{ (or) } \frac{n_1}{n_2} = \frac{1}{n_2/n_1}$$

**b) Chain rule:**

$$n_{32} = n_{31}\, n_{12} \text{ (or) } \frac{n_3}{n_2} = \frac{n_3}{n_1} \times \frac{n_1}{n_2}$$

**Apparent depth**

It is a common observation that the bottom of a tank filled with water appears raised. An equation could be derived for the apparent depth for viewing in the near normal direction.

Light from the object *O* at the bottom of the tank passes from denser medium (water) to rarer medium (air) to reach our eyes. It deviates away from the normal in the rarer medium at the point of incidence *B*. The refractive index of the denser medium is $n_1$ and rarer medium is $n_2$. Here, $n_1 > n_2$. The angle of incidence in the denser medium is *i* and the angle of refraction in the rarer medium is *r*. The lines and *OD* are parallel. Thus angle ∠*DIB* is also *r*. The angles *i* and *r* are very small as the diverging light from *O* entering the eye is very narrow. The Snell's law in product form for this refraction is,

$$n_1 \sin i = n_2 \sin r$$

As the angles *i* and *r* are small, we can approximate, sin $i \approx$ tan $I$ and sin r ≈ tan r.

$$n_1 \tan i = n_2 \tan i$$

In triangles Δ*DOB* and Δ*DIB*,

$$\tan(i) = \frac{DB}{DO} \text{ and } \tan(r) = \frac{DB}{DI}$$

$$n_1 = \frac{DB}{DO} = n_2 \frac{DB}{DI}$$

*DB* is cancelled on both sides, *DO* is the actual depth *d* and *DI* is the apparent depth *d'*.

$$n_1 \frac{1}{d} = n_2 \frac{1}{d'}$$

$$\frac{d'}{d} = \frac{n_2}{n_1}$$

Rearranging the above equation for the apparent depth d',

$$d' = \frac{n_2}{n_1} d$$

As the rarer medium is air and its refractive index $n_2$ can be taken as 1, ($n_2$=1). And the refractive index $n_1$ of denser medium could then be taken as n, ($n_1$=n). Now, the equation for apparent depth becomes,

$$d' = \frac{d}{n}$$

The bottom appears to be elevated by d– d′

$$d - d' = d - \frac{d}{n} \text{ or } d - d' = d\left(1 - \frac{1}{n}\right)$$

Atmospheric refraction: Due to refraction of light through different layers of atmosphere which vary in refractive index, the path of light deviates continuously when it passes through atmosphere. For example, the Sun is visible a little before the actual sunrise and also until a little after the actual sunset due to refraction of light through the atmosphere. By actual sunrise what we mean is the actual crossing of the sun at the horizon. Figure shows the actual and apparent positions of the sun with respect to the horizon. The figure is highly exaggerated to show the effect. The apparent shift in the direction of the sun is around half a degree and the corresponding time difference between actual and apparent positions is about 2 minutes. Sun appears flattened (oval shaped) during sun rise and sunset due to the same phenomenon.
The same is also applicable for the positions of stars as shown in Figure. The stars actually do not twinkle. They appear twinkling because of the movement of the atmospheric layers with varying refractive indices which is clearly seen in the night sky.

**Critical angle and total internal reflection**

When a ray passes from an optically denser medium to an optically rarer medium, it bends away from normal. Because of this, the angle of refraction *r* on the rarer medium is greater than the corresponding angle of incidence i in the denser medium. As angle of incidence *i* is gradually increased, *r* rapidly increases and at a certain stage it becomes 90° or gracing the boundary. The angle of incidence in the denser medium for which the refracted ray graces the boundary is called critical angle $i_c$.

If the angle of incidence in the denser medium is increased beyond the critical angle, there is no refraction possible in to the rarer medium. The entire light is reflected back into the denser medium itself. This phenomenon is called total internal reflection.

The two conditions for total internal reflection are,

1. light must travel from denser to rarer medium,
2. angle of incidence in the denser medium must be greater than critical angle ($i > ic$).

For critical angle of incidence, the Snell's law in the product form, becomes,

$n_1 \sin ic = n_2 \sin 90^\circ$

$n_1 \sin i_c = n_2$

$\sin i_c = \frac{n_2}{n_1}$

Here, $n_1 > n_2$

If the rarer medium is air, then its refractive index $n_2$ is 1, ($n_2 = 1$) and the refractive index of the denser medium $n_1$ is taken as n itse lf, ($n_1 = n$) then,

$$\sin i_c = \frac{1}{n} \text{ (or) } i_c = \sin^{-1}\left(\frac{1}{n}\right)$$

The critical angle ic depends on the refractive index n of the medium. refractive index and the critical angle for different materials.

**Refractive index and critical angle for different media:**

| Material | Refractive Index | Critical Angle |
|---|---|---|
| Ice | 1.310 | 49.8° |
| Water | 1.333 | 48.6° |
| Fused Quartz ($SiO_2$) | 1.458 | 43.3° |
| Crown Glass | 1.541 | 40.5° |
| Flint Glass | 1.890 | 31.9° |
| Calcite ($CaCO_2$) | 1.658 | 37.0° |
| Diamond | 2.417 | 24.4° |
| Strontium Titanate (Sr $TiO_3$) | 2.417 | 24.4° |
| Rutile | 2.621 | 22.4° |

**Effects due to total internal reflection**
**Glittering of diamond**

Diamond appears dazzling because the total internal reflection of light happens inside the diamond. The refractive index of only diamond is about 2.417. It is much larger than that for ordinary glass which is about only 1.5. The critical angle of diamond is about 24.4°. It is much less than that of glass. A skilled diamond cutter makes use of this larger range of angle of incidence (24.4° to 90° inside the diamond), to ensure that light entering the diamond is total internally reflected from the many cut faces before getting out. This gives a sparkling effect for diamond.

**Mirage and looming**

The refractive index of air increases with its density. In hot places, air near the ground is hotter than air at a height. Hot air is less dense. Hence, in still air the refractive index of air increases with height. Because of this, light from tall objects like a tree, passes through a medium whose refractive index decreases towards the ground. Hence, a ray of light successively deviates away from the normal at different layers of air and undergoes total internal reflection when the angle of incidence near the ground exceeds the critical angle. This gives an illusion as if the light comes from somewhere below the ground. For of the shaky nature of the layers of air, the observer feels as if the object is getting reflected by a pool of water or wet surface beneath the object. This phenomenon is called mirage.

In the cold places the refractive index increases towards the ground because the temperature of air close to the ground is lesser than the temperature above the surface of earth. Thus, the density and refractive index of air near the ground is greater than at a height. In the cold regions like glaciers and frozen lakes and seas, the reverse effect of mirage will happen. Hence, an inverted image is formed little above the surface. This phenomenon is called looming. It is also called as superior mirage, towering and stooping.

**Prisms making using of total internal reflection**

Prisms can be designed to reflect light by 90° or by 180° by making use of total internal reflection. In the first two cases, the critical angle ic for the material of the prism must be less than 45°. This is true for both crown glass and flint glass. Prisms are also used to invert images without changing their size.

**Radius of illumination (Snell's window)**

When a light source like electric bulb is kept inside a water tank, the light from the source travels in all direction inside the water. The light that is incident on the water surface at an angle less than the critical angle will undergo refraction and emerge out from the water. The light incident at an angle greater than critical angle will undergo total internal reflection. The light falling particularly at critical angle graces the surface.

On the other hand, when light entering the water from outside is seen from inside the water, the view is restricted to a particular angle equal to the critical angle ic. The restricted illuminated circular area is called Snell's window.

The angle of view for water animals is restricted to twice the critical angle $2i_c$. The critical angle for water is 48.6°. Thus the angle of view is 97.2°. The radius $R$ of the circular area depends on the depth $d$ from which it is seen and also the refractive indices of the media. The radius of Snell's window can be deduced with the illustration.

Light is seen from a point $A$ at a depth $d$. The Snell's law in product form, equation for the refraction happening at the point $B$ on the boundary between the two media is,

$$n_1 \sin i_c = n_2 \sin 90^\circ$$

$$n_1 \sin i_c = n_2 \qquad . \ \sin 90^\circ = 1$$

$$\sin i_c = \frac{n_2}{n_1}$$

From the right angle triangle ΔABC

$$\sin i_c = \frac{CB}{AB} = \frac{R}{\sqrt{R^2 + R^2}}$$

Equating the above two equation and equation $\frac{R}{\sqrt{d^2 + R^2}} = \frac{n_2}{n_1}$

squaring on both sides, $\frac{R^2}{R^2 + d^2} = \left(\frac{n_2}{n_1}\right)^2$

Taking reciprocal, $\frac{R^2+d^2}{R^2} = \left(\frac{n_1}{n_2}\right)^2$

Further simplifying,

$$1+\frac{R^2}{d^2} = \left(\frac{n_1}{n_2}\right)^2 ; \qquad \frac{R^2}{d^2} = \left(\frac{n_1}{n_2}\right)^2 - 1;$$

$$\frac{d^2}{R^2} = \frac{n_1^2}{n_2^2} - 1 = \frac{n_1^2 - n_2^2}{n_2^2}$$

Againg taking reciprocal and rearranging,

$$\frac{R^2}{d^2} = \frac{n_2^2}{n_1^2 - n_2^2} ; \qquad R^2 = d^2 = \left(\frac{n_2^2}{n_1^2 - n_2^2}\right)$$

After taking the squre root, the radius of illumination is,

$$R = d\sqrt{\frac{n_2^2}{\left(n_1^2 - n_2^2\right)}}$$

If the rarer medium outside is air, then, $n_2 = 1$, and we can take $n_1 = n$

$$R = d\left(\frac{1}{\sqrt{n_2 - 1}}\right) \text{ (or) } R = \frac{d}{\sqrt{n_2 - 1}}$$

**Optical Fiber**

Transmitting signals through optical fibres is possible due to the phenomenon of total internal reflection. Optical fibres consists of inner part called core and outer part called cladding (or) sleeving. The refractive index of the material of the core must be higher than that of the cladding for total internal reflection to happen. Signal in the form of light is made to incident inside the core-cladding boundary at an angle greater than the critical angle. Hence, it undergoes repeated total internal reflections along the length of the fibre without undergoing any refraction. The light travels inside the core with no appreciable loss in the intensity of the light. Even while bending the optic fiber, it is done in such a way that the condition for total internal reflection is ensured at every reflection.

**Acceptance angle in optical fibre**

To ensure the critical angle incidence in the core-cladding boundary inside the optical fibre, the light should be incident at a certain angle at the end of the optical fiber while entering in to it. This angle is called *acceptance angle*. It depends on the refractive indices of the core *n*1, cladding *n*2 and the outer medium *n*3. Assume the light is incident at an angle called acceptance angle *ia*at the outer medium and core boundary at *A*.

The Snell's law in the product form, equation (6.19) for this refraction at the point *A*.

$$n_3 \sin i_a = n_1 \sin r_a$$

To have the total internal reflection inside optical fibre, the angle of incidence at the core-cladding interface at B should be atleast critical angle ic. Snell's law in the product form, for the refraction at point B is,

$$n_1 \sin i_c = n_2 \sin 90^\circ$$

$$n_1 \sin i_c = n_2 \qquad . \sin 90^\circ = 1$$

$$\therefore \sin i_c = \frac{n_2}{n_1}$$

From the right angle triangle ΔABC,

$$i_c = 90^\circ - r_a$$

Now, equation becomes,

$$\sin (90^\circ - r_a) = \frac{n_2}{n_1} \text{ (or) } \cos r_a = \frac{n_2}{n_1}$$

$$\sin ra = \sqrt{1-\cos^2 r_a}$$

Substituting for cos $r_a$

$$\sin \sqrt{1-\left(\frac{n_2}{n_1}\right)^2} = \sqrt{\frac{n_1^2-n_2^2}{n_1^2}}$$

Substituting this in equation

$$n_3 \sin i_a = n_1 \sqrt{\frac{n_1^2-n_2^2}{n_1^2}} = \sqrt{n_1^2-n_2^2}$$

On further simplification,

$$\sin i_a = \sqrt{\frac{n_1^2-n_2^2}{n_3}} \text{ (or) } \sin i_a = \sqrt{\frac{n_1^2-n_2^2}{n_3^2}}$$

$$i_a = \sin^{-1}\left(\sqrt{\frac{n_1^2-n_2^2}{n_3^2}}\right)$$

If outer medium is air, then $n_3 = 1$. The acceptance angle ia becomes,

$$ia = \sin^{-1}\left(\sqrt{n_1^2-n_2^2}\right)$$

Light can have any angle of incidence from 0 to ia with the normal at the end of the optical fibre forming a conical shape called acceptance cone called numerical aperture NA of the optical fibre.

$$NA = n_3 \sin i_a = \sqrt{n_1^2-n_2^2}$$

If outer medium is air, then $n_3 = 1$. The numerical aperture *NA* becomes,

$$NA = \sin i_a = \sqrt{n_1^2-n_2^2}$$

An endoscope is an instrument used by doctors which has a bundle of optical fibres that are used to see inside a patient's body. Endoscopes work on the phenomenon of total internal reflection. The optical fibres are inserted in to the body through mouth, nose or a special hole made in the body. Even operations could be carried out with the endoscope cable which has the necessary instruments attached at their ends

## Refraction in glass slab

When a ray of light passes through a glass slab it refracts at two refracting surfaces. When the light ray enters the slab it travels from rarer medium (air) to denser medium (glass). This results in deviation of ray towards the normal. When the light ray leaves the slab it travels from denser medium to rarer medium resulting in deviation of ray away from the normal. After the two refractions, the emerging ray has the same direction as that of the incident ray on the slab with a lateral displacement or shift *L*. i.e. There is no change in the direction of ray but the path of the incident ray and refracted ray are different and parallel to each other. To calculate the lateral displacement, a perpendicular is drawn in between the paths of incident ray and refracted ray.

Consider a glass slab of thickness t and refractive index n is kept in air medium. The path of the light is ABCD and the refractions occur at two points B and C in the glass slab. The angles of incidence i and refraction r are measured with respect to the normal $N_1$ and $N_2$ at the two points B and C respectively. The lateral displacement L is the perpendicular distance CE drawn between the path of light and the undeviated path of light at point C.
In the right angle triangle ΔBCE,

$$\sin(i - r) = \frac{L}{BC}; BC = \frac{L}{\sin(i-r)}$$

In the right angle triangle ΔBCF,

$$\cos(r) = \frac{t}{BC}; BC = \frac{t}{\cos(r)}$$

Equating equations

$$\frac{L}{\sin(i-r)} = \frac{t}{\cos(r)}$$

After rearranging

$$L = t\left(\frac{\sin(i-r)}{\cos(r)}\right)$$

The lateral displacement depends upon (i) the thickness of the slab, (ii) the angle of incidence and (iii) the refractive index of the slab which decides the angel of refraction. Thicker the slab, larger will be the lateral displacement. Greater the angle of incidence, larger will be the lateral displacement. Higher the refractive index, larger will be the lateral displacement.

## Refraction at Single Spherical Surface

We have so far studied only the refraction at plane surface. The refraction can also take place at spherical surface between two transparent media. The laws of refraction hold good at every point on the spherical surface. The normal at the point of incidence is perpendicular drawn to the tangent plane of the spherical surface at that point. Therefore, the normal always passes through its center of curvature. The study of refraction at single spherical surface paves way to the understanding of thin lenses which consist of two refracting surfaces.

The following assumptions are made while considering refraction at spherical surfaces.

1. The incident light is assumed to be monochromatic (single colour)
2. The incident ray of light is very close to the principal axis (paraxial rays).

The sign conventions are similar to that of the spherical mirrors.

**Equation for refraction at single spherical surface**

Let us consider two transparent media having refractive indices $n_1$ and $n_2$ are separated by a spherical surface. Let C be the centre of curvature of the spherical surface. Let a point object O be in the medium $n_1$. The line OC cuts the spherical surface at the pole P of the surface. As the rays considered are paraxial rays, the perpendicular dropped for the point of incidence to the principal axis is very close to the pole or passes through the pole itself.

Light from *O* falls on the refracting surface at *N*. The normal drawn at the point of incidence passes through the centre of curvature *C*. As $n_2 > n_1$, light in the denser medium deviates towards the normal and meets the principal axis at *I* where the image is formed.

Snell's law in product form for the refraction at the point *N* could be written as,

$$n_1 \sin i = n_2 \sin r$$

As the angles are small, sine of the angle could be approximated to the angle itself.

$$n_1 i = n_2 r$$

Let the angles,

$$\angle NOP = \alpha, \angle NCP = \beta, \angle NIP = \gamma$$

From the right angle triangles ΔNOP, ΔNCP and ΔNIP,

$$\tan \alpha = \frac{PN}{PO}; \ \tan \beta = \frac{PN}{PC}; \tan \gamma = \frac{PN}{PI}$$

As these angles are small, tan of the angle could be approximated to the angle itself.

$$\alpha = \frac{PN}{PO}; \beta = \frac{PN}{PC}; \gamma = \frac{PN}{PI}$$

For the triangle, ΔONC,

$$i = \alpha + \beta$$

For the triangle, ΔINC,

$$\beta = r + \gamma \text{ (or) } r = \beta - \gamma$$

Substituting for *i* and *r* from equations.

$$n_1 (\alpha + \beta) = n_2 (\beta - \gamma)$$

After Rearringing,

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$

Substituting for α, β and γ from equation

$$n_1\left(\frac{PN}{PO}\right) + n_2\left(\frac{PN}{PI}\right) = (n_2 - n_1)\left(\frac{PN}{PC}\right)$$

Further simplifying by cancelling PN,

$$\frac{n_1}{PO} + \frac{n_2}{PI} = \frac{n_2 - n_1}{PC}$$

Following sign conventions, PO = -u, PI = +v and PC = +R in equation

$$\frac{n_1}{-u} - \frac{n_2}{v} = \frac{(n_2 - n_1)}{R}$$

After rearranging, finally we get,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$$

**Thin Lens**

A lens is formed by a transparent material bounded between two spherical surfaces or one plane and another spherical surface. In a thin lens, the distance between the surfaces is very small. If there are two spherical surfaces, then there will be two centres of curvature $C_1$ and $C_2$ and correspondingly two radii of curvature $R_1$ and $R_2$. A plane surface has its center of curvature $C$ at infinity and its radius of curvature $R$ is infinity ($R = \infty$). The terminologies of spherical mirrors also hold good very much for thin lens except for focal length.

**Primary and Secondary focal points**

As the thin lens is formed by two spherical surfaces, the lens may separate two different media. i.e. the media to the left and right of the lens may be different. Hence, we have two focal lengths.

The primary focus $F_1$ is defined as a point where an object should be placed to give parallel emergent rays to the principal axis. For a convergent lens, such an object is a real object and for a divergent lens, it is a virtual object. The distance $PF_1$ is the primary focal length $f_1$.

The secondary focus $F_2$ is defined as a point where all the parallel rays travelling close to the principal axis converge to form an image on the principal axis. For a convergent lens, such an image is a real image and for a divergent lens, it is a virtual image. The distance $PF_2$ is the secondary focal length $f_2$.

If the media on the two sides of a thin lens have same refractive index, then the two focal lengths are equal. We will mostly be using the secondary focus $F_2$ in our further discussions.

**Sign conventions for lens on focal length**

The sign conventions for thin lenses differ only in the signs followed for focal lengths.

1. The sign of focal length is not decided on the direction of measurement of the focal length from the pole of the lens as they have two focal lengths, one to the left and another to the right (primary and secondary focal lengths on either side of the lens).

2. The focal length of the thin lens is taken as positive for a converging lens and negative for a diverging lens.

The other sign conventions for object distance, image distance, radius of curvature, object height and image height (except for the focal lengths as mentioned above) remain the same for thin lenses as that of spherical mirrors.

**Lens maker's formula and lens equation**

Let us consider a thin lens made up of a medium of refractive index $n_2$ is placed in a medium of refractive index $n_1$. Let $R_1$ and $R_2$ be the radii of curvature of two spherical surfaces ① and ② respectively and $P$ be the pole. Consider a point object $O$ on the principal axis. The ray which falls very close to $P$, after refraction at the surface ① forms image at $I'$. Before it does so, it is again refracted by the surface ②. Therefore the final image is formed at $I$.

The general equation for the refraction at a single spherical surface is given by the equation

$$\frac{n_2}{v} = \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$$

For the refracting surface ①, the light goes from $n1$ to $n2$.

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R_1}$$

For the refracting surface ②, the light goes from medium $n2$ to $n1$.

$$\frac{n_1}{v} - \frac{n_2}{v'} = \frac{(n_1 - n_2)}{R_2}$$

For surface ②, I' acts as virtual object. Adding the above two

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

On further simplifying and rearranging,

$$\frac{1}{v} - \frac{1}{u} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the object is at infinity, the image is formed at the focus of the lens. Thus, for $u = \infty$, $v = f$. Then the equation becomes.

$$\frac{1}{f} - \frac{1}{\infty} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the lens is kept in air, then we can take $n_1 = 1$ and $n_2 = n$. So the becomes,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The above equation is called the lens maker's formula, because it tells the lens manufactures what curvature is needed to make a lens of desired focal length with a material of particular refractive index. This formula holds good also for a concave lens.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

The above equation is known as lens equation which relates the object distance u and image distance v with the focal length f of the lens. This formula holds good for a any type of lens.

**Lateral magnification in thins lens**

Let us consider an object OO' of height h1 placed on the principal axis with its height perpendicular to the principal axis as shown in. The ray OP passing through the pole of the lens goes undeviated. The inverted real image formed has a height $h_2$.

The lateral or transverse magnification *m* is defined as the ratio of the height of the image to that of the object.

$$m = \frac{II'}{OO'}$$

From the two similar triangles ΔPOO' and ΔPII', we can write,

$$\frac{II'}{OO'} = \frac{PI}{PO}$$

On applying sign convention,

$$\frac{-h'}{h} = \frac{v}{-u}$$

Substituting this in the for magnification,

$$m = \frac{h'}{h} = \frac{v}{u}$$

After rearranging,

$$m = \frac{h'}{h} = \frac{v}{u}$$

The magnification is negative for real image and positive for virtual image. In the case of a concave lens, the magnification is always positive and less than one.

We can also have the equations for magnification by combining the lens equation with the formula for magnification as,

$$m = \frac{h'}{h} = \frac{f}{f+u} \,(or)\, m = \frac{h'}{h} = \frac{f-v}{f}$$

**Power of a lens**

Power of lens is the measurement of deviating strength of a lens i.e. when a ray is incident on a lens then the degree with which the lens deviates the ray is determined by the power of the lens. Power of the lens is inversely proportional to focal length i.e. greater the power of lens, greater will be the deviation of ray and smaller will be the focal length. The lens (b) has greater deviating strength than that of (a). As (b) has greater deviating strength, its focal length is less and vice versa. In other words, the power of a lens is a measure of the degree of convergence or divergence of light falling on it. The power of a lens P is defined as the reciprocal of its focal length is less and vice versa.

In other words, the power of a lens is a measure of the degree of convergence (or) divergence the lens produces on the light falling on it. The power of a lens P is the reciprocal of its focal length in meter.

$$P = \frac{1}{f}$$

The unit of power is diopter D. 1 D = 1 $m^{-1}$. Power is positive for converging lens and negative for diverging lens.

From the lens maker's formula, can be written for power as ,

$$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The outcome of this equation of power is that larger the value of refractive index, greater is the power of lens and vice versa. Also for lenses with small radius of curvature (bulky) the power is large and for lenses with large the radius of curvature (skinny), the power is small.

**Focal length of lenses in contact**

Let us consider two lenses ① and ② of focal length $f_1$ and $f_2$ are placed coaxially in contact with each other so that they have a common principal axis. For an object placed at O beyond the focus of the first lens ① on the principal axis, an image is formed by it at $I'$. This image $I'$ acts as an object for the second lens ② and the final image is formed at I. As these two lenses are thin, the measurements are done with respect to the common optical centre $P$ in the middle of the two lenses.

For the lens ①, the object distance *PO* is *u* and the image distance *PI'* is *v'*. For the lens ②, the object distance *PI'* is *v'* and the image distance *PI* is *v*.

Writing the lens

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}$$

Writing the lens equation

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2}$$

Adding the above two equations

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

If the combination acts as a single lens of focal length *f* so that for an object at the position *O* it forms the image at *I* then,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Comparing the equations

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The above equation can be extended for any number of lenses in contact as,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \frac{1}{f_4} + \ldots..$$

The above equation can be written in terms of power of the lenses as,

$$P = P_1 + P_2 + P_3 + P_4 + \ldots.$$

Where, P is the net power of the lens combination of lenses in contact. One should note that the sum in is an algebraic sum. The powers of individual lenses may be positive (for convex lenses) or negative (for concave lenses). Combination of lenses helps to obtain diverging or converging lenses of desired magnification. Also, combination of lenses enhances the sharpness of the images. As the image formed by the first lens becomes the object for the second and so on, the total magnification m of the combination is a product of magnification of individual lenses. We can write,

$$m = m_1 \times m_2 \times m_3$$

Where $m_1$, $m_2$, $m_3$ . . . . . are magnification of individual lenses.

**Silvered lenses**

If one of the surfaces of a lens is silvered from outside, then such a lens is said to be a silvered lens. A silvered lens is a combination of a lens and a mirror. Light can enter through the transparent front surface of the lens and get reflected by the silver coated rear surface. Hence, light travels two times through the lens

The power P of the silvered lens is,

$$P = P_l + P_m + P_l$$

$$P = 2P_l + P_m$$

Here, $P_l$ is the power of the lens and $P_m$ is the power of the mirror. We know that the power of a lens is the reciprocal of its focal length. But, the power of a mirror is negative of the reciprocal of its focal length. This is because, a concave mirror which has negative focal length is a converging mirror with positive power. Also, a silvered lens is basically a modified mirror. Thus,

$$P = \frac{1}{-f}; P_1 = \frac{1}{f_1}; P_m = \frac{1}{-f_m}$$

Now,

$$\left(\frac{1}{-f}\right) = \left(\frac{2}{f_1}\right) + \left(\frac{1}{-f_m}\right)$$

Proper sign conventions are to be followed for.

Suppose the object distance u and image distance v are to be found, we can very well use the mirror, since the silvered lens is a modified mirror.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

**PRISM**

A prism is a triangular block of glass or plastic. It is bounded by the three plane faces not parallel to each other. Its one face is grounded which is called base of the prism. The other two faces are polished which are called refracting faces of the prism. The angle between the two refracting faces is called angle of prism (or) refracting angle (or) apex angle of the prism represented as A As the height of the image is positive, the image is erect, and it is real.

**Angle of deviation produced by prism**

Let light ray PQ is incident on one of the refracting faces of the prism.The angles of incidence and refraction at the first face AB are i1 and r1. The path of the light inside the prism is QR. The angle of incidence and refraction at the second face AC is r2 and i2 respectively. RS is the ray emerging from the second face. Angle i2 is also called angle of emergence. The angle between the direction of the incident ray PQ and the emergent ray RS is called the angle of deviation d. The two normals drawn at the point of incidence Q and emergence R are QN and RN. They meet at point N. The incident ray and the emergent ray meet at a point M.

The angle of deviation $d_1$ at surface $AB$ is,

$$\angle RQM = d_1 = i_1 - r_1$$

The angle of deviation $d_2$ at surface AC is,

$$QRM = d_2 = i_2 - r_2$$

Total angle of deviation *d* produced is,

$$d = d_1 + d_2$$

Substituting $d_1$ and $d_2$, in

$$d = (i_1 - r_1) + (i_2 - r_2)$$

After rearranging,

$$d = (i_1 + i_2) + (r_1 + r_2)$$

In the quadrilateral AQNR, two of the angles (at the vertices Q and R) are right angles. Therefore, the sum of the other angles of the quadrilateral is 180°.

$$\angle A + \angle QNR = 180°$$

In the triangle ΔQNR,

$$r_1 + r_2 + \angle QNR = 180°$$

Comparing the two we get,

$$r_1 + r_2 = A$$

Substituting this in for angle of deviation,

$$d = i_1 + i_2 - A$$

Thus, the angle of deviation depends on the angle of incidence angle of emergence and the angle for the prism. For a given angle of incidence the angle of emergence is decided by the refractive index of the material of the prism. Hence, the angle of deviation depends on these following factors.

1. the angle of incidence
2. the angle of the prism
3. the material of the prism
4. the wave length of the light

**Angle of minimum deviation**

A graph plotted between the angle of incidence and angle of deviation. One could observe that the angle of deviation decreases with increase in angle of incidence and reaches a minimum value and then continues to increase.

The minimum value of angle of deviation is called angle of minimum deviation *D*. At minimum deviation,

1. the angle of incidence is equal to the angle of emergence, $i_1=i_2$.
2. the angle of refraction at the face one and face two are equal, ($r_1=r_2$).
3. the refracted ray inside the prism is parallel to its base of the prism.

**Refractive index of the material of the prism**

At minimum deviation, $i_1 = i_2 = i$ and $r_1 = r_2 = r$

Now, the equation becomes,

$$D = i_1 + i_2 - A = 2i - A \text{ (or) } i = \frac{(A+D)}{2}$$

The equation becomes

$$r_1 + r_2 = A \Rightarrow 2r = A \text{ (or) } r = \frac{A}{2}$$

Substituting i and r in Snell's law,

$$n = \frac{\sin i}{\sin r}$$

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

The above equation is used to find the refractive index of the material of the prism. The angles A and D can be measured experimentally.

**Dispersion of white light through prism**

So far the angle of deviation produced by a prism is discussed for monochromatic light (i.e. light of single colour). When white light enter in to a prism, the effect called dispersion takes place. Dispersion is splitting of white light into its constituent colours. This band of colours of light is called its spectrum. When a narrow beam of parallel rays of white light is incident on the face of a prism and the refracted beam is received on a white screen, a band of colours is obtained in the order, recollected by the word: VIBGYOR i.e., Violet, Indigo, Blue, Green, Yellow, Orange and Red.Violet is the most deviated and red is the least deviated colour.

The colours obtained in a spectrum depend on the nature of the source of the light used. Each colour of light is associated with a definite wavelength. Red light is at the longer wavelength end (700 nm) while the violet colour has the shortest wavelength of 400 nm in vacuum. Though all the colours have different wavelengths, they all travel with the same speed in vacuum. The speed of light is independent of wavelength in vacuum. Therefore, vacuum is a non-dispersive medium.

But, when the white light enters a medium the red colour travels with the highest speed and violet colour travels with least speed. Hence, the wavelengths of colours in a medium are no longer the same as they are in vacuum. Actually, the dispersion takes place in a medium because of the difference in speed for different colours in a medium. In other words, the refractive index of the material of the prism is different for different colours. For violet colour, the refractive index is the highest and for red colour the refractive index is the least. The refractive index of two different glasses for different colours

**Refractive indices for different wavelengths**

| Colour | Wavelength (nm) | Crown glass | Flint glass |
|---|---|---|---|
| Violet | 396.9 | 1.533 | 1.663 |
| Blue | 486.1 | 1.523 | 1.639 |
| Yellow | 589.3 | 1.517 | 1.627 |
| Red | 656.3 | 1.515 | 1.622 |

**Dispersive Power**

Consider a beam of white light passing through a prism. It gets dispersed into its constituent colours.

If the angle of prism is small of the order of 10o, the prism is said to be a small angle prism. When rays of light pass through such prisms, the angle of minimum deviation also becomes small. Let A be the angle of a small angle prism and δ be its angle of minimum deviation, then becomes,

Rainbow appears in sky during mild shower (or) near the fountains/falls where there are water droplets remain suspended in air. A rainbow is seen when the sun is at the back of the observer. Dispersion occurs when sunlight enters a water droplet and the white light is split into its constituent seven colours. A primary rainbow is formed when the light entering a droplet undergoes one total internal reflection inside it. Sometimes, a secondary rainbow is also formed enveloping the primary rainbow as shown in the figure. The secondary rainbow is formed when light entering a raindrop undergoes two total internal reflections. The order of colour in primary rainbow is from violet to red whereas in secondary rainbow it is from red to violet. The angle of view in primary rainbow from violet to red is from 40˚ to 42˚. The angle of view for secondary rainbow from red to violet is from 52˚ to 54˚.

$$n = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

For small angles of A and $\delta_m$,

$$\sin\left(\frac{A+\delta}{2}\right) \approx \left(\frac{A+\delta}{2}\right)$$

$$\sin\left(\frac{A}{2}\right) \approx \left(\frac{A}{2}\right)$$

$$\therefore n = \frac{\left(\frac{A+\delta}{2}\right)}{\left(\frac{A}{2}\right)} = \frac{A+\delta}{A} + 1 + \frac{\delta}{A}$$

On Further simplifying $\frac{\delta}{A} = n - 1$

$\delta = (n - 1)A$

When white light enters the prism, the deviation is different for different colours. Thus, the refractive index is also different for different colours.

For Viotet Light, $\delta_v = (n_v - 1)A$

For Red light, $\delta_R = (n_R - 1)A$

As, angle of deviation for violet colour $\delta_V$ is greater than angle of deviation for red colour $\delta_R$, the refractive index for violet colour $n_V$ is greater than the refractive index for red colour $n_R$.

Subtracting $\delta V$ from $\delta R$ we get,

$$\delta_v - \delta_R = (n_v - n_R)A$$

The angular separation between the two extreme colours (violet and red) in the spectrum ($\delta_V - \delta_R$) is called the angular dispersion.

If we take $\delta$ is the angle of deviation for any middle ray (green or yellow) and $n$the corresponding refractive index. Then,

$\delta = (n - 1) A$

Dispersive power (ω) is the ability of the material of the prism to cause dispersion. It is defined as the ratio of the angular dispersion for the extreme colours to the deviation for any mean colour.

Dispersive power (ω),

$$\omega = \frac{angular\,dispersion}{mean\,deviation} = \frac{\delta_v - \delta_R}{\delta}$$

substituting for ($\delta_v$ - $\delta_R$) and ($\delta$)

$$\omega = \frac{(n_v - n_R)}{(n - 1)}$$

The Dispersive power is a dimensionless quality. It has no unit. Dispersive power is always positive. The dispersive power of a prism depends only on the nature of material of the prism and it is independent of the angle of the prism.

**Scattering of sunlight**

When sunlight enters the atmosphere of the earth, the atmospheric particles present in the atmosphere change the direction of the light. This process is known as scattering of light.

If the scattering of light is by atoms and molecules which have size a very less than that of the wave length λ of light $a << \lambda$, the scattering is called Rayleigh's scattering. The intensity of Rayleigh's scattering is inversely proportional to fourth power of wavelength.

$$I \propto \frac{1}{\lambda^4}$$

According to, during day time, violet colour which has the shortest wavelength gets more scattered then the other colours. The next scattered colour is blue. As our eyes are more sensitive to blue colour than violet colour, the sky appears blue during day time as shown in Figure 6.45(b). But, during sunrise and sunset, the light from sun travels a greater distance through the atmosphere. Hence, the blue light which has shorter wavelength is scattered away and the red light which has longer wavelength and less-scattered manages to reach our eye. This is the reason for the reddish appearance of sky during sunrise and sunset.

If light is scattered by large particles like dust and water droplets present in the atmosphere which have size a greater than the wavelength $\lambda$ of light, ($a \gg \lambda$), the intensity of scattering is equal for all the colours. This non-Rayleigh's scattering is independent of wavelength. It happens in clouds which contains large amount of dust and water droplets. Thus, in clouds all the colours get equally scattered. This is the reason for the whitish appearance of cloud as. But, the rain clouds appear dark because of the condensation of water droplets on dust particles that makes the cloud opaque.

If earth has no atmosphere there would not have been any scattering and the sky would appear dark. That is why sky appears dark for the astronauts who could see the sky from above the atmosphere.