# (Nuclear physics Part –II )

# $11^{TH}$ PHYSICS
# Unit - 7

# DUAL NATURE OF RADIATION AND MATTER

## INTRODUCTION

We are familiar with the concepts of particle and wave in our everyday experience. Marble balls, grains of sand, atoms, electrons and so on are some examples of particles while the examples of waves are sea waves, ripples in a pond, sound waves and light waves.

Particle is a material object which is considered as a tiny concentration of matter (localized in space and time) whereas wave is a broad distribution of energy (not localized in space and time). They, both particles and waves, have the ability to carry energy and momentum from one place to another.

Classical physics which describes the motion of the macroscopic objects treats particles and waves as separate components of physical reality. The mechanics of particles and the optics of waves are traditionally independent subjects, each with its own experiments and principles.

Electromagnetic radiations are regarded as waves because they exhibit wave phenomena such as interference, diffraction and polarization under some suitable circumstances. Similarly, under other circumstances like black body radiation and photo electric effect, electromagnetic radiations behave as though they consist of stream of particles.

When electrons, protons and other particles are discovered, they are considered as particles because they possess mass and charge. However later experiments showed that under certain circumstance, they exhibit wave-like properties.

In this unit, the particle nature of waves (radiation) and then the wave nature of particles (matter) – that is, wave –particle duality of radiation and matter are discussed with the relevant experimental observations which support this dual nature.

**Electron emission**

In metals the electrons in the outer most shells are loosely bound to the nucleus. Event at room temperature, there are a large number of free electrons which are moving inside the metal in a random manner. Thought they move freely inside the metal they cannot leave the surface of the metal. The reason is that when free electrons reach the surface of the metal, they are attracted by the positive nuclei of the metal. It is this attractive pull which will not allow free electrons to leave the metallic surface at room temperature.

In order to leave the metallic surface the free electrons must cross a potential barrier created by the positive nuclei of the metal. The potential barrier which prevents free electrons from leaving the metallic surface is called surface barrier.

Free electrons possess some kinetic energy and this is different for different electrons. The kinetic energy of the free electrons is not sufficient overcome the surface barrier. Whenever an additional energy is given to the free electrons, they will have sufficient energy to cross the surface barrier. And they escape from the metallic surface. The liberations of electrons from any surface of a substance is called electron emission.

The minimum energy needed for an electron to escape from the metal surface is called work function of that metal. The work function of the metal is denoted by φ0 and is measured in electron volt (eV).

| Metal | Symbol | Work function ($eV$) | Metal | Symbol | Work function ($eV$) |
|---|---|---|---|---|---|
| Cesium | Cs | 2.14 | Aluminium | Al | 4.28 |
| Potassium | K | 2.30 | Mercury | Hg | 4.49 |
| Sodium | Na | 2.75 | Copper | Cu | 4.65 |
| Calcium | Ca | 3.20 | Silver | Ag | 4.70 |
| Molypdenum | Mo | 4.17 | Nickel | Ni | 5.15 |
| Lead | Pb | 4.25 | Platinum | Pt | 5.65 |

So the metal selected for electron emission should have low work function. The electron emission is categorized into different types depending upon the form of energy being utilized. There are mainly four types of electron emission which are given below.

**Thermionic emission**

When a metal is heated to a high temperature, the free electrons on the surface of the metal get sufficient energy in the form of thermal energy so that they are emitted from the metallic. This type of emission is known as thermionic emission.

The intensity of the thermionic emission (the number of electrons emitted) depends on the metal used and its temperature. **Examples:** cathode ray tubes, electron microscopes, X-ray tubes etc.

**Field emission**

Electric field emission occurs when a very strong electric field is applied across the metal. This strong field pulls the free electrons and helps them to overcome the surface barrier of the metal (Figure 7.3). **Examples:** Field emission scanning electron microscopes, Field-emission display etc.

**Photo electric emission**

When an electromagnetic radiation of suitable frequency is incident on the surface of the metal, the energy is transferred from the radiation to the free electrons. Hence, the free electrons get sufficient energy to cross the surface barrier and the photo electric emission takes place (Figure 7.4). The number of electrons emitted depends on the intensity of the incident radiation.

**Examples:** Photo diodes, photo electric cells etc.

**Secondary emission**

When a beam of fast moving electrons strikes the surface of the metal, the kinetic energy of the striking electrons is transferred to the free electrons on the metal surface. Thus the free electrons get sufficient kinetic energy so that the secondary emission of electron occurs (Figure 7.5).

**Examples:** Image intensifiers, photo multiplier tubes etc.

## PHOTO ELECTRIC EFFECT

### Hertz, Hallwachs and Lenard's observation

### Hertz observation

Maxwell's theory of electromagnetism predicted the existence of electromagnetic waves and concluded that light itself is just an electromagnetic wave. Then the experimentalists tried to generate and detect electromagnetic waves through various experiments.

In 1887, Heinrich Hertz first became successful in generating and detecting electromagnetic wave with his high voltage induction coil to cause a spark discharge between two metallic spheres (we have learnt this in Unit 5 of XII standard physics). When a spark is formed, the charges will oscillate back and forth rapidly and the electromagnetic waves are produced.

The electromagnetic waves thus produced were detected by a detector that has a copper wire bent in the shape of a circle. Although the detection of waves is successful, there is a problem in observing the tiny spark produced in the detector.

In order to improve the visibility of the spark, Hertz made many attempts and finally noticed an important thing that small detector spark became more vigorous when it was exposed to ultraviolet light.

The reason for this behaviour of the spark was not known at that time. Later it was found that it is due to the photoelectric emission. Whenever ultraviolet light is incident on the metallic sphere, the electrons on the outer surface are emitted which caused the spark to be more vigorous.

It is interesting to note that the experiment of Hertz confirmed that light is an electromagnetic wave. But the same experiment also produced the first evidence for particle nature of light.

**Hallwachs' observation**

In 1888, Wilhelm Hallwachs, a German physicist, confirmed that the strange behaviour of the spark is due to the action of ultraviolet light with his simple experiment.

A clean circular plate of zinc is mounted on an insulating stand and is attached to a gold leaf electroscope by a wire. When the uncharged zinc plate is irradiated by ultraviolet light from an arc lamp, it becomes positively charged and the leaves will open.

Further, if the negatively charged zinc plate is exposed to ultraviolet light, the leaves will close as the charges leaked away quickly (Figure 7.6(b)). If the plate is positively charged, it becomes more positive upon UV rays irradiation and the leaves will open further (Figure 7.6(c)). From these observations, it was concluded that negatively charged electrons were emitted from the zinc plate under the action of ultraviolet light.

**Lenard's observation**

In 1902, Lenard studied this electron emission phenomenon in detail. His simple experimental setup is as shown in Figure 7.7. The apparatus consists of two metallic plates *A* and *C* placed in an evacuated quartz bulb. The galvanometer *G* and battery *B* are connected in the circuit.

When ultraviolet light is incident on the negative plate *C*, an electric current flows in the circuit that is indicated by the deflection in the galvanometer. On other hand, if the positive plate is irradiated by the ultraviolet light, no current is observed in the circuit.

From these observations, it is concluded that when ultraviolet light falls on the negative plate, electrons are ejected from it which are attracted by the positive plate *A*. On reaching the positive plate through the evacuated bulb, the circuit is completed and the current flows in it. Thus, the ultraviolet light falling on the negative plate causes the electron emission from the surface of the plate.

**Photoelectric effect**

The ejection of electrons from a metal plate when illuminated by light or any other electromagnetic radiation of suitable wavelength (or frequency) is called photoelectric effect. Although these electrons are not different from all other electrons, it is customary to call them as photoelectrons and the corresponding current as photoelectric current or photo current.

Metals like cadmium, zinc, magnesium etc show photoelectric emission for ultraviolet light while some alkali metals lithium, sodium, caesium respond well even to larger wavelength radiation like visible light. The materials which eject photoelectrons upon irradiation of electromagnetic wave of suitable wavelength are called photosensitive materials.

**Effect of intensity of incident light on photoelectric current**
**Experimental setup**

The apparatus shown in Figure 7.8 is employed to study the phenomenon of photoelectric effect in detail. *S* is a source of electromagnetic waves of known and variable frequency $\nu$ and intensity *I*. C is the cathode (negative electrode) made up of photosensitive material and is used to emit electrons. The anode (positive electrode) *A* collects the electrons emitted from *C*. These electrodes are taken in an evacuated glass envelope with a quartz window that permits the passage of ultraviolet and visible light.

The necessary potential difference between *C* and *A* is provided by high tension battery *B* which is connected across a potential divider arrangement *PQ* through a key *K*. *C* is connected to the centre terminal while *A* to the sliding contact *J* of the potential divider. The plate *A* can be maintained at a desired positive or negative potential with respect to *C*. To measure both positive and negative potential of *A* with respect to *C*, the voltmeter is designed to have its zero marking at the centre and is connected between *A* and *C*. The current is measured by a micro ammeter *mA* in series.

If there is no light falling on the cathode *C*, no photoelectrons are emitted and the micro ammeter reads zero. When ultraviolet or visible light is allowed to fall on *C*, the photoelectrons are liberated and are attracted towards anode. As a result, the photoelectric current is set up in the circuit which is measured using micro ammeter.

The variation of photocurrent with respect to (i) intensity of incident light (ii) the potential difference between the electrodes (iii) the nature of the material and (iv) frequency of incident light can be studied with the help of this apparatus.

**Effect of intensity of incident light on photoelectric current**

To study the effect of intensity of incident light on photoelectric current, the frequency of the incident light and the accelerating potential V of the anode are kept constant. Here the potential of A is kept positive with respect to that of C so that the electrons emitted from C are attracted towards A. Now, the intensity of the incident light is varied and the corresponding photoelectric current is measured.

A graph is drawn between light intensity along x-axis and the photocurrent along y-axis. From the graph in Figure 7.9, it is evident that photocurrent – the number of electrons emitted per second – is directly proportional to the intensity of the incident light.

**Effect of potential difference on photoelectric current**

To study the effect of potential difference V between the electrodes on photoelectric current, the frequency and intensity of the incident light are kept constant. Initially the potential of A is kept positive with respect to C and the cathode is irradiated with the given light.

Now, the potential of A is increased and the corresponding photocurrent is noted. As the potential of A is increased, photocurrent is also increased. However a stage is reached where photocurrent reaches a saturation value (saturation current) at which all the photoelectrons from C are collected by A. This is represented by the flat portion of the graph between potential of A and photocurrent.

When a negative (retarding) potential is applied to A with respect to C, the current does not immediately drop to zero because the photoelectrons are emitted with some definite and different kinetic energies. The kinetic energy of some of the photoelectrons is such that they could overcome the retarding electric field and reach the electrode A.

When the negative (retarding) potential of A is gradually increased, the photocurrent starts to decrease because more and more photoelectrons are being repelled away from reaching the electrode A. The photocurrent becomes zero at a particular negative potential $V_0$, called stopping or cut-off potential.

Stopping potential is that the value of the negative (retarding) potential given to the collecting electrode A which is just sufficient to stop the most energetic photoelectrons emitted and make the photocurrent zero.

At the stopping potential, even the most energetic electron is brought to rest. Therefore, the initial kinetic energy of the fastest electron ($K_{max}$) is equal to the work done by the stopping potential to stop it ($eV_0$).

$$K_{max} = \frac{1}{2} m \upsilon_{max}^2 = eV_0$$

where $\upsilon_{max}$ is the maximum speed of the emitted photoelectron.

$$\upsilon_{max} = \sqrt{\frac{2eV_0}{m}}$$

$$\upsilon_{max} = \sqrt{\frac{2 \times 1.602 \times 10^{-19}}{9.1 \times 10^{-31}} \times V_0}$$

$$= 5.93 \times 10^5 \sqrt{V_0}$$

$$K_{max} = eV_0 \ (in\ joule) \quad \text{(or)}$$

$$K_{max} = V_0 \ (in\ eV)$$

From the Figure 7.10, when the intensity of the incident light alone is increased, the saturation current also increases but the value of $V_0$ remains constant.

Thus, for a given frequency of the incident light, the stopping potential is independent of intensity of the incident light. This also implies that the maximum kinetic energy of the photoelectrons is independent of intensity of the incident light.

## Effect of frequency of incident light on stopping potential

To study the effect of frequency of incident light on stopping potential, the intensity of the incident light is kept constant. The variation of photocurrent with the collector electrode potential is studied for radiations of different frequencies and a graph drawn between them is shown in Figure 7.11. From the graph, it is clear that stopping potential vary over different frequencies of incident light.

Greater the frequency of the incident radiation, larger is the corresponding stopping potential. This implies that as the frequency is increased, the photoelectrons are emitted with greater kinetic energies so that the retarding potential needed to stop the photoelectrons is also greater.

Now a graph is drawn between frequency and the stopping potential for different metals (Figure 7.12). From this graph, it is found that stopping potential varies linearly with frequency. Below a certain frequency called threshold frequency, no electrons are emitted; hence stopping potential is zero for that reason. But as the frequency is increased above threshold value, the stopping potential varies linearly with the frequency of incident light.

## Laws of photoelectric effect

The above detailed experimental investigations of photoelectric effect revealed the following results:

1. For a given frequency of incident light, the number of photoelectrons emitted is directly proportional to the intensity of the incident light. The saturation current is also directly proportional to the intensity of incident light.
2. Maximum kinetic energy of the photo electrons is independent of intensity of the incident light.
3. Maximum kinetic energy of the photo electrons from a given metal is directly proportional to the frequency of incident light.
4. For a given surface, the emission of photoelectrons takes place only if the frequency of incident light is greater than a certain minimum frequency called the threshold frequency.
5. There is no time lag between incidence of light and ejection of photoelectrons

Once photoelectric phenomenon has been thoroughly examined through various experiments, the attempts were made to explain it on the basis of wave theory of light.

## Concept of quantization of energy

## Failures of classical wave theory

From Maxwell's theory (Refer unit 5 of volume 1), we learnt that light is an electromagnetic wave consisting of coupled electric and magnetic oscillations that move with the speed of light and exhibit typical wave behaviour. Let us try to explain the experimental observations of photoelectric effect using wave picture of light.

When light is incident on the target, there is a continuous supply of energy to the electrons. According to wave theory, light of greater intensity should impart greater kinetic energy to the liberated electrons (Here, Intensity of light is the energy delivered per unit area per unit time).

But this does not happen. The experiments show that maximum kinetic energy of the photoelectrons does not depend on the intensity of the incident light.

According to wave theory, if a sufficiently intense beam of light is incident on the surface, electrons will be liberated from the surface of the target, however low the frequency of the radiation is.

From the experiments, we know that photoelectric emission is not possible below a certain minimum frequency. Therefore, the wave theory fails to explain the existence of threshold frequency.

Since the energy of light is spread across the wave front, the electrons which receive energy from it are large in number. Each electron needs considerable amount of time (a few hours) to get energy sufficient to overcome the work function and to get liberated from the surface.

But experiments show that photoelectric emission is almost instantaneous process (the time lag is less than $10^{-9}$ $s$ after the surface is illuminated) which could not be explained by wave theory.

Thus, the experimental observations of photoelectric emission could not be explained on the basis of the wave theory of light.

For the photoelectric emission from cesium, show that wave theory predicts that
**i)** maximum kinetic energy of the photoelectrons ($K_{max}$) depends on the intensity $I$ of the incident light
**ii)** $K_{\max}$ does not depend on the frequency of the incident light and
**iii)** the time interval between the incidence of light and the ejection of photoelectrons is very long

(Given : The work function for cesium is 2.14 $eV$ and the power absorbed per unit area is $1.60\times10^{-6}$.$Wm^{-2}$ which produces a measurable photocurrent in cesium.)

**Solution**

- According to wave theory, the energy in a light wave is spread out uniformly and continuously over the wave front. For the sake of simplicity, the following assumptions are made.
- Light is absorbed in the top atomic layer of the metal
- For a given element, each atom absorbs an equal amount of energy and this energy is proportional to its cross-sectional area A
- Each atom gives this energy to one of the electrons.
- The energy absorbed by each electron in time t is given by

$$E = IAt$$

With this energy absorbed, the most energetic electron is released with $K_{\max}$ by overcoming the surface energy barrier or work function $\phi 0$ and this is expressed as

$$K_{\max} = IAt - \phi_0$$

Thus, wave theory predicts that for a unit time, at low light intensities when $IA < \phi_0$, no electrons are emitted. At higher intensities, when $IA \geq \phi_0$, electrons are emitted. This implies that higher the light intensity, greater will be $Kmax$.

Therefore, the predictions of wave theory contradict experimental observations at both very low and very high light intensities.

$K_{\max}$ is dependent only on the intensity under given conditions – that is, by suitably increasing the intensity, one can produce photoelectric effect even if the frequency is less than the threshold frequency. So the concept of threshold frequency does not even exist in wave theory.

- According to wave theory, the intensity of a light wave is proportional to the square of the amplitude of the electric field($E_0^2$).The amplitude of this electric field increases with increasing intensity and imparts an increasing acceleration and kinetic energy to an electron.
- Now $I$ is replaced with a quantity proportional to ($E_0^2$) in equation (1). This means that $K_{\max}$ should not depend at all on the frequency of the classical light wave which again contradicts the experimental results.
- If an electron accumulates light energy just enough to overcome the work function, then it is ejected out of the atom with zero kinetic energy. Therefore, from equation (1),

$$0 = IAt - \phi_0$$

$$t = \frac{\phi_0}{IA} = \frac{\phi_0}{I(\pi r^2)}$$

$$t = \frac{2.14 \times 1.6 \times 10^{-19}}{1.60 \times 10^{-6} \times 3.14 \times (1 \times 10^{-10})^2}$$

$$= 0.68 \times 10^7\, s \approx 79\, days$$

Thus, wave theory predicts that there is a large time gap between the incidence of light and the ejection of photoelectrons but the experiments show that photo emission is an instantaneous process.

**Concept of quantization of energy**

Max Planck proposed quantum concept in 1900 in order to explain the thermal radiations emitted by a black body and the shape of its radiation curves. According to Planck, matter is composed of a large number of oscillating particles (atoms) which vibrate with different frequencies. Each atomic oscillator - which vibrates with its characteristic frequency - emits or absorbs electromagnetic radiation of the same frequency. It also says that

- If an oscillator vibrates with frequency $v$, its energy can have only certain discrete values, given by the equation.

$$E_n = nh\nu \qquad n=1,2,3....$$

where $h$ is a constant, called Planck's constant.

The oscillators emit or absorb energy in small packets or quanta and the energy of each quantum is $E = h\nu$.

This implies that the energy of the oscillator is quantized – that is, energy is not continuous as believed in the wave picture. This is called quantization of energy.

## Particle nature of light: Einstein's explanation

Einstein extended Planck's quantum concept to explain the photoelectric effect in 1905. According to Einstein, the energy in light is not spread out over wave fronts but is concentrated in small packets or energy quanta. Therefore, light (or any other electromagnetic waves) of frequency $v$ from any source can be considered as a stream of quanta and the energy of each light quantum is given by $E = h\nu$.

He also proposed that a quantum of light has linear momentum and the magnitude of that linear momentum is $p = \frac{h\nu}{c}$ . The individual light quantum of definite energy and momentum can be associated with a particle. The light quantum can behave as a particle and this is called photon. Therefore, photon is nothing but particle manifestation of light.

## Characteristics of photons:

According to particle nature of light, photons are the basic constituents of any radiation and possess the following characteristic properties:

The photons of light of frequency $\nu$ and wavelength $\lambda$ will have energy, given by

$$E = h\nu = \frac{hc}{\lambda}.$$

The energy of a photon is determined by the frequency of the radiation and not by its intensity and the intensity has no relation with the energy of the individual photons in the beam.

The photons travel with the velocity of light and its momentum is given by

$$p=\frac{h}{\lambda}=\frac{h\nu}{c}$$

Since photons are electrically neutral, they are unaffected by electric and magnetic fields.

When a photon interacts with matter (photon-electron collision), the total energy, total linear momentum and angular momentum are conserved. Since photon may be absorbed or a new photon may be produced in such interactions, the number of photons may not be conserved.

**Einstein's explanation of photoelectric equation**

When a photon of energy $h\nu$ is incident on a metal surface, it is completely absorbed by a single electron and the electron is ejected. In this process, a part of the photon energy is used for the ejection of the electrons from the metal surface (photoelectric work function $\phi 0$ ) and the remaining energy as the kinetic energy of the ejected electron. From the law of conservation of energy,

$$h\nu=\phi_0+\frac{1}{2}m\upsilon^2$$

where $m$ is the mass of the electron and $\upsilon$ its velocity.

If we reduce the frequency of the incident light, the speed or kinetic energy of photo electrons is also reduced. At some frequency $\nu 0$ of incident radiation, the photo electrons are ejected with almost zero kinetic energy (Figure 7.13(b)). Then the equation (7.6) becomes

$$h\nu_0=\phi_0$$

where $\nu_0$ is the threshold frequency. By rewriting the equation (7.6), we get

$$h\nu = h\nu_0 + \frac{1}{2}m\upsilon^2$$

If the electron does not lose energy by internal collisions, then it is emitted with maximum kinetic energy $K_{max}$. Then

$$K_{max} = \frac{1}{2}m\upsilon^2_{max}$$

where $\upsilon_{max}$ is the maximum velocity of the electron ejected. The equation (7.6) is rearranged as follows:

$$K_{max} = h\nu - \phi_0$$

A graph between maximum kinetic energy $K_{max}$ of the photoelectron and frequency $\nu$ of the incident light is a straight line as shown in Figure 7.14. The slope of the line is $h$ and its y-intercept is $-\phi_0$. Einstein's equation was experimentally verified by R.A. Millikan. He drew *Kmax* versus $\nu$ graph for many metals (cesium, potassium, sodium and lithium) as shown in Figure 7.15 and found that the slope is independent of the metals.

Millikan also calculated the value of Planck's constant ($h$=6.626×10–34 $Js$) and work function of many metals (Cs, K, Na, Ca); these values are in agreement with the theoretical prediction.

**Explanation for the photoelectric effect:**

The experimentally observed facts of photoelectric effect can be explained with the help of Einstein's photoelectric equation.

As each incident photon liberates one electron, then the increase of intensity of the light (the number of photons per unit area per unit time) increases the number of electrons emitted thereby increasing the photocurrent. The same has been experimentally observed.

From , it is evident that $K_{max}=hv-ø_0$ is proportional to the frequency of the light and is independent of intensity of the light.

As given in equation (7.7), there must be minimum energy (equal to the work function of the metal) for incident photons to liberate electrons from the metal surface. Below which, emission of electrons is not possible. Correspondingly, there exists minimum frequency called threshold frequency below which there is no photoelectric emission.

According to quantum concept, the transfer of photon energy to the electrons is instantaneous so that there is no time lag between incidence of photons and ejection of electrons.

Thus, the photoelectric effect is explained on the basis of quantum concept of light.

**The nature of light: wave - particle duality**

We have learnt that wave nature of light explains phenomena such as interference, diffraction and polarization. Certain phenomena like black body radiation, photoelectric effect can be explained by assigning particle nature to light. Therefore, both theories have enough experimental evidences.

In the past, many scientific theories have been either revised or discarded when they contradicted with new experimental results. Here, two different theories are needed to answer the question: what is nature of light? It is therefore concluded that light possesses dual nature, that of both particle and wave. It behaves like a wave at some circumstances and it behaves like a particle at some other circumstances.

In other words, light behaves as a wave during its propagation and behaves as a particle during its interaction with matter. Both theories are necessary for complete description of physical phenomena. Hence, the wave nature and quantum nature complement each other.

**Photo electric cells and their applications**
**Photo cell**

Photo electric cell or photo cell is a device which converts light energy into electrical energy. It works on the principle of photo electric effect. When light is incident on the photosensitive materials, their electric properties will get affected, based on which photo cells are classified into three types. They are

**Photo emissive cell:**

Its working depends on the electron emission from a metal cathode due to irradiation of light or other radiations.

**Photo voltaic cell:**

Here sensitive element made of semiconductor is used which generates voltage proportional to the intensity of light or other radiations.

**Photo conductive cell:**

In this, the resistance of the semiconductor changes in accordance with the radiant energy incident on it.

In this section, we discuss about photo emissive cell and its applications.

**Photo emissive cell**
**Construction:**

It consists of an evacuated glass or quartz bulb in which two metallic electrodes – that is, a cathode and an anode are fixed.

A reader may find it difficult to understand how light can be both a wave and a stream of particle. This is the case even for great scientist like Albert Einstein.
Einstein once wrote a letter to his friend Michel Besso in 1954 expressing his frustration:
“All these fifty years of conscious brooding have brought me no closer to answer the question, ‘What are light quanta?’ Of course today everyone thinks he knows the answer, but he is deluding himself”.

The cathode *C* is semi-cylindrical in shape and is coated with a photo sensitive material. The anode *A* is a thin rod or wire kept along the axis of the semi-cylindrical cathode. A potential difference is applied between the anode and the cathode through a galvanometer *G*.

**Working:**

When cathode is illuminated, electrons are emitted from it. These electrons are attracted by anode and hence a current is produced which is measured by the galvanometer. For a given cathode, the magnitude of the current depends on

i) the intensity to incident radiation and
ii) the potential difference between anode and cathode.

**Applications of photo cells:**

Photo cells have many applications, especially as switches and sensors. Automatic lights that turn on when it gets dark use photocells, as well as street lights. that switch on and off according to whether it is night or day.

Photo cells are used for reproduction of sound in motion pictures and are used as timers to measure the speeds of athletes during a race. Photo cells of exposure meters in photography are used to measure the intensity of the given light and to calculate the exact time of exposure.

A radiation of wavelength 300 *nm* is incident on a silver surface. Will photoelectrons be observed?

**Solution:**

Energy of the incident photon is

$$E = h\nu = \frac{hc}{\lambda} \text{ (in joules)}$$

$$E = \frac{hc}{\lambda e} \text{ (in } eV\text{)}$$

Substituting the known values, we get

$$E = \frac{6.626\times10^{-34}\times3\times10^{8}}{300\times10^{-9}\times1.6\times10^{-19}}$$

$$E = 4.14\ eV$$

From Table 7.1, the work function of silver = 4.7 $eV$. Since the energy of the incident photon is less than the work function of silver, photoelectrons are not observed in this case.

## MATTER WAVES

### Introduction - Wave nature of particles

So far, we learnt that the characteristics of particles and waves are different. A wave is specified by its frequency, wavelength, wave velocity, amplitude and intensity. It spreads out and occupies a relatively large region of space. A particle specified by its mass, velocity, momentum and energy occupies a definite position in space and is very small in size.

Classical physics treated particles and waves as distinct entities. But quantum theory suggested dual character for radiations – that is, radiation behaves as a wave at times and as a particle at other times.

From this wave – particle duality of radiation, the concept of wave nature of matter arises which we will see in this section.

### De Broglie wave:

The wave–particle duality of radiation was extended to matter by a French physicist Louis de Broglie (pronounced as de Broy) in 1924.

Greatly influenced by the symmetry in nature, de Broglie suggested that if radiation like light can act as particles at times, then matter particles like electrons should also act as waves at times.

According to de Broglie hypothesis, all matter particles like electrons, protons, neutrons in motion are associated with waves. These waves are called de Broglie waves or matter waves.

### De Broglie wave length:

The momentum of photon of frequency $\nu$ is given by

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \quad \text{since } c = \nu\lambda$$

The wavelength of a photon in terms of its momentum is

$$\lambda = \frac{h}{p}$$

According to de Broglie, the above equation is completely a general one and this is applicable to material particles as well. Therefore, for a particle of mass m travelling with speed υ , the wavelength is given by

$$\lambda = \frac{h}{m\upsilon} = \frac{h}{p}$$

This wavelength of the matter waves is known as **de Broglie wavelength**. This equation relates the wave character (the wave length λ) and the particle character (the momentum *p*) through Planck's constant.

**De Broglie wave length of electrons:**

An electron of mass *m* is accelerated through a potential difference of *V* volt. The kinetic energy acquired by the electron is given by

$$\frac{1}{2}m\upsilon^2 = eV$$

Therefore, the speed υ of the electron is

$$\upsilon = \sqrt{\frac{2eV}{m}}$$

$$\lambda = \frac{h}{m\upsilon} = \frac{h}{\sqrt{2emV}}$$

Substituting the known values in the above equation, we get

$$\lambda = \frac{6.626\times10^{-34}}{\sqrt{2V\times1.6\times10^{-19}\times9.11\times10^{-31}}}$$

$$= \frac{12.27\times10^{-10}}{\sqrt{V}}\text{ meter (or)}$$

$$\lambda = \frac{12.27}{\sqrt{V}}\,\mathring{A}$$

For example, if an electron is accelerated through a potential difference of $100V$, then its de Broglie wavelength is 1.227 $\mathring{A}$.

Since the kinetic energy of the electron, $K = eV$, then the de Broglie wavelength associated with electron can be also written as

$$\lambda = \frac{h}{\sqrt{2mK}}$$

## Davisson – Germer experiment

De Broglie hypothesis of matter waves was experimentally confirmed by Clinton Davisson and Lester Germer in 1927. They demonstrated that electron beams are diffracted when they fall on crystalline solids. Since crystal can act as a three-dimensional diffraction grating for matter waves, the electron waves incident on crystals are diffracted off in certain specific directions. Figure 7.17 shows a schematic representation of the apparatus for the experiment.

The filament $F$ is heated by a low tension (L.T.) battery. Electrons are emitted from the hot filament by thermionic emission. They are then accelerated due to the potential difference between the filament and the anode aluminium cylinder by a high tension (H.T.) battery. Electron beam is collimated by using two thin aluminium diaphragms and is allowed to strike a single crystal of Nickel.

The electrons scattered by Ni atoms in different directions are received by the electron detector which measures the intensity of scattered electron beam. The detector is rotatable in the plane of the paper so that the angle $\phi$ between the incident beam and the scattered beam can be changed at our will. The intensity of the scattered electron beam is measured as a function of the angle $\theta$.

Variation of intensity of the scattered electrons with the angle $\theta$ for the accelerating voltage of $54V$. For a given accelerating voltage $V$, the scattered wave shows a peak or maximum at an angle of 50° to the incident electron beam. This peak in intensity is attributed to the constructive interference of electrons diffracted from various atomic layers of the target material. From the known value of inter planar spacing of Nickel, the wavelength of the electron wave has been experimentally calculated as 1.65Å.

The wavelength can also be calculated from de Broglie relation for V = 54 V from equation (7.18) as

$$\lambda = \frac{12.27}{\sqrt{V}} \mathring{A} = \frac{12.27}{\sqrt{54}}$$

$$\lambda = 1.67\ \mathring{A}$$

This value agrees well with the experimentally observed wavelength of 1.65Å. Thus this experiment directly verifies de Broglie's hypothesis of the wave nature of moving particles.

## Electron Microscope

### Principle

This is the direct application of wave nature of particles. The wave nature of the electron is used in the construction of microscope called electron microscope. The resolving power of a microscope is inversely proportional to the wavelength of the radiation used for illuminating the object under study. Higher magnification as well as higher resolving power can be obtained by employing the waves of shorter wavelengths.

De Broglie wavelength of electron is very much less than (a few thousands less) that of the visible light being used in optical microscopes. As a result, the microscopes employing de Broglie waves of electrons have very much higher resolving power than optical microscope. Electron microscopes giving magnification more than 2,00,000 times are common in research laboratories.

### Working

The construction and working of an electron microscope is similar to that of an optical microscope except that in electron microscope focussing of electron beam is done by the electrostatic or magnetic lenses. The electron beam passing across a suitably arranged either electric or magnetic fields undergoes divergence or convergence thereby focussing of the beam is done (Figure 7.19).

The electrons emitted from the source are accelerated by high potentials. The beam is made parallel by magnetic condenser lens. When the beam passes through the sample whose magnified image is needed, the beam carries the image of the sample.

With the help of magnetic objective lens and magnetic projector lens system, the magnified image is obtained on the screen. These electron microscopes are being used in almost all branches of science.

## X – RAYS

### Introduction

Quantum theory of radiation explains photoelectric effect in which the electrons are emitted due to the incidence of photons and the energy is transferred from photons to the electrons. Immediately, a question arises: Is the reverse process also possible?

This means that whether electron kinetic energy can be transformed into photon energy or not. The phenomenon which answers this question has already been discovered, even before Planck's quantum theory of radiation.

### Discovery of x-rays

Wilhelm Roentgen in 1895 discovered that whenever fast moving electrons fall on certain materials, a highly penetrating radiation is emitted. Since their origin was not known at that time, they were called x-rays.

X-rays are electromagnetic waves of short wavelength ranging from 0.1 to 100$\mathring{A}$. They travel along straight lines with the velocity of light and are not affected by electric and magnetic fields. X-ray photons are highly energetic because of its high frequency or short wavelength. Therefore, they can pass through materials which are opaque to visible light.

The quality of x-rays is measured in terms of their penetrating power which depends on the velocity with which the electrons strike the target material and the atomic number of target material. The intensity of x-rays is dependent on the number of electrons striking the target.

## Production of x-rays

X-rays are produced in x-ray tube which is essentially a discharge tube as shown in Figure 7.20. A tungsten filament *F* is heated to incandescence by a battery. As a result, electrons are emitted from it by thermionic emission.

The electrons are accelerated to high speeds by the voltage applied between the filament *F* and the anode. The target materials like tungsten, molybdenum are embedded in the face of the solid copper anode. The face of the target is inclined at an angle with respect to the electron beam so that x-rays can leave the tube through its side.

When high-speed electrons strike the target, they are decelerated suddenly and lose their kinetic energy. As a result, x-ray photons are produced. Since most of the kinetic energy of the bombarding electrons gets converted into heat, targets made of high-melting-point metals and a cooling system are usually employed.

## X-ray spectra

X-rays are produced when fast moving electrons strike the metal target. The intensity of the x-rays when plotted against its wavelength gives a curve called **x-ray spectrum** (Figure 7.21(a) and (b)). X-ray spectra consist of two parts: a continuous spectrum and a series of peaks superimposed on it.

The continuous spectrum consists of radiations of all possible wavelengths with a certain minimum wavelength $\lambda_0$ which depends on the voltage across the electrodes. The peaks are characteristics of the material of the target and hence they are called characteristic spectrum. Figure 7.21(a) depicts the x-ray spectra of tungsten at various accelerating voltages and Figure 7.21(b) shows the x-ray spectra of tungsten and molybdenum at a particular accelerating voltage.

Though classical electromagnetic theory suggests the emission of radiations from accelerating electrons, it could not explain two features exhibited by x-ray spectra. These features are given below.

- For a given accelerating voltage, the lower limit for the wavelength of continuous x-ray spectra is same for all targets. This minimum wavelength is called cut-off wavelength.

- The intensity of x-rays is significantly increased at certain well-defined wavelengths as shown in the case of characteristic x-ray spectra for molybdenum (Figure 7.21(b)).
- But these two features could be explained on the basis of photon theory of radiation.

## Continuous x-ray spectra

When a fast moving electron penetrates and approaches a target nucleus, the interaction between the electron and the nucleus either accelerates or decelerates it which results in a change of path of the electron. The radiation produced from such decelerating electron is called Bremsstrahlung or braking radiation.

The energy of the photon emitted is equal to the loss of kinetic energy of the electron. Since an electron may lose part or all of its energy to the photon, the photons are emitted with all possible energies (or frequencies). The continuous x-ray spectrum is due to such radiations.

When an electron gives up all its energy, then the photon is emitted with highest frequency $\nu_0$ (or lowest wavelength $\lambda_0$). The initial kinetic energy of an electron is given by $eV$ where $V$ is the accelerating voltage. Therefore, we have

$$h\nu_0 = eV \quad \text{(or)} \quad \frac{hc}{\lambda_0} = eV$$

$$\lambda_0 = \frac{hc}{eV}$$

where $\lambda_0$ is the cut-off wavelength. Substituting the known values in the above equation, we get

$$\lambda_0 = \frac{12400}{V}\ \text{Å}$$

The relation given by equation (7.14) is known as the Duane – Hunt formula.

The value of $\lambda_0$ depends only on the accelerating potential and is same for all targets. This is in good agreement with the experimental results. Thus, the production of continuous x-ray spectrum and the origin of cut – off wavelength can be explained on the basis of photon theory of radiation.

**Characteristic x – ray spectra:**

X – ray spectra show some narrow peaks at some well – defined wavelengths when the target is hit by fast electrons. The line spectrum showing these peaks is called characteristic x – ray spectrum. This x – ray spectrum is due to the electronic transitions within the atoms.

When an energetic electron penetrates into the target atom and removes some of the *K*-shell electrons. Then the electrons from outer orbits jump to fill up the vacancy so created in the *K*-shell. During the downward transition, the energy difference between the levels is given out in the form of x– ray photon of definite wavelength. Such wavelengths, characteristic of the target, constitute the line spectrum.

From the Figure 7.23, it is evident that *K*-series of lines in the x-ray spectrum of an element arises due to the electronic transitions from *L, M, N*, . . levels to the *K*-level. Similarly, the longer wavelength *L*-series originates when an *L*-electron is knocked out of the atom and the corresponding vacancy is filled by the electronic transitions from *M, N, O*,... and so on.

The $K_\alpha$ and $K_\beta$ of the *K*-series of molybdenum are shown by the two peaks in its x-ray spectrum in Figure 7.21(b).

Applications of x-rays:

X-rays are being used in many fields. Let us list a few of them.

**Medical diagnosis**

X-rays can pass through flesh more easily than through bones. Thus an x-ray radiograph containing a deep shadow of the bones and a light shadow of the flesh may be obtained. X-ray radiographs are used to detect fractures, foreign bodies, diseased organs etc.

**Medical therapy**

Since x-rays can kill diseased tissues, they are employed to cure skin diseases, malignant tumours etc.

**Industry**

X-rays are used to check for flaws in welded joints, motor tyres, tennis balls and wood. At the custom post, they are used for detection of contraband goods.

**Scientific research**

X-ray diffraction is important tool to study the structure of the crystalline materials – that is, the arrangement of atoms and molecules in crystals.

Calculate the cut-off wavelength and cut-off frequency of x-rays from an x –ray tube of accelerating potential 20,000 $V$.

**Solution**

The cut-off wavelength of the characteristic x-rays is

$$\lambda_0 = \frac{12400}{V} Å = \frac{12400}{20000} Å$$
$$= 0.62 Å$$

The corresponding frequency is

$$\nu_0 = \frac{c}{\lambda_0} = \frac{3\times10^8}{0.62\times10^{-10}} = 4.84\times10^{18}\,\text{Hz}$$

# Unit 8 ATOMIC AND NUCLEAR PHYSICS

## INTRODUCTION

In earlier classes, we have studied that anything which occupies space is called matter. Matter can be classified into solids, liquids and gases. In our daily life, we use water for drinking, petrol for vehicles, we inhale oxygen, stainless steel vessels for cooking, etc. Experiences tell us that behaviour of one material is not same as another, this means that the physical and chemical properties are different for different materials. In order to understand this, we need to know the fundamental constituents of materials.

When an object is divided repeatedly, the process of division could not be done beyond a certain stage in a similar way and we end up with a small speck. This small speck was defined as an atom. The word atom in Greek means 'without division or indivisible'. The size of an atom is very very small. For an example, the size of hydrogen atom (simplest among other atoms) is arround 10–10 m. An American Physicist Richard P. Feynman said that if the atom becomes the size of an apple, then the apple becomes the size of the earth as shown in Figure 8.1. Such a small entity is an atom.

In this unit, we first discuss the theoretical models of atom to understand its structure. The Bohr atom model is more successful than J. J. Thomson and Rutherford atom models. It explained many unsolved issues in those days and also gave better understanding of chemistry.

Later, scientists observed that even the atom is not the fundamental entity. It consists of electrons and nucleus. Around 1930, scientists discovered that nucleus is also made of proton and neutron. Further research discovered that even the proton and neutron are made up of fundamental entities known as quarks.

In this context, the remaining part of this unit is written to understand the structure and basic properties of nucleus. Further how the nuclear energy is produced and utilized are discussed.

## ELECTRIC DISCHARGE THROUGH GASES

Gases at normal atmospheric pressure are poor conductors of electricity because they do not have free electrons for conduction. But by special arrangement, one can make a gas to conduct electricity.

A simple and convenient device used to study the conduction of electricity through gases is known as gas discharge tube. The arrangement of discharge tube is shown in Figure 8.2. It consists of a long closed glass tube (of length nearly 50 cm and diameter of 4 cm) inside of which the gas in pure form is filled usually. The small opening in the tube is connected to a high vacuum pump and a low-pressure gauge. This tube is fitted with two metallic plates known as electrodes which are connected to secondary of an induction coil. The electrode connected to positive of secondary is known as anode and the electrode to the negative of the secondary is cathode. The potential of secondary is maintained about 50 kV.

Suppose the pressure of the gas in discharge tube is reduced to around 110 mm of Hg using vacuum pump, it is observed that no discharge takes place. When the pressure is kept near 100 mm of Hg, the discharge of electricity through the tube takes place. Consequently, irregular streaks of light appear and also crackling sound is produced. When the pressure is reduced to the order of 10 mm of Hg, a luminous column known as positive column is formed from anode to cathode.

When the pressure reaches to around 0.01 mm of Hg, positive column disappears. At this time, a dark space is formed between anode and cathode which is often called Crooke’s dark space and the walls of the tube appear with green colour. At this stage, some invisible rays emanate from cathode called cathode rays, which are later found be a beam of electrons.

### Properties of cathode rays

Cathode rays possess energy and momentum and travel in a straight line with high speed of the order of 107m s-1. It can be deflected by application of electric and magnetic fields. The direction of deflection indicates that they are negatively charged particles.

When the cathode rays are allowed to fall on matter, they produce heat. They affect the photographic plates and also produce fluorescence when they fall on certain crystals and minerals.

When the cathode rays fall on a material of high atomic weight, x-rays are produced.

Cathode rays ionize the gas through which they pass.

The speed of cathode rays is up to $\left(\frac{1}{10}\right)^{th}$ of the speed of light.

## Determination of specific charge $\left(\frac{e}{m}\right)$ of an electron – Thomson's experiment

Thomson's experiment is considered as one among the landmark experiments for the birth of modern physics. In 1887, J. J. Thomson made remarkable improvement in the scope of study of gases in discharge tubes. In the presence of electric and magnetic fields, the cathode rays are deflected. By the variation of electric and magnetic fields, mass normalized charge or the specific charge (charge per unit mass) of the cathode rays is measured.

The arrangement of J. J. Thomson's experiment is shown in Figure 8.3. A highly evacuated discharge tube is used and cathode rays (electron beam) produced at cathode are attracted towards anode disc A. Anode disc is made with pin hole in order to allow only a narrow beam of cathode rays. These cathode rays are now allowed to pass through the parallel metal plates, maintained at high voltage as shown in Figure 8.3. Further, this gas discharge tube is kept in between pole pieces of magnet such that both electric and magnetic fields are perpendicular to each other. When the cathode rays strike the screen, they produce scintillation and hence bright spot is observed. This is achieved by coating the screen with zinc sulphide.

For a fixed electric field between the plates, the magnetic field is adjusted such that the cathode rays (electron beam) strike at the original position O (Figure 8.3). This means that the magnitude of electric force is balanced by the magnitude of force due to magnetic field as shown in Figure 8.4. Let *e* be the charge of the cathode rays, then

$$eE\ eB=\upsilon$$

$$\Rightarrow \upsilon = \frac{E}{B}$$

**Determination of specific charge**

Since the cathode rays (electron beam) are accelerated from cathode to anode, the potential energy of the electron beam at the cathode is converted into kinetic energy of the electron beam at the anode. Let *V* be the potential difference between anode and cathode, then the potential energy is *eV*. Then from law of conservation of energy,

$$eV = \frac{1}{2}m\upsilon^2 \Rightarrow \frac{e}{m} = \frac{\upsilon^2}{2V}$$

Substituting the value of velocity from equation (8.1), we get

$$\frac{e}{m} = \frac{1}{2V}\frac{E^2}{B^2}$$

Substituting the values of *E, B* and *V*, the specific charge can be determined as

$$\frac{e}{m} = 1.7 \times 10^{11}\,\mathrm{C\,kg^{-1}}$$

**Deflection of charge only due to uniform electric field**

When the magnetic field is turned off, the deflection is only due to electric field. The deflection in vertical direction is due to the electric force.

$$F_e = eE$$

Let *m* be the mass of the electron and by applying Newton's second law of motion, acceleration of the electron is

$$a_e = \frac{1}{m}F_e$$

Substituting equation (8.4) in equation

$$a_e = \frac{1}{m} eE = \frac{e}{m} E$$

Let y be the deviation produced from original position on the screen as shown in Figure 8.5. Let the initial upward velocity of cathode ray be u = 0 before entering the parallel electric plates. Let t be the time taken by the cathode rays to travel in electric field. Let l be the length of one of the plates, then the time taken is

$$t = \frac{l}{\upsilon}$$

Hence, the deflection y′ of cathode rays is

$$y' = ut + \frac{1}{2}at^2 \Rightarrow y' = ut + \frac{1}{2}a_e t^2$$
$$= \frac{1}{2}\left(\frac{e}{m}E\right)\left(\frac{l}{\upsilon}\right)^2$$

$$y' = \frac{1}{2}\frac{e}{m}\frac{l^2 B^2}{E}$$

Therefore, the deflection y on the screen

$$y \propto y' \Rightarrow y = Cy'$$

where C is proportionality constant which depends on the geometry of the discharge tube and substituting y′ value in equation 8.6, we get

$$y = C\frac{1}{2}\frac{e}{m}\frac{l^2 B^2}{E}$$

$$\frac{e}{m} = \frac{2yE}{Cl^2 B^2}$$

Substituting the values on RHS, the value of specific charge is calculated as $\frac{e}{m} = 1.7 \times 10^{11} Ckg^{-1}$

## Deflection of charge only due to uniform magnetic field

Suppose that the electric field is switched off and only the magnetic field is switched on. Now the deflection occurs only due to magnetic field. The force experienced by the electron in uniform magnetic field applied perpendicular to its path is

$$F_m = e\upsilon B$$

Since this force provides the centripetal force, the electron beam undergoes a semi- circular path. Therefore, we can equate $F_m$ to centripetal force $\frac{mv^2}{R}$

$$F_m = e\upsilon B = m\frac{\upsilon^2}{R}$$

where $v$ is the velocity of electron beam at the point where it enters the magnetic field and $R$ is the radius of the circular path traversed by the electron beam.

$$eB = m\frac{\upsilon}{R} \Rightarrow \frac{e}{m} = \frac{\upsilon}{BR}$$

Further, substituting equation (8.1) in equation (8.10), we get

$$\frac{e}{m} = \frac{E}{B^2R}$$

By knowing the values of electric field, magnetic field and the radius of circular path, the value of specific charge $\left(\frac{e}{m}\right)$ can be calculated, which is also consistant with other two methods

## Determination of charge of an electron – Millikan’s oil drop experiment

Millikan’s oil drop experiment is another important experiment in modern physics which is used to determine one of the fundamental constants of nature known as charge of an electron (Figure 8.6 (a)).

By adjusting electric field suitably, the motion of oil drop inside the chamber can be controlled – that is, it can be made to move up or down or even kept balanced in the field of view for sufficiently long time.

The experimental arrangement is shown in Figure 8.6 (b). The apparatus consists of two horizontal circular metal plates A and B each with diameter around 20 cm and are separated by a small distance 1.5 cm.

These two parallel plates are enclosed in a chamber with glass walls. Further, plates A and B are given a high potential difference around 10 $kV$ such that electric field acts vertically downward. A small hole is made at the centre of the upper plate A and atomizer is kept exactly above the hole to spray the liquid. When a fine droplet of highly viscous liquid (like glycerine) is sprayed using atomizer, it falls freely downward through the hole of the top plate only under the influence of gravity.

Few oil drops in the chamber can acquire electric charge (negative charge) because of friction with air or passage of x-rays in between the parallel plates. Further the chamber is illuminated by light which is passed horizontally and oil drops can be seen clearly using microscope placed perpendicular to the light beam. These drops can move either upwards or downward.

Let $m$ be the mass of the oil drop and $q$ be its charge. Then the forces acting on the droplet are

1. gravitational force $F_g$ =mg
2. electric force $F_e$ =qE

3. buoyant force $F_b$
4. viscous force $F_v$

**Determination of radius of the droplet**

When the electric field is switched off, the oil drop accelerates downwards. Due to the presence of air drag forces, the oil drops easily attain its terminal velocity and moves with constant velocity. This velocity can be carefully measured by noting down the time taken by the oil drop to fall through a predetermined distance. The free body diagram of the oil drop is shown in Figure 8.7 (a), we note that viscous force and buoyant force balance the gravitational force.

Let the gravitational force acting on the oil drop (downward) be $F_g = mg$

Let us assume that oil drop to be spherical in shape. Let ρ be the density of the oil drop, and *r* be the radius of the oil drop, then the mass of the oil drop can be expressed in terms of its density as

$$\rho = \frac{m}{V}$$

$$\Rightarrow m = \rho\left(\frac{4}{3}\pi r^3\right) \qquad \left(\because \text{volume of the sphere}, V = \frac{4}{3}\pi r^3\right)$$

The gravitational force can be written in terms of density as

$$F_g = mg \Rightarrow F_g = \rho\left(\frac{4}{3}\pi r^3\right)g$$

Let σ be the density of the air, the up thrust force experienced by the oil drop due to displaced air is

$$F_b = \sigma\left(\frac{4}{3}\pi r^3\right)g$$

Once the oil drop attains a terminal velocity *υ*, the net downward force acting on the oil drop is equal to the viscous force acting opposite to the direction of motion of the oil drop. From Stokes law, the viscous force on the oil drop is

$$F_v = 6\pi r \upsilon \eta$$

From the free body diagram as shown in Figure 8.7 (a), the force balancing equation is

$$F_g = F_b + F_\upsilon$$

$$\rho\left(\frac{4}{3}\pi r^3\right)g = \sigma\left(\frac{4}{3}\pi r^3\right)g + 6\pi r\upsilon\eta$$

$$\frac{4}{3}\pi r^3(\rho-\sigma)g = 6\pi r\upsilon\eta$$

$$\frac{2}{3}\pi r^3(\rho-\sigma)g = 3\pi r\upsilon\eta$$

$$\boxed{r = \left[\frac{9\eta\upsilon}{2(\rho-\sigma)g}\right]^{\frac{1}{2}}}$$

**Determination of electric charge**

When the electric field is switched on, charged oil drops experience an upward electric force (*qE*). Among many drops, one particular drop can be chosen in the field of view of microscope and strength of the electric field is adjusted to make that particular drop to be stationary. Under these circumstances, there will be no viscous force acting on the oil drop. Then, from the free body diagram shown Figure 8.7 (b), the net force acting on the oil droplet is

$$F_e + F_b = F_g$$

$$\Rightarrow qE + \frac{4}{3}\pi r^3\sigma g = \frac{4}{3}\pi r^3\rho g$$

$$\Rightarrow qE = \frac{4}{3}\pi r^3(\rho-\sigma)g$$

$$\boxed{\Rightarrow q = \frac{4}{3E}\pi r^3(\rho-\sigma)g}$$

Substituting equation (8.11) in equation (8.12), we get

$$q = \frac{18\pi}{E}\left(\frac{\eta^3\upsilon^3}{2(\rho-\sigma)g}\right)^{\frac{1}{2}}$$

Millikan repeated this experiment several times and computed the charges on oil drops. He found that the charge of any oil drop can be written as integral multiple of a basic value, $-1.6\times10^{-19}$C which is nothing but the charge of an electron.

## ATOM MODELS

### Introduction

Around 400 B.C, Greek philosophers Leucippus and Democretus proposed the concept of atom, 'Every object on continued subdivision ultimately yields atoms'. Later, many physicists and chemists tried to understand the nature with the idea of atoms. Many theories were proposed to explain the properties (physical and chemical) of bulk materials on the basis of atomic model.

For instance, J. J. Thomson proposed a theoretical atom model which is based on static distribution of electric charges. Since this model fails to explain the stability of atom, one of his students E. Rutherford proposed the first dynamic model of an atom. Rutherford gave atom model which is based on results of an experiment done by his students (Geiger and Marsden). But this model also failed to explain the stability of the atom.

Later, Niels Bohr who is also a student of Rutherford proposed an atomic model for hydrogen atom which is more successful than other two models. Niels Bohr atom model could explain the stability of the atom and also the origin of line spectrum. There are other atom models, such as Sommerfeld's atom model and atom model from wave mechanics (quantum mechanics). But we will restrict ourselves only to very simple (mathematically simple) atom model in this section.

### J. J. Thomson's Model (Water melon model)

In this model, the atoms are visualized as homogeneous spheres which contain uniform distribution of positively charged particles (Figure 8.8 (a)). The negatively charged particles known as electrons are embedded in it like seeds in water melon as shown in Figure 8.8 (b).

The atoms are electrically neutral, this implies that the total positive charge in an atom is equal to the total negative charge. According to this model, all the charges are assumed to be at rest. But from classical electrodynamics, no stable equilibrium points exist in electrostatic configuration (this is known as Earnshaw's theorem) and hence such an atom cannot be stable. Further, it fails to explain the origin of spectral lines observed in the spectrum of hydrogen atom and other atoms.

**Rutherford's model**

In 1911, Geiger and Marsden did a remarkable experiment based on the advice of their teacher Rutherford, which is known as scattering of alpha particles by gold foil.

The experimental arrangement is shown in Figure 8.9. A source of alpha particles (radioactive material, example polonium) is kept inside a thick lead box with a fine hole as seen in Figure 8.9. The alpha particles coming through the fine hole of lead box pass through another fine hole made on the lead screen. These particles are now allowed to fall on a thin gold foil and it is observed that the alpha particles passing through gold foil are scattered through different angles. A movable screen (from 0° to 180°) which is made up of zinc sulphide (ZnS) is kept on the other side of the gold foil to collect the alpha particles. Whenever alpha particles strike the screen, a flash of light is observed which can be seen through a microscope.

Rutherford proposed an atom model based on the results of alpha scattering experiment. In this experiment, alpha particles (positively charged particles) are allowed to fall on the atoms of a metallic gold foil. The results of this experiment are given below and are shown in Figure 8.10, Rutherford expected the nuclear model to be as seen in Figure 8.10 (a) but the experiment showed the model as in Figure 8.10 (b).

- Most of the alpha particles are un-deflected through the gold foil and went straight.
- Some of the alpha particles are deflected through a small angle.
- A few alpha particles (one in thousand) are deflected through the angle more than 90°
- Very few alpha particles returned back (back scattered) –that is, deflected back by 180°

In Figure 8.10 (c), the dotted points are the alpha scattering experiment data points obtained by Geiger and Marsden and the solid curve is the prediction from

Rutherford’s nuclear model. It is observed that the Rutherford’s nuclear model is in good agreement with the experimental data.

**Conclusion made by Rutherford based on the above observation**

From the experimental observations, Rutherford proposed that an atom has a lot of empty space and contains a tiny matter known as nucleus whose size is of the order of 10-14m. The nucleus is positively charged and most of the mass of the atom is concentrated in nucleus. The nucleus is surrounded by negatively charged electrons. Since static charge distribution cannot be in a stable equilibrium, he suggested that the electrons are not at rest and they revolve around the nucleus in circular orbits like planets revolving around the sun.

When an alpha particle moves straight towards the nucleus, it reaches a point where it comes to rest momentarily and returns back as shown in Figure 8.11. The minimum distance between the centre of the nucleus and the alpha particle just before it gets reflected back through 180° is defined as the distance of closest approach r0 (also known as contact distance). At this distance, all the kinetic energy of the alpha particle will be converted into electrostatic potential energy (Refer unit 1, volume 1 of +2 physics text book).

$$\frac{1}{2}m\upsilon_0^2 = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{r_0}$$

$$\Rightarrow r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{\left(\frac{1}{2}m\upsilon_0^2\right)} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{E_k}$$

where *Ek* is the kinetic energy of the alpha particle. This is used to estimate the size of the nucleus but size of the nucleus is always lesser than the distance of closest approach. Further, Rutherford calculated the radius of the nucleus for different nuclei and found that it ranges from $10^{-14}$ m to $10^{-15}$ m.

The impact parameter (b) (see Figure 8.12) is defined as the perpendicular distance between the centre of the gold nucleus and the direction of velocity vector of alpha particle when it is at a large distance. The relation between impact parameter and scattering angle can be shown as

$$b \propto \cot\left(\frac{\theta}{2}\right) \Rightarrow b = K \cot\left(\frac{\theta}{2}\right)$$

where $K = \frac{1}{4\pi \sum_0} \frac{2Ze^2}{m\upsilon_0^2}$ and θ is called scattering angle. Equation (8.13) implies that when impact parameter increases, the scattering angle decreases. Smaller the impact parameter, larger will be the deflection of alpha particles.

**Drawbacks of Rutherford model**

Rutherford atom model helps in the calculation of the diameter of the nucleus and also the size of the atom but has the following limitations:

This model fails to explain the distribution of electrons around the nucleus and also the stability of the atom. Hence, Rutherford model could not account for the stability of atoms.

According to this model, emission of radiation must be continuous and must give continuous emission spectrum but experimentally we observe only line (discrete) emission spectrum for atoms.

**Bohr atom model**

In order to overcome the limitations of the Rutherford atom model in explaining the stability and also the line spectrum observed for a hydrogen atom (Figure 8.14), Niels Bohr made modifications of Rutherford atom model. He is the first person to give better theoretical model of the structure of an atom to explain the line spectrum of hydrogen atom. The following are the assumptions (postulates) made by Bohr.

**Postulates of Bohr atom model:**

The electron in an atom moves around nucleus in circular orbits under the influence of Coulomb electrostatic force of attraction. This Coulomb force gives necessary centripetal force for the electron to undergo circular motion.

Electrons in an atom revolve around the nucleus only in certain discrete orbits called stationary orbits where it does not radiate electromagnetic energy. Only those discrete orbits allowed are stable orbits.

The angular momentum of the electron in these stationary orbits are quantized – that is, $\hbar$ it can be written as integer or integral multiple of $\frac{h}{2\pi r}$ called as reduced Planck's constant – that is, (read it as h-bar) and the integer n is called as principal quantum number of the orbit.

$$l = n\hbar \qquad \text{where } \hbar = \frac{h}{2\pi}$$

This condition is known as angular momentum quantization condition.

According to quantum mechanics, particles like electrons have dual nature (Refer unit 7, volume 2 of +2 physics text book). The standing wave pattern of the de Broglie wave associated with orbiting electron in a stable orbit is shown in Figure 8.15.

The circumference of an electron's orbit of radius $r$ must be an integral multiple of de Broglie wavelength – that is,

$$2\pi r = n\lambda \qquad (8.14)$$

where n = 1,2,3,......

But the de Broglie wavelength ($\lambda$) for an electron of mass $m$ moving with velocity $\upsilon$ is $\lambda = \frac{h}{m\upsilon}$ where $h$ is called Planck's constant. Thus from equation (8.14),

$$2\pi r = n\left(\frac{h}{m\upsilon}\right)$$

$$m\upsilon r = n\frac{h}{2\pi}$$

For any particle of mass $m$ undergoing circular motion with radius $r$ and velocity $\upsilon$, the magnitude of angular momentum $l$ is given by

$$l = r(m\upsilon)$$
$$m\upsilon r = l = n\hbar$$

Energy of orbits are not continuous but discrete. This is called the quantization of energy. An electron can jump from one orbit to another orbit by absorbing or emitting a photon whose energy is equal to the difference in energy (ΔE) between the two orbital levels (Figure 8.16)

$$\Delta E = E_{final} - E_{initial} = hv = h\frac{c}{\lambda}$$

where $c$ is the speed of light and λ is the wavelength of the radiation used and $v$ is the frequency of the radiation. Thus, the frequency of the radiation emitted is related only to change in atom's energy and it does not depend on frequency of electron's orbital motion.

The radius of the 5th orbit of hydrogen atom is 13.25 Å. Calculate the wavelength of the electron in the 5th orbit.

$$2\pi r = n\lambda$$
$$2 \times 3.14 \times 13.25\text{Å} = 5 \times \lambda$$
$$\therefore \lambda = 16.64\text{Å}$$

Find the (i) angular momentum (ii) velocity of the electron in the 5th orbit of hydrogen atom.

($h = 6.6 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg)

Angular momentum is given by

$$l = n\hbar = \frac{nh}{2\pi}$$
$$= \frac{5 \times 6.6 \times 10^{-34}}{2 \times 3.14} = 5.25 \times 10^{-34}\ \text{kgm}^2\text{s}^{-1}$$

Velocity is given by

$$\text{Velocity } \upsilon = \frac{l}{mr}$$

$$= \frac{(5.25\times10^{-34}\ \text{kgm}^2\text{s}^{-1})}{(9.1\times10^{-31}\ \text{kg})(13.25\times10^{-10}\ \text{m})}$$

$$\upsilon = 4.4\times10^{5}\ \text{ms}^{-1}$$

**Radius of the orbit of the electron and velocity of the electron**

Consider an atom which contains the nucleus at rest and an electron revolving around the nucleus in a circular orbit of radius *rn* as shown in Figure 8.17. Nucleus is made up of protons and neutrons. Since proton is positively charged and neutron is electrically neutral, the charge of a nucleus is purely the total charge of protons.

Let $Z$ be the atomic number of the atom, then $+Ze$ is the charge of the nucleus. Let $-e$ be the charge of the electron. From Coulomb's law, the force of attraction between the nucleus and the electron is

$$\vec{F}_{coloumb} = \frac{1}{4\pi\varepsilon_0}\frac{(+Ze)(-e)}{r_n^2}\hat{r}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n^2}\hat{r}$$

This force provides necessary centripetal force

$$\vec{F}_{centripetal} = \frac{m\upsilon_n^2}{r_n}\hat{r}$$

where $m$ be the mass of the electron that moves with a velocity $\upsilon_n$ in a circular orbit. Therefore,

$$\left|\vec{F}_{coloumb}\right| = \left|\vec{F}_{centripetal}\right|$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n^2} = \frac{m\upsilon_n^2}{r_n}$$

$$r_n = \frac{4\pi\varepsilon_0 (m\upsilon_n r_n)^2}{Zme^2}$$

From Bohr's assumption, the angular momentum quantization condition $m\upsilon_n = l_n = n\hbar$

$$\therefore r_n = \frac{4\pi\varepsilon_0 (m\upsilon_n r_n)^2}{Zme^2}$$

$$r_n = \frac{4\pi\varepsilon_0 (n\hbar)^2}{Zme^2} = \frac{4\pi\varepsilon_0 n^2\hbar^2}{Zme^2}$$

$$r_n = \left(\frac{\varepsilon_0 h^2}{\pi me^2}\right)\frac{n^2}{Z} \quad (\because \hbar = \frac{h}{2\pi}) \quad (8.16)$$

where $n \in \mathbb{N}$ . Since, ε0, *h*, *e* and π are constants. Therefore, the radius of the orbit becomes

$$r_n = a_0 \frac{n^2}{Z}$$

$$\text{where} \quad a_0 = \frac{\varepsilon_0 h^2}{\pi me^2} = 0.529\,\text{Å}.$$

This is known as Bohr radius which is the smallest radius of the orbit in an atom. Bohr radius is also used as unit of length called Bohr. 1 Bohr = 0.53 Å. For hydrogen atom (Z = 1), the radius of nth orbit is

$$r_n = a_0 n^2$$

For $n = 1$ (first orbit or ground state),

$$r_1 = a_0 = 0.529\ \text{Å}$$

For n = 2 (second orbit or first excited state),

$$r_2 = 4a_0 = 2.116\ \text{Å}$$

For n = 3 (third orbit or second excited state),

$$r_3 = 9a_0 = 4.761\ \text{Å}$$

Thus the radius of the orbit from centre increases with *n*, that is, $r_n \propto n^2$ as shown in Figure 8.18.

Further, Bohr's angular momentum quantization condition leads to

$$m\upsilon_n r_n = m\upsilon_n a_0 n^2 = n\frac{h}{2\pi}$$

$$\upsilon_n = \frac{h}{2\pi m a_0}\frac{Z}{n}$$

$$\upsilon_n \propto \frac{1}{n}$$

Note that the velocity of electron decreases as the principal quantum number increases as shown in Figure 8.19. This curve is the rectangular hyperbola. This implies that the velocity of electron in ground state is maximum when compared to excited states.

**The energy of an electron in the $n^{th}$ orbit**

Since the electrostatic force is a conservative force, the potential energy for the $n^{th}$ orbit is

$$U_n = \frac{1}{4\pi\varepsilon_0}\frac{(+Ze)(-e)}{r_n} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n}$$

$$= -\frac{1}{4\varepsilon_0^2}\frac{Z^2me^4}{h^2n^2} \quad \left(\because r_n = \frac{\varepsilon_0 h^2}{\pi m e^2}\frac{n^2}{Z}\right)$$

The kinetic energy for the $n^{th}$ orbit is

$$KE_n = \frac{1}{2}m\upsilon_n^2 = \frac{me^4}{8\varepsilon_0^2h^2}\frac{Z^2}{n^2}$$

This implies that $U_n = -2\ KE_n$. Total energy in the nth orbit is

$$E_n = KE_n + U_n = KE_n - 2KE_n = -KE_n$$

$$E_n = -\frac{me^4}{8\varepsilon_0^2h^2}\frac{Z^2}{n^2}$$

For hydrogen atom (Z = 1),

$$E_n = -\frac{me^4}{8\varepsilon_0^2h^2}\frac{1}{n^2}\ \text{joule}$$

where *n* stands for principal quantum number. The negative sign in equation (8.17) indicates that the electron is bound to the nucleus.

Substituting the values of mass and charge of an electron (*m* and *e*), permittivity of free space ε0 and Planck's constant *h* and expressing in terms of *eV*, we get

$$E_n = -13.6\frac{1}{n^2}\ eV$$

For the first orbit (ground state), the total energy of electron is E1= – 13.6 *eV*.
For the second orbit (first excited state), the total energy of electron is E2= –3.4 *eV*.
For the third orbit (second excited state), the total energy of electron is E3= –1.51 *eV* and so on.

Notice that the energy of the first excited state is greater than the ground state, second excited state is greater than the first excited state and so on. Thus, the orbit which is closest to the nucleus ($r_1$) has lowest energy (minimum energy compared with other orbits). So, it is often called ground state energy (lowest energy state). The ground state energy of hydrogen (–13.6 $eV$ ) is used as a unit of energy called Rydberg (1 Rydberg = –13.6 $eV$ ).

The negative value of this energy is because of the way the zero of the potential energy is defined. When the electron is taken away to an infinite distance (very far distance) from nucleus, both the potential energy and kinetic energy terms vanish and hence the total energy also vanishes.

The energy level diagram along with the shape of the orbits for increasing values of $n$ are shown in Figure 8.20. It shows that the energies of the excited states come closer and closer together when the principal quantum number $n$ takes higher values.

1. Show that the ratio of velocity of an electron in the first Bohr orbit to the speed of light $c$ is a dimensionless number.
2. Compute the velocity of electrons in ground state, first excited state and second excited state in Bohr atom model.

The velocity of an electron in n<sup>th</sup> orbit is

$$\upsilon_n = \frac{h}{2\pi m a_0}\frac{Z}{n}$$

$$\upsilon_n \propto \frac{1}{n}$$

Note that the velocity of electron decreases as the principal quantum number increases as shown in Figure 8.19. This curve is the rectangular hyperbola. This implies that the velocity of electron in ground state is maximum when compared to excited states.

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$$= -\frac{1}{4\varepsilon_0^2}\frac{Z^2me^4}{h^2n^2} \quad \left(\because r_n = \frac{\varepsilon_0 h^2}{\pi me^2}\frac{n^2}{Z}\right)$$

The kinetic energy for the $n^{th}$ orbit is

$$KE_n = \frac{1}{2}m\upsilon_n^2 = \frac{me^4}{8\varepsilon_0^2h^2}\frac{Z^2}{n^2}$$

This implies that $U_n = -2\,KE_n$. Total energy in the nth orbit is

$$E_n = KE_n + U_n = KE_n - 2KE_n = -KE_n$$

$$E_n = -\frac{me^4}{8\varepsilon_0^2h^2}\frac{Z^2}{n^2}$$

For hydrogen atom (Z = 1),

$$E_n = -\frac{me^4}{8\varepsilon_0^2h^2}\frac{1}{n^2}\text{ joule}$$

where n stands for principal quantum number. The negative sign in equation (8.17) indicates that the electron is bound to the nucleus.

Substituting the values of mass and charge of an electron (*m* and *e*), permittivity of free space $\varepsilon_0$ and Planck's constant *h* and expressing in terms of *eV*, we get

$$E_n = -13.6\frac{1}{n^2}\,eV$$

For the first orbit (ground state), the total energy of electron is E1= – 13.6 *eV*.
For the second orbit (first excited state), the total energy of electron is E2= –3.4 *eV*.

For the third orbit (second excited state), the total energy of electron is E3= –1.51 $eV$ and so on.

Notice that the energy of the first excited state is greater than the ground state, second excited state is greater than the first excited state and so on. Thus, the orbit which is closest to the nucleus ($r1$) has lowest energy (minimum energy compared with other orbits). So, it is often called ground state energy (lowest energy state). The ground state energy of hydrogen (–13.6 $eV$ ) is used as a unit of energy called Rydberg (1 Rydberg = –13.6 $eV$ ).

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The energy level diagram along with the shape of the orbits for increasing values of $n$ are shown in Figure 8.20. It shows that the energies of the excited states come closer and closer together when the principal quantum number $n$ takes higher values.

1. Show that the ratio of velocity of an electron in the first Bohr orbit to the speed of light $c$ is a dimensionless number.
2. Compute the velocity of electrons in ground state, first excited state and second excited state in Bohr atom model.

**Solution**

The velocity of an electron in nth orbit is

$$\upsilon_n = \frac{h}{2\pi m a_0}\frac{Z}{n}$$

where $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$ Bohr radius. Substituting for $a_0$ in $v_n$

$$\upsilon_n = \frac{e^2}{2\varepsilon_0 h}\frac{Z}{n} = c\left(\frac{e^2}{2\varepsilon_0 hc}\right)\frac{Z}{n} = \frac{\alpha c Z}{n}$$

where c is the speed of light in free space or vacuum and its value is c=3× $10^8$ms$^1$and α is called fine structure constant.

For a hydrogen atom, $Z$ = 1 and for the first orbit, $n$ = 1, the ratio of velocity of electron in first orbit to the speed of light in vacuum or free space is

$$\alpha = \frac{(1.6\times10^{-19}\mathrm{C})^2}{2\times(8.854\times10^{-12}\mathrm{C^2N^{-1}m^{-2}})}$$

$$\frac{(1.6\times10^{-19}\mathrm{C})^2}{(6.6\times10^{-34}\mathrm{Nms})\times(3\times10^{8}\mathrm{ms^{-1}})}$$

$$\frac{\upsilon_1}{c} = \alpha = \frac{e^2}{2\varepsilon_0 hc}$$

$\approx \frac{1}{136.9} = \frac{1}{137}$ is a dimensionless number

$$\Rightarrow \alpha = \frac{1}{137}$$

Using fine structure constant, the velocity of electron can be written as

$$\upsilon_n = \frac{\alpha c Z}{n}$$

For hydrogen atom (Z = 1) the velocity of electron in $n$th orbit is

$$\upsilon_n = \frac{c}{137}\frac{1}{n} = (2.19\times10^{6})\frac{1}{n}\ \mathrm{ms^{-1}}$$

For the first orbit (ground state), the velocity of electron is

$$\upsilon_1 = 2.19\times10^{6}\mathrm{ms^{-1}}$$

For the second orbit (first excited state), the velocity of electron is

$$\upsilon_2 = 1.095\times10^{6}\mathrm{ms^{-1}}$$

For the third orbit (second excited state), the velocity of electron is

$$\upsilon_3 = 0.73 \times 10^6 \, \text{ms}^{-1}$$

$$\text{Here, } \upsilon_1 > \upsilon_2 > \upsilon_3$$

The Bohr atom model is derived with the assumption that the nucleus of the atom is stationary and only electrons revolve around the nucleus. Suppose the nucleus is also in motion, then calculate the energy of this new system.

**Solution**

Let the mass of the electron be *m* and mass of the nucleus be *M*. Since there is no external force acting on the system, the centre of mass of hydrogen atom remains at rest. Hence, both nucleus and electron move about the centre of mass as shown in figure.

Let *V* be the velocity of the nuclear motion and *υ* be the velocity of electron motion. Since the total linear momentum of the system is zero,

$$-m\upsilon + M\upsilon = 0 \text{ or}$$

$$MV = m\upsilon = p$$

$$\vec{p}_e + \vec{p}_n = \vec{0} \text{ or}$$

$$\left|\vec{p}_e\right| = \left|\vec{p}_n\right| = p$$

Hence, the kinetic energy of the system is

$$KE = \frac{p_n^2}{2M} + \frac{p_e^2}{2m} = \frac{p^2}{2}\left(\frac{1}{M} + \frac{1}{m}\right)$$

$$\text{Let } \frac{1}{M} + \frac{1}{m} = \frac{1}{\mu_m}\text{. Here the reduced mass}$$

$$\text{is, } \mu_m = \frac{mM}{M+m}$$

Therefore, the kinetic energy of the system

$$KE = \frac{p^2}{2\mu_m}$$

Since the potential energy of the system is same, the total energy of the hydrogen can be expressed by replacing mass by reduced mass, which is

$$E_n = -\frac{\mu_m e^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2}$$

Since the nucleus is very heavy compared to the electron, the reduced mass is closer to the mass of the electron.

**Excitation energy and excitation potential**

The energy required to excite an electron from lower energy state to any higher energy state is known as excitation energy.

The excitation energy for an electron from ground state (n = 1) to first excited state (n = 2) is called first excitation energy, which is

$$E_I = E_2 - E_1 = -3.4\ eV - (-13.6\ eV) = 10.2\ eV$$

Similarly, the excitation energy for an electron from ground state (n = 1) to second excited state (n = 3) is called second excitation energy, which is

$$E_{II} = E_3 - E_1 = -1.51\ eV - (-13.6\ eV) = 12.1\ eV$$

Excitation potential is defined as excitation energy per unit charge. First excitation potential is,

$$E_I = eV_I \Rightarrow V_I = \frac{1}{e}E_I = 10.2\ \text{volt}$$

Second excitation potential is,

$$E_{II} = eV_{II} \Rightarrow V_{II} = \frac{1}{e}E_{II} = 12.1\ \text{volt}$$

**Ionization energy and ionization potential**

An atom is said to be ionized when an electron is completely removed from the atom – that is, it reaches the state with energy En→∞. The minimum energy required to remove an electron from an atom in the ground state is known as binding energy or ionization energy.

$$E_{\text{ionizaation}} = E_{\infty} - E_{1} = 0 - (-13.6eV)$$
$$= 13.6eV$$

When an electron is in nth state of an atom, the energy spent to remove an electron from that state – that is, its ionization energy is

$$E_{\text{ionization}} = E_{\infty} - E_{n} = 0 - \left(-\frac{13.6}{n^2} Z^2\, eV\right)$$
$$= \frac{13.6}{n^2} Z^2\, eV$$

At normal room temperature, the electron in a hydrogen atom (Z=1) spends most of its time in the ground state. The amount of energy spent to remove an electron from the ground state of an atom (E = 0 for n→∞) is known as first ionization energy (13.6 eV). Then, the hydrogen atom is said to be in ionized state or simply called as hydrogen ion, denoted by *H*+. If we supply more energy than the ionization energy, the excess energy will be the kinetic energy of the free electron.

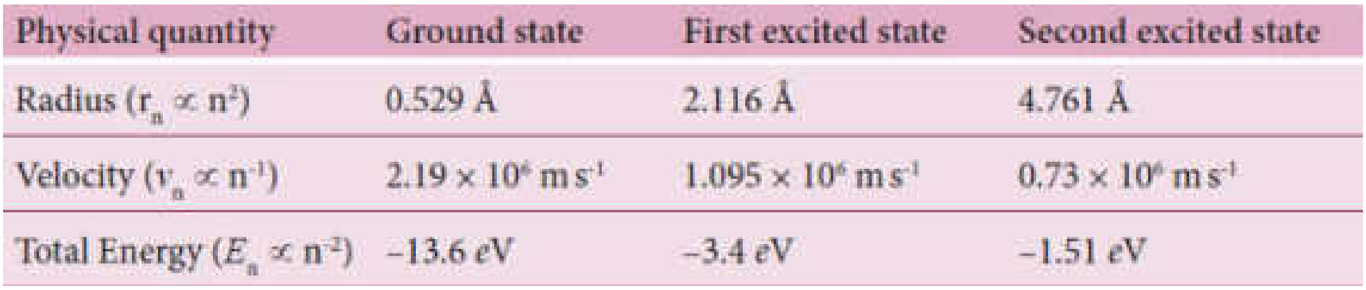
Ionization potential is defined as ionization energy per unit charge.

$$V_{\text{ionization}} = \frac{1}{e} E_{\text{ionization}} = \frac{13.6}{n^2} Z^2\, V$$

Thus, for a hydrogen atom (Z =1), the ionization potential is

$$V = \frac{13.6}{n^2} \text{ volt}$$

The radius, velocity and total energy in ground state, first excited state and second excited state is listed in Table 8.1.



| Physical quantity | Ground state | First excited state | Second excited state |
|---|---|---|---|
| Radius ($r_n \propto n^2$) | 0.529 Å | 2.116 Å | 4.761 Å |
| Velocity ($v_n \propto n^{-1}$) | $2.19 \times 10^6$ m s$^{-1}$ | $1.095 \times 10^6$ m s$^{-1}$ | $0.73 \times 10^6$ m s$^{-1}$ |
| Total Energy ($E_n \propto n^{-2}$) | –13.6 *e*V | –3.4 *e*V | –1.51 *e*V |

Suppose the energy of a hydrogen–like atom is given as $E_n = \frac{54.4}{n^2} ev$ where $n \in \mathbb{N}$ . Calculate the following:

1. Sketch the energy levels for this atom and compute its atomic number.
2. If the atom is in ground state, compute its first excitation potential and also its ionization potential.
3. When a photon with energy 42 *e*V and another photon with energy 56 *e*V are made to collide with this atom, does this atom absorb these photons?
4. Determine the radius of its first Bohr orbit.
5. Calculate the kinetic and potential energies in the ground state.

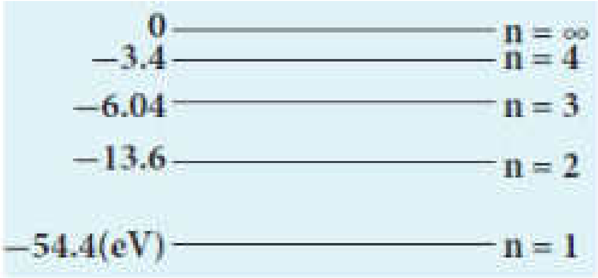
**Solutions**

1. Given that

$$E_n = -\frac{54.4}{n^2}\, eV$$

For n = 1, the ground state energy $E_1$ = –54.4 eV and for n = 2, $E_2$ = –13.6 eV. Similarly, $E_3$ = –6.04 eV, $E_4$ = –3.4 eV and so on.

For large value of principal quantum number – that is, $n = \infty$, we get $E\infty = 0$ *e*V.



2. For a hydrogen-like atom, ground state energy is

$$E_1 = -\frac{13.6}{n^2} Z^2 \, eV$$

where $Z$ is the atomic number. Hence, comparing this energy with given energy, we get, $-13.6 Z^2 = -54.4 \Rightarrow Z = \pm 2$. Since, atomic number cannot be negative number, $Z = 2$.

3. The first excitation energy is

$$\begin{aligned} E_1 &= E_2 - E_1 = -13.6 eV - (-54.4 eV) \\ &= 40.8 eV \end{aligned}$$

Hence, the first excitation potential is

$$\begin{aligned} V_1 &= \frac{1}{e} E_1 = \frac{(40.8 eV)}{e} \\ &= 40.8 \text{ volt} \end{aligned}$$

The first ionization energy is

$$\begin{aligned} E_{\text{ionization}} &= E_{\infty} - E_1 = 0 - (-54.4 eV) \\ &= 54.4 eV \end{aligned}$$

Hence, the first ionization potential is

$$V_{\text{ionization}} = \frac{1}{e} E_{\text{ionization}} = \frac{(54.4\,eV)}{e}$$
$$= 54.4\,\text{volt}$$

4. Consider two photons to be A and B.

Given that photon A with energy 42 *e*V and photon B with energy 51 *e*V

From Bohr assumption, difference in energy levels is equal to photon energy, then atom will absorb energy, otherwise, not.

$$E_2 - E_1 = -13.6\,eV - (-54.4\,eV)$$
$$= 40.8\,eV \approx 41\,eV$$

Similarly,

$$E_3 - E_1 = -6.04\,eV - (-54.4\,eV)$$
$$= 48.36\,eV$$

$$E_4 - E_1 = -3.4\,eV - (-54.4\,eV)$$
$$= 51\,eV$$

$$E_3 - E_2 = -6.04\,eV - (-13.6\,eV)$$
$$= 7.56\,eV$$

and so on.

But note that E2 – E1 ≠ 42 eV, E3 – E1 ≠ 42 eV, E4 – E1 ≠ 42 eV and E3 – E2 ≠ 42 eV.

For all possibilities, no difference in energy is an integer multiple of photon energy. Hence, photon A is not absorbed by this atom. But for Photon B, *E4 – E1* = 51 *e*V, which means, Photon B can be absorbed by this atom.

Since total energy is equal to negative of kinetic energy in Bohr atom model, we get

$$KE_n = -E_n = -\left(-\frac{54.4}{n^2}eV\right)$$
$$= \frac{54.4}{n^2}eV$$

Potential energy is negative of twice the kinetic energy, which means,

$$U_n = -2KE_n = -2\left(\frac{54.4}{n^2}eV\right)$$
$$= -\frac{108.8}{n^2}eV$$

For a ground state, put n =1
Kinetic energy is KE1 = 54.4 eV and Potential energy is $U_1$ = –108.8 eV

**Atomic spectra**

Materials in the solid, liquid and gaseous states emit electromagnetic radiations when they are heated up and these emitted radiations usually belong to continuous spectrum. For example, when white light is examined through a spectrometer, electromagnetic radiations of all wavelengths are observed which is a continuous spectrum.

In early twentieth century, many scientists spent considerable time in understanding the characteristic radiations emitted by the atoms of individual elements exposed to a flame or electrical discharge. When they are viewed or photographed, instead of a continuous spectrum, the radiation contains of a set of discrete lines, each with characteristic wavelength. In other words, the wavelengths of the light obtained are well defined and the positions and intensities are characteristic of the element as shown in Figure 8.21.

This implies that these spectra are unique to each element and can be used to identify the element of the gas (like finger print used to identify a person) – that is, it varies from one gas to another gas. This uniqueness of line spectra of elements made the scientists to determine the composition of stars, sun and also used to identify the unknown compounds.

**Hydrogen spectrum**

When the hydrogen gas enclosed in a tube is heated up, it emits electromagnetic radiations of certain sharply-defined characteristic wavelength (line spectrum), called hydrogen emission spectrum (Refer unit 5, volume 1 of +2 physics text book). The emission spectra of hydrogen are shown in Figure 8.22(a).

When any gas is heated up, the thermal energy is supplied to excite the electrons. Similarly by passing light on the atoms, electrons can be excited by absorbing photons. Once the electrons get sufficient energy as given by Bohr's postulate (c), it absorbs energy with particular wavelength (or frequency) and jumps from its stationary state (original state) to higher energy state. Those wavelengths (or frequencies) for which the colours are not observed are seen as dark lines in the absorption spectrum as shown in Figure 8.22 (b).

Since electrons in excited states have very small life time, these electrons jump back to ground state through spontaneous emission in a short duration of time (approximately 10–8 s) by emitting the radiation with same wavelength (or frequency) corresponding to the colours it absorbed (Figure 8.22 (a)). This is called emission spectroscopy.

The wavelengths of these lines can be calculated with great precision. Further, the emitted radiation contains wavelengths both lesser and greater than the visible spectrum.

Notice that the spectral lines of hydrogen as shown in Figure 8.23 are grouped in separate series. In each series, the distance of separation between the consecutive wavelengths decreases from higher wavelength to the lower wavelength, and also wavelength in each series approach a limiting value known as the series limit. These series are named as Lyman series, Balmer series, Paschen series, Brackett series, Pfund series, etc. The wavelengths of these spectral lines perfectly agree with the equation derived from Bohr atom model.

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = \bar{v}$$

where $\bar{v}$ is known as wave number which is inverse of wavelength, $R$ is known as Rydberg constant whose value is 1.09737 x $10^7$ $m^{-1}$and $m$ and $n$ are positive integers such that $m > n$. The various spectral series are discussed below:

**Lyman series**

Put $n$ = 1 and $m$ = 2,3,4....... in equation (8.18). The wave number or wavelength of spectral lines of Lyman series which lies in ultra-violet region is

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right)$$

**Balmer series**

Put n = 2 and m = 3,4,5....... in equation (8.18). The wave number or wavelength of spectral lines of Balmer series which lies in visible region is

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right)$$

**Paschen series**

Put n = 3 and m = 4,5,6....... in equation (8.18). The wave number or wavelength of spectral lines of Paschen series which lies in infra-red region (near IR) is

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{m^2}\right)$$

**Brackett series**

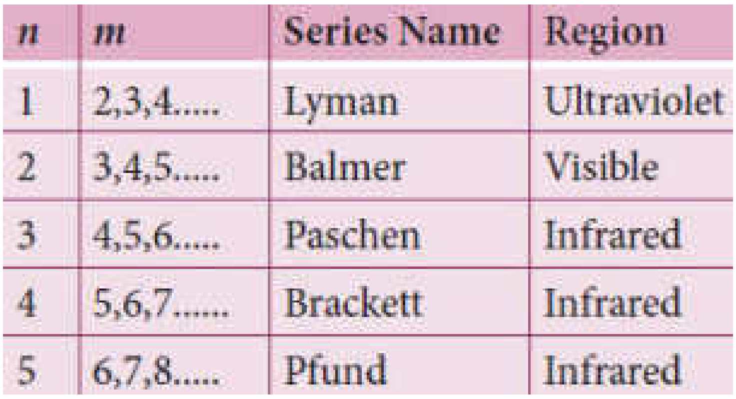
Put $n$ = 4 and $m$ = 5,6,7........ in equation (8.18). The wave number or wavelength of spectral lines of Brackett series which lies in infra-red region (middle IR) is

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{m^2}\right)$$

**Pfund series**

Put $n$ = 5 and $m$ = 6,7,8........ in equation (8.18). The wave number or wavelength of spectral lines of Pfund series which lies in infra-red region (far IR) is

$$\bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{m^2}\right)$$



| n | m | Series Name | Region |
|---|---|---|---|
| 1 | 2,3,4..... | Lyman | Ultraviolet |
| 2 | 3,4,5..... | Balmer | Visible |
| 3 | 4,5,6..... | Paschen | Infrared |
| 4 | 5,6,7...... | Brackett | Infrared |
| 5 | 6,7,8..... | Pfund | Infrared |

**Limitations of Bohr atom model**

The following are the drawbacks of Bohr atom model

- Bohr atom model is valid only for hydrogen atom or hydrogen like-atoms but not for complex atoms.
- When the spectral lines are closely examined, individual lines of hydrogen spectrum is accompanied by a number of faint lines. These are often called fine structure. This is not explained by Bohr atom model.
- Bohr atom model fails to explain the intensity variations in the spectral lines.
- The distribution of electrons in atoms is not completely explained by Bohr atom model.

## NUCLEI

### Introduction

In the previous section, we have discussed various preliminary atom models, Rutherford's alpha particle scattering experiment and Bohr atom model. These played a vital role to understand the structure of the atom and the nucleus. In this section, the structure and the properties of the nucleus, and its classifications are discussed.

### Composition of nucleus

Atoms have a nucleus surrounded by electrons. The nucleus contains protons and neutrons. The neutrons are electrically neutral ($q = 0$) and the protons have positive charge ($q = +e$) equal in magnitude of the charge of the electron ($q = -e$). The number of protons in the nucleus is called the atomic number and it is denoted by Z. The number of neutrons in the nucleus is called neutron number (N). The total number of neutrons and protons in the nucleus is called the mass number and it is denoted by A. Hence, $A = Z+N$.

The two constituents of nucleus namely neutrons and protons, are collectively called nucleons. The mass of a proton is $1.6726\times10^{-27}$ kg which is roughly 1836 times the mass of the electron. The mass of a neutron is slightly greater than the mass of the proton and it is equal to $1.6749\times10^{-27}$ kg.

To specify the nucleus of any element, we use the following general notation

$$^{A}_{Z}X$$

where X is the chemical symbol of the element, A is the mass number and Z is the atomic number. For example, the nitrogen nucleus is represented by $^{15}_{7}N$ . It implies that nitrogen contains 15 nucleons of which 7 are protons ($Z = 7$) and 8 are neutrons ($N = A - Z = 8$). Note that once the element is specified, the value of Z is known and subscript Z is sometimes omitted. For example, nitrogen nucleus is simply denoted as $^{15}N$ and we call it as 'nitrogen fifteen'.

Since the nucleus is made up of positively charged protons and electrically neutral neutrons, the overall charge of the nucleus is positive and it has the value of $+Ze$. But the atom is electrically neutral which implies that the number of electrons in the atom is equal to the number of protons in the nucleus.

**Isotopes, isobars, and isotones**

**Isotopes:**

In nature, there are atoms of a particular element whose nuclei have same number of protons but different number of neutrons. These kinds of atoms are called isotopes. In other words, isotopes are atoms of the same element having same atomic number Z, but different mass number A. For example, hydrogen has three isotopes and they are represented as $^{1}_{1}H$ (hydrogen), $^{2}_{1}H$ (deuterium),and $^{3}_{1}H$ (tritium). Note that all the three nuclei have one proton and, hydrogen has no neutron, deuterium has 1 neutron and tritium has 2 neutrons.

The number of isotopes for the particular element and their relative abundances (percentage) vary with each element. For example, carbon has four main isotopes: ${}^{11}_{6}C, {}^{12}_{6}C, {}^{13}_{6}C$ and ${}^{14}_{6}C$. But in nature, the percentage of ${}^{12}_{6}C$ is approximately 98.9%, that of ${}^{13}_{6}C$ is 1.1% and that of ${}^{14}_{6}C$ is 0.0001%. The other carbon isotope ${}^{11}_{6}C$ do not occur naturally and it can be produced only in nuclear reactions in the laboratory or by cosmic rays.

The chemical properties of any atom are determined only by electrons, the isotopes of any element have same electronic structure and same chemical properties. So the isotopes of the same element are placed in the same location in the periodic table.

**Isobars:**

Isobars are the atoms of different elements having the same mass number A, but different atomic number Z. In other words, isobars are the atoms of different chemical element which has same number of nucleon. For example ${}^{40}_{16}S, {}^{40}_{17}Cl, {}^{40}_{18}Ar, {}^{40}_{19}K$, and ${}^{40}_{20}Ca$ are isobars having same mass number 40 and different atomic number. Unlike isotopes, isobars are chemically different elements. They have different physical and chemical properties.

**Isotones:**

Isotones are the atoms of different elements having same number of neutrons. ${}^{12}_{5}B$ and ${}^{13}_{6}C$ are examples of isotones which 7 neutrons.

**Atomic and nuclear masses**

The mass of nuclei is very small when it expressed in SI units (about $10^{-25}$ kg or less). Therefore, it is more convenient to express it in terms of another unit namely, the atomic mass unit (u). One atomic mass unit (u) is defined as the 1/12th of the mass of the isotope of carbon ${}^{12}_{6}C$, the most abundant naturally occurring isotope of carbon.

$$1\,u = \frac{\text{mass of } {}^{12}_{6}C \text{ atom}}{12} = \frac{1.9926\times10^{-26}}{12}$$
$$= 1.660\times10^{-27}\,\text{kg}$$

In terms of this atomic mass unit, the mass of the neutron = 1.008665 $u$, the mass of the proton =1.007276 $u$, the mass of the hydrogen atom = 1.007825 $u$ and the mass of ${}^{12}_{6}C$ =12u. Note that usually mass specified is the mass of the atoms, not mass of the nucleus. To get the nuclear mass of particular nucleus, the mass of electrons has to be subtracted from the corresponding atomic mass. Experimentally the atomic mass is determined by the instrument called Bainbridge mass spectrometer. If we determine the atomic mass of the element without considering the effect of its isotopes, we get the mass averaged over different isotopes weighted by their abundances.

Calculate the average atomic mass of chlorine if no distinction is made between its different isotopes?

**Solution**

The element chlorine is a mixture of 75.77% of and 24.23% ${}^{35}_{17}Cl$. So the average atomic mass will be

$$\frac{75.77}{100}\times 34.96885u + \frac{24.23}{100}\times 36.96593u$$
$$= 35.453u$$

In fact, the chemist uses the average atomic mass or simply called chemical atomic weight (35.453 u for chlorine) of an element. So it must be remembered that the atomic mass which is mentioned in the periodic table is basically averaged atomic mass.

**Size and density of the nucleus**

The alpha particle scattering experiment and many other measurements using different methods have been carried out on the nuclei of various atoms. The nuclei are found to be approximately spherical in shape. It is experimentally found that radius of nuclei for $Z > 10$, satisfies the following empirical formula.

$$R = R_0 A^{\frac{1}{3}}$$

Here $A$ is the mass number of the nucleus and the constant R0 = 1.2 F, where 1 F = 1 × $10^{-15}$ m.The unit fermi (F) is named after Enrico Fermi.
Calculate the radius of ${}^{197}_{79}Au$ nucleus.

**Solution**

According to the equation (8.19),

$$R = 1.2 \times 10^{-15} \times (197)^{\frac{1}{3}} = 6.97 \times 10^{-15} m$$
$$\text{Or} \quad R = 6.97 \text{ F}$$

Calculate the density of the nucleus with mass number $A$.

From equation (8.19), the radius of the nuclei satisfy the equation $= R_0 A^{\frac{1}{3}}$. Then the volume of the nucleus

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

By ignoring the mass difference between the proton and neutron, the total mass of the nucleus having mass number $A$ is equal to $A.m$ where $m$ is mass of the proton and is equal to 1.6726 x $10^{-27}$ kg.

Nuclear density

$$\rho = \frac{\text{mass of the nuclei}}{\text{Volume ofthe nuclei}} = \frac{A.m}{\frac{4}{3}\pi R_0^3 A} = \frac{m}{\frac{4}{3}\pi R_0^3}$$

The above expression shows that the nuclear density is independent of the mass number $A$. In other words, all the nuclei (Z > 10) have the same density and it is an important characteristics of the nuclei.

We can calculate the numerical value of this density by substituting the corresponding values.

$$\rho = \frac{1.67\times10^{-27}}{\frac{4}{3}\pi\times(1.2\times10^{-15})^3} = 2.3\times10^{17}\,\text{kg m}^{-3}.$$

It implies that nucleons are extremely tightly packed in the nucleus and compare this density with the density of water which is $10^3$ kg $m^{-3}$.

A single teaspoon of nuclear matter would weigh about trillion tons.

**Mass defect and binding energy**

It is experimentally found out that the mass of any nucleus is always less than the sum of the mass of its individual constituents. For example, consider the carbon-12 nucleus which is made up of 6 protons and 6 neutrons. Mass of 6 neutrons = 6×1.00866 u=6.05196u Mass of 6 protons =6 ×1.00727u = 6.04362u Mass of 6 electrons =6 × 0.00055u = 0.0033u

The expected mass of carbon-12 nucleus =6.05196u + 6.04362u = 12.09558u

But using mass spectroscopy, the atomic mass of carbon-12 atom is found to be 12 u. So if we subtract the mass of 6 electrons (0.0033 u) from 12 u, we get the carbon-12 nuclear mass which is equal to 11.9967 u. Note that the experimental mass of carbon-12 nucleus is less than the total mass of its individual constituents by Δm = 0.09888u . This difference in mass Δm is called mass defect. In general, if M, $m_p$, and $m_n$ are mass of the nucleus $\left({}^{A}_{Z}X\right)$ the mass of a proton and the mass of a neutron respectively, then the mass defect is given by

$$\Delta m = \left(Zm_p + Nm_n\right) - M$$

Where has this mass disappeared? The answer was provided by Albert Einstein with the help of famous mass-energy relation($E = mc^2$). According to this relation, the mass can be converted into energy and energy can be converted into mass. In the case of the carbon-12 nucleus, when 6 protons and 6 neutrons combine to form carbon-12 nucleus, mass equal to mass defect disappears and the corresponding energy is released. This is called the binding energy of the nucleus

(BE) and is equal to $(\Delta m)c^2$. In fact, to separate the carbon-12 nucleus into individual constituents, we must supply the energy equal to binding energy of the nucleus.

We can write the equation (8.20) in terms of binding energy

$$BE = \left(Zm_p + Nm_n - M\right)c^2$$

It is always convenient to work with the mass of the atom than the mass of the nucleus. Hence by adding and subtracting the mass of the $Z$ electrons, we get

$$BE = \left(Zm_p + Zm_e + Nm_n - M - Zm_e\right)c^2$$

$$BE = \left[Z\left(m_p + m_e\right) + Nm_n - M - Zm_e\right]c^2$$

where $m_p + m_e = m_H$ (mass of hydrogen atom)

$$BE = \left[Zm_H + Nm_n - (M + Zm_e)\right]c^2$$

It is always convenient to work with the mass of the atom than the mass of the nucleus. Hence by adding and subtracting the mass of the Z electrons, we get

$$BE = \left(Zm_p + Zm_e + Nm_n - M - Zm_e\right)c^2$$

$$BE = \left[Z\left(m_p + m_e\right) + Nm_n - M - Zm_e\right]c^2$$

where $m_p + m_e = m_H$ (mass of hydrogen atom)

$$BE = \left[Zm_H + Nm_n - (M + Zm_e)\right]c^2$$

Here $M + Zm_e = M_A$ where $M_A$ is the mass of the atom of an element ${}^{A}_{Z}X$.

Finally, the binding energy in terms of the atomic masses is given by

$$BE=\left[Zm_H+Nm_n-M_A\right]c^2$$

**E X A M P L E**

Compute the binding energy of ${}^{2}_{4}He$ nucleus using the following data: Atomic mass of Helium atom, $M_A$ (He) = 4.00260u and that of hydrogen atom, $m_H$= 1.00785 u.

**Solution:**

Binding energy $BE=[Zm_H+Nm_n-M_A]c^2$

For helium nucleus, Z = 2, N = A–Z = 4–2 = 2

Mass defect

$$\Delta m=\left[(2\times1.00785u)+(2\times1.008665u)\right.$$
$$\left.-4.00260u\right]\ \Delta m=0.03038u$$
$$B.E=0.03038u\times c^2$$
$$B.E=0.03038\times931MeV=28\,MeV$$
$$\left[\because 1uc^2=931MeV\right]$$

The binding energy of the ${}^{2}_{4}He$ nucleus is 28 MeV.

**Binding energy curve**

In the previous section, the origin of the binding energy is discussed. Now we can find the average binding energy per nucleon $\overline{BE}$ . It is given by

$$\overline{BE}=\frac{\left[Zm_H+Nm_n-M_A\right]c^2}{A}$$

The average binding energy per nucleon is the energy required to separate single nucleon from the particular nucleus. $\overline{BE}$ is plotted against A of all known nuclei. It gives a curve

Important inferences from of the average binding energy curve:

The value of $\overline{BE}$ rises as the mass number increases until it reaches a maximum value of 8.8 MeV for A = 56 (iron) and then it slowly decreases

The average binding energy per nucleon is about 8.5 MeV for nuclei having mass number between A = 40 and 120. These elements are comparatively more stable and not radioactive.

For higher mass numbers, the curve reduces slowly and BE for uranium is about 7.6 MeV. They are unstable and radioactive.

From Figure 8.24, if two light nuclei with A<28 combine with a nucleus with A<56, the binding energy per nucleon is more for final nucleus than initial nuclei. Thus, if the lighter elements combine to produce a nucleus of medium value A, a large amount of energy will be released. This is the basis of nuclear fusion and is the principle of the hydrogen bomb.

If a nucleus of heavy element is split (fission) into two or more nuclei of medium value A, the energy released would again be large. The atom bomb is based on this principle and huge energy of atom bombs comes from this fission when it is uncontrolled. Fission is explained in the section 8.7

**EXAMPLE**

Compute the binding energy per nucleon of ${}^{2}_{4}He$ nucleus.

**Solution**

From example 8.9, we found that the BE of ${}^{2}_{4}He$ =28 Mev

Binding energy per nucleon = $\overline{B.E}$. = 28 MeV/4 = 7 MeV.

**NUCLEAR FORCE**

Nucleus contains protons and neutrons. From electrostatics, we learnt that like charges repel each other. In the nucleus, the protons are separated by a distance of about a few Fermi ($10^{-15}$m), they must exert on each other a very strong repulsive force. For example, the electrostatic repulsive force between two protons separated by a distance $10^{-15}$m

$$F = k \times \frac{q^2}{r^2} = 9 \times 10^9 \times \frac{\left(1.6 \times 10^{-19}\right)^2}{\left(10^{-15}\right)^2} \approx 230N$$

The acceleration experienced by a proton due to the force of 230 N is

$$a = \frac{F}{m} = \frac{230N}{1.67 \times 10^{-27} kg} \approx 1.4 \times 10^{29} m s^{-2} .$$

This is nearly $10^{28}$ times greater than the acceleration due to gravity. So if the protons in the nucleus experience only the electrostatic force, then the nucleus would fly apart in an instant. Then how protons are held together in nucleus?

From this observation, it was concluded that there must be a strong attractive force between protons to overcome the repulsive Coulomb's force. This attractive force which holds the nucleus together is called strong nuclear force. The properties of strong nuclear force were understood through various experiments carried out between 1930s and 1950s. A few properties of strong nuclear force are

1. The strong nuclear force is of very short range, acting only up to a distance of a few Fermi. But inside the nucleus, the repulsive Coulomb force or attractive gravitational forces between two protons are much weaker than the strong nuclear force between two protons. Similarly, the gravitational force between two neutrons is also much weaker than strong nuclear force between the neutrons. So nuclear force is the strongest force in nature.
2. The strong nuclear force is attractive and acts with an equal strength between proton-proton, proton-neutron, and neutron – neutron.
3. Strong nuclear force does not act on the electrons. So it does not alter the chemical properties of the atom.

## RADIOACTIVITY

In the binding energy curve, the stability of the nucleus that has Z > 82 starts to decrease and these nuclei are called unstable nuclei. Some of the unstable nuclei decay naturally by emitting some kind of particles to form a stable nucleus. The elements of atomic number Z > 82 and isotopes of lighter nuclei belong to naturally-occurring radioactive nuclei. Each of these radioactive nuclei decays to another nucleus by the emission of ${}^{2}_{4}He$ nucleus (α-decay) or electron or positron (β-decay) or gamma rays ( γ -decay).

The phenomenon of spontaneous emission of highly penetrating radiations such as α, β and γ rays by an element is called radioactivity and the substances which emit these radiations are called radioactive elements. These radioactive elements can be heavy elements (Z > 82), isotopes of lighter and heavy elements and these isotopes are called radioisotopes. For example, carbon isotope ${}^{14}_{6}C$ is radioactive but ${}^{12}_{6}C$ is not.

Radioisotopes have a variety of applications such as carbon dating, cancer treatment, etc. When radioactive nucleus undergoes decay, the mass of the system decreases – that is, the mass of the initial nucleus before decay is always greater than the sum of the mass of the final nucleus and that of the emitted particle. When this difference in mass $\Delta m < 0$ , it appears as the energy according to Einstein's relation $E = \Delta m\, c^2$.

The phenomenon of radioactivity was first discovered by Henri Becquerel in 1896. Later, Marie Curie and her husband Pierre Curie did a series of experiments in detail to understand the phenomenon of radioactivity. In India, Saha Institute of Nuclear Physics (SINP), Kolkata is the premier institute pursuing active research in nuclear physics.

### Alpha decay

When unstable nuclei decay by emitting an α-particle (${}^{2}_{4}He$ nucleus), it loses two protons and two neutrons. As a result, its atomic number Z decreases by 2, the mass number decreases by 4. We write the alpha decay process symbolically in the following way

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

Here X is called the parent nucleus and Y is called the daughter nucleus.

**Example:**

Decay of Uranium $^{238}_{92}U$ to thorium $^{234}_{90}Th$ with the emission of $^{2}_{4}He$ nucleus (α-particle)

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

As already mentioned, the total mass of the daughter nucleus and $^{2}_{4}He$ nucleus is always less than that of the parent nucleus. The difference in mass ($\Delta m = m_X - m_Y - m_\alpha$) is released as energy called disintegration energy Q and is given by

$$Q = (m_X - m_Y - m_\alpha)c^2$$

Note that for spontaneous decay (natural radioactivity) Q >0. In alpha decay process, the disintegration energy is certainly positive (Q > 0). In fact, the disintegration energy Q is also the net kinetic energy gained in the decay process or if the parent nucleus is at rest, Q is the total kinetic energy of daughter nucleus and the $^{2}_{4}He$ nucleus. Suppose Q < 0, then the decay process cannot occur spontaneously and energy must be supplied to induce the decay.

**EXAMPLE**

Calculate the disintegration energy when stationary $^{232}_{92}U$ nucleus decays to thorium $^{238}_{90}Th$ with the emission of α particle. The atomic masses are of $^{232}_{92}U$ = 232.037156u , $^{238}_{90}Th$ = 228.028741uand $^{2}_{4}He$ = 4.002603u

Calculate kinetic energies of $^{238}_{90}Th$ and α-particle and their ratio

**Solution**

The difference in masses $\Delta m = (m_U - m_{Th}\ m_\alpha)$ = (232.037156–228.028741 – 4.002603)u

The mass lost in this decay = 0.005812 u Since 1u = 931MeV, the energy Q released is

$$Q = (0.005812u)\times(931MeV/u)$$
$$= 5.41\,MeV$$

This disintegration energy Q appears as the kinetic energy of α particle and the daughter nucleus.

In any decay, the total linear momentum must be conserved.

Total linear momentum of the parent nucleus = total linear momentum of the daughter nucleus +α particle

Since before decay, the uranium nucleus is at rest, its momentum is zero.

By applying conservation of momentum, we get

$$0 = m_{Th}\bar{\upsilon}_{Th} + m_{\alpha}\bar{\upsilon}_{\alpha}$$
$$m_{\alpha}\bar{\upsilon}_{\alpha} = -m_{Th}\bar{\upsilon}_{Th}$$

It implies that the alpha particle and daughter nucleus move in opposite directions.

In magnitude $m_{\alpha}v_{\alpha} = m_{Th}v_{Th}$

The velocity of α particle

$$\upsilon_{\alpha} = \frac{m_{Th}}{m_{\alpha}}\upsilon_{Th}$$

Note that $\frac{m_{Th}}{m_{\alpha}} > 1, so\ v_{\alpha} > v_{Th}$. The ratio of the kinetic energy of α particle to the daughter nucleus

$$\frac{K.E_{\alpha}}{K.E_{Th}}=\frac{\frac{1}{2}m_{\alpha}\upsilon^2_{\alpha}}{\frac{1}{2}m_{Th}\upsilon^2_{Th}}$$

By substituting, the value of υα into the above equation, we get

$$\frac{K.E_{\alpha}}{K.E_{Th}}=\frac{m_{Th}}{m_{\alpha}}=\frac{228.02871}{4.002603}=57$$

The kinetic energy of α particle is 57 times greater than the kinetic energy of the daughter nucleus ($^{228}_{90}Th$).

The disintegration energy Q = total kinetic energy of products

$$K.E_{\alpha}+K.E_{Th}=5.41\,MeV$$

$$57K.E_{Th}+K.E_{Th}=5.41MeV$$

$$K.E_{Th}=\frac{5.41}{58}MeV=0.093MeV$$

$$K.E_{\alpha}=57K.E_{Th}=57\times0.093=5.301\,MeV$$

In fact, 98% of total kinetic energy is taken by the α particle.

**Beta decay**

In beta decay, a radioactive nucleus emits either electron or positron. If electron ($e^-$) is emitted, it is called $\beta^-$ decay and if positron ($e^+$) is emitted, it is called $\beta^+$ decay. The positron is an anti-particle of an electron whose mass is same as that of electron and charge is opposite to that of electron – that is, +e. Both positron and electron are referred to as beta particles.

**$\beta^-$ decay:**

In $\beta^-$ decay, the atomic number of the nucleus increases by one but mass number remains the same. This decay is represented by

$$ {}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + e^- + \bar{\nu} $$

It implies that the element X becomes Y by giving out an electron and antineutrino ($\bar{v}$). In other words, in each β- decay, one neutron in the nucleus of X is converted into a proton by emitting an electron ($e^-$) and antineutrino. It is given by

$$ n \rightarrow p + e^- + \bar{\nu} $$

Where p -proton, $\bar{v}$ -antineutrino.

Example: Carbon (${}_{6}^{14}C$) is converted into nitrogen (${}_{7}^{14}N$) through β- decay.

$$ {}_{6}^{14}C \rightarrow {}_{7}^{14}N + e^- + \bar{\nu} $$

**$\beta^+$decay:**

In $\beta^+$ decay, the atomic number is decreased by one and the mass number remains the same. This decay is represented by

$$ {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + e^+ + \nu $$

It implies that the element X becomes Y by giving out an positron and neutrino (ν). In other words, for each $\beta^+$ decay, a proton in the nucleus of X is converted into a neutron by emitting a positron ($e^+$) and a neutrino. It is given by

$$ p \rightarrow n + e^+ + \nu $$

However a single proton (not inside any nucleus) cannot have $\beta^+$ decay due to energy conservation, because neutron mass is larger than proton mass. But a single neutron (not inside any nucleus) can have $\beta^-$decay.

The smoke detector uses around 0.2 mg of man-made weak radioactive isotope called americium($^{241}_{95}Am$). This radioactive source is placed between two oppositely charged metal plates and α radiations from $^{241}_{95}Am$ continuously ionize the nitrogen, oxygen molecules in the air space between the plates. As a result, there will be a continuous flow of small steady current in the circuit. If smoke enters, the radiation is being absorbed by the smoke particles rather than air molecules. As a result, the ionization and along with it the current is reduced. This drop in current is detected by the circuit and alarm starts. The radiation dosage emitted by americium is very much less than safe level, so it can be considered harmless.

Example: Sodium $\left(^{22}_{11}Na\right)$ is converted into neon $\left(^{22}_{10}Ne\right)$ through$\beta^+$ decay.

$$^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + e^+ + \nu$$

It is important to note that the electron or positron which comes out from nuclei during beta decay never present inside the nuclei rather they are produced during the conversion of neutron into proton or proton into neutron inside the nucleus.

**Neutrino:**

Initially, it was thought that during beta decay, a neutron in the parent nucleus is converted to the daughter nuclei by emitting only electron as given by

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^-$$

But the kinetic energy of electron coming out of the nucleus did not match with the experimental results. In alpha decay, the alpha particle takes only certain allowed discrete energies whereas in beta decay, it was found that the beta particle (i.e, electron) have a continuous range of energies. But the conservation of energy and momentum gives specific single values for electron energy and the recoiling nucleus Y. It seems that the conservation of energy, momentum are violated and could not be explained why energy of beta particle have continuous range of values. So beta decay remained as a puzzle for several years.

After a detailed theoretical and experimental study, in 1931 W.Pauli proposed a third particle which must be present in beta decay to carry away missing energy and momentum. Fermi later named this particle the neutrino (little neutral one) since it has no charge, have very little mass. For many years, the neutrino (symbol v , Greek nu) was hypothetical and could not be verified experimentally. Finally, the neutrino was detected experimentally in 1956 by Fredrick Reines and Clyde Cowan. Later Reines received Nobel prize in physics in the year 1995 for his discovery.

The neutrino has the following properties

1. It has zero charge
2. It has an antiparticle called anti-neutrino.
3. Recent experiments showed that the neutrino has very tiny mass.
4. It interacts very weakly with the matter. Therefore, it is very difficult to detect. In fact, in every second, trillions of neutrinos coming from the sun are passing through our body without any interaction.

**Gamma decay**

In α and β decay, the daughter nucleus is in the excited state most of the time. The typical life time of excited state is approximately 10-11s. So this excited state nucleus immediately returns to the ground state or lower energy state by emitting highly energetic photons called γ rays. In fact, when the atom is in the excited state, it returns to the ground state by emitting photons of energy in the order of few eV. But when the excited state nucleus returns to its ground state, it emits a highly energetic photon (γ rays) of energy in the order of MeV. The gamma decay is given by

$$ {}_{Z}^{A}X^{*} \rightarrow {}_{Z}^{A}X + \text{gamma}\left(\gamma\right)\text{rays} $$

Here the asterisk(*) means excited state nucleus. In gamma decay, there is no change in the mass number or atomic number of the nucleus.

Boron (${}_{5}^{12}B$) has two beta decay modes

1. it undergoes beta decay directly into ground state carbon (${}_{6}^{12}C$ ) by emitting an electron of maximum of energy 13.4 MeV.
2. it undergoes beta decay to an excited state of carbon (${}_{6}^{12}C^{*}$) by emitting an electron of maximum energy 9.0 MeV followed by gamma decay to ground state by emitting a photon of energy 4.4 MeV.

It is represented by

$$^{12}_{5}B \rightarrow {}^{12}_{6}C + e^{-} + \overline{\nu}$$

$$^{12}_{6}C^{*} \rightarrow {}^{12}_{6}C + \gamma$$

**Law of radioactive decay**

In the previous section, the decay process of a single radioactive nucleus was discussed. In practice, we have bulk material of radioactive sample which contains a vast number of the radioactive nuclei and not all the radioactive nucleus in a sample decay at the same time. It decays over a period of time and this decay is basically a random process. It implies that we cannot predict which nucleus is going to decay or rather we can determine like probabilistic basis (like tossing a coin). We can calculate approximately how many nuclei in a sample are decayed over a period of time.

At any instant t, the number of decays per unit time, called rate of decay $\frac{dN}{dt}$ is proportional to the number of nuclei ( N ) at the same instant.

$$\frac{dN}{dt} \propto N$$

By introducing a proportionality constant, the relation can be written as

$$\frac{dN}{dt} = -\lambda N$$

Here proportionality constant λ is called decay constant which is different for different radioactive sample and the negative sign in the equation implies that the N is decreasing with time.

By rewriting the equation

$$dN = -\lambda N dt$$

Here dN represents the number of nuclei decaying in the time interval dt.

Let us assume that at time t = 0 s, the number of nuclei present in the radioactive sample is $N_0$ . By integrating the equation (8.33), we can calculate the number of undecayed nuclei N at any time t.

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_{0}^{t} \lambda dt$$

$$\left[\ln N\right]_{N_0}^{N} = -\lambda t$$

$$\ln\left[\frac{N}{N_0}\right] = -\lambda t$$

Taking exponentials on both sides, we get

$$N = N_0 e^{-\lambda t}$$

[Note: $e^{\ln x} = e^{y} \Rightarrow x = e^{y}$]

Equation (8.35) is called the law of radioactive decay. Here N denotes the number of undecayed nuclei present at any time t and $N_0$ denotes the number of nuclei at initial time t=0. Note that the number of atoms is decreasing exponentially over the time. This implies that the time taken for all the radioactive nuclei to decay will be infinite.

We can also define another useful quantity called activity (R) or decay rate which is the number of nuclei decayed per second and it is denoted as $R = \left|\frac{dN}{dt}\right|$. Note that activity R is a positive quantity.

$$R = \left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$$

$$R = R_0 e^{-\lambda t}$$

$$\text{where } R_0 = \lambda N_0$$

The equation (8.37) is also equivalent to radioactive law of decay. Here $R_0$ is the activity of the sample at t=0 and R is the activity of the sample at any time t. From equation (8.37), activity also shows exponential decay behavior. The activity R also can be expressed in terms of number of undecayed atoms present at any time t.

From equation (8.37), since $N = N_0 e^{-\lambda t}$

$$R = \lambda N$$

Equation (8.35) implies that the activity at any time t is equal to the product of decay constant and number of undecayed nuclei at the same time t. Since N decreases over time, R also decreases

The SI unit of activity R is Becquerel and one Becquerel (Bq) is equal to one decay per second. There is also another standard unit for the activity called Curie(Ci).

$$1 \text{ Curie} = 1 \text{ Ci} = 3.7\times10^{10} \text{ decays per second}$$

$$1 \text{ Ci} = 3.7\times10^{10} \text{ Bq}$$

**Half-life**

It is difficult to calculate the time taken by a given a sample of N atoms to decay. However, we can calculate the time taken by the given sample of atoms to reduce some fraction of the initial amount.

We can define the half-life $T_{1/2}$ as the time required for the number of atoms initially present to reduce to one half of the initial amount.

The half-life is the important characteristic of every radioactive sample. Some radioactive nuclei are known to have half-life as long as 1014 years and some nucleus have very shorter life time ($10^{-14}$s).

We can express half-life in terms of the decay constant. At t = $T_{1/2}$ , the number of undecayed nuclei $N = \frac{N_0}{2}$

By substituting this value in to the equation (8.35), we get

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}} \text{ or } e^{\lambda T_{1/2}} = 2$$

Taking logarithm on both sides and rearranging the terms,

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{\lambda}$$

If the number of atoms present at t=0 is $N_0$, then $\frac{N_0}{2}$ atoms remain undecayed in first half-life and $\frac{N_0}{4}$ atoms remain undecayed after second half life and so on. In general, aft er n half-lives, the number of nuclei remaining undecayed is given by

$$N = \left(\frac{1}{2}\right)^n N_0$$

where n can be integer or non-integer. Since the activity of radioactive sample also obeys the exponential decay law, we can also write an equation for an activity similar to equation

After n half-lives, the activity or decay rate of any radioactive sample is

$$R = \left(\frac{1}{2}\right)^n R_0$$

**Mean life (τ):**

When the radioactive nucleus undergo the decay, the nucleus which disintegrates first has zero life time and the nucleus which decay last has an infinite lifetime. The actual life time for each nucleus varies from zero to infinity. Therefore, it is meaningful to define average life or mean life time t, that the nucleus survives before it decays.

The mean life time of the nucleus is the ratio of sum or integration of life times of all nuclei to the total number nuclei present initially.

The total number of nuclei decaying in the time interval from t to t +Δt is equal to $R\Delta t = \lambda N_0 e^{-\lambda t}\Delta t$. It implies that until the time t, this RΔt number of nuclei lived. So the life time of these RΔt nuclei is equal to be tRΔt. In the limit Δt®0 , the total life time of all the nuclei would be the integration of tRdt from the limit t =0 to t = ∞ .

$$\tau = \frac{\int_0^\infty t[Rdt]}{N_0} = \frac{\int_0^\infty t\left[\lambda N_0 e^{-\lambda t}\, dt\right]}{N_0}$$

After a few integration (refer box item), the expression for mean life time,

$$\tau = \frac{1}{\lambda}$$

Note that mean life and decay constant is inversely proportional to each other. Using mean life, the half-life can be rewritten as

$$T_{1/2} = \tau \ln 2 = 0.6931\tau$$

**Mean life : Not for examination**

The integration in the equation (8.42) can be performed using integration by parts.

$$\tau=\frac{\int_0^\infty \lambda N_0 t e^{-\lambda t}dt}{N_0}=\frac{\lambda N_0\int_0^\infty t e^{-\lambda t}dt}{N_0}$$

$$\tau=\lambda\int_0^\infty t e^{-\lambda t}dt$$

$$u=t \qquad dv=e^{-\lambda t}\,dt$$

$$\tau=\lambda\int_0^\infty t e^{-\lambda t}dt=\lambda\left[\frac{t e^{-\lambda t}}{-\lambda}\right]_0^\infty-\lambda\int_0^\infty\left[\frac{e^{-\lambda t}}{-\lambda}\right]dt$$

By substituting the limits, the first term in the above equation becomes zero.

$$\tau=\int_0^\infty e^{-\lambda t}\,dt=-\frac{1}{\lambda}\left[e^{-\lambda t}\right]_0^\infty=\frac{1}{\lambda}$$

**EXAMPLE**

Calculate the number of nuclei of carbon-14 undecayed after 22,920 years if the initial number of carbon-14 atoms is 10,000. The half-life of carbon-14 is 5730 years.

**Solution**

To get the time interval in terms of halflife,

$$n=\frac{t}{T_{1/2}}=\frac{22,920\,yr}{5730\,yr}=4$$

The number of nuclei remaining undecayed after 22,920 years,

$$N = \left(\frac{1}{2}\right)^n N_0 = \left(\frac{1}{2}\right)^4 \times 10,000$$

$$N = 625$$

**E X A M P L E**

A radioactive sample has 2.6μg of pure ${}^{13}_{7}N$ which has a half-life of 10 minutes.
(a) How many nuclei are present initially?
(b) What is the activity initially?
(c) What is the activity after 2 hours?
(d) Calculate mean life of this sample.

**Solution**

a. To find $N_0$, we have to find the number of ${}^{13}_{7}N$ atoms in 2.6μg . The atomic mass of nitrogen is 13. Therefore, 13 g of ${}^{13}_{7}N$ contains Avogadro number ($6.02 \times 10^{23}$) of atoms.

   In 1 g, the number of ${}^{13}_{7}N$ is equal to be $\frac{6.02\times10^{23}}{13}$ atoms . So the number of ${}^{13}_{7}N$ atoms in 2.6μg is $N_0 = \frac{6.02\times10^{23}}{13} \times 2.6\times10^{-6} = 12.04\times10^{16}$ atoms

b. To find the initial activity R0 , we have to evaluate decay constant λ

$$\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{10\times60} = 1.155\times10^{-3}\,\mathrm{s}^{-1}$$

   Therefore

$$R_0 = \lambda N_0 = 1.155\times10^{-3}\times12.04\times10^{16}$$
$$= 13.90\times10^{13}\,\text{decays/s}$$
$$= 13.90\times10^{13}\,\text{Bq}$$

   In terms of a curie,

$$R_0 = \frac{13.90\times10^{13}}{3.7\times10^{10}} = 3.75\times10^{3}\ \text{Ci}$$

$$\text{since } 1\text{Ci} = 3.7\times10^{10}\text{Bq}$$

c. Activity after 2 hours can be calculated in two different ways:

Method 1: $R = R_0e^{-\lambda t}$

At $t = 2\,\text{hr} = 7200\ \text{s}$

$$R = 3.75\times10^{3}\times e^{-7200\times1.155\times10^{-3}}$$

$$R = 3.75\times10^{3}\times2.4\times10^{-4} = 0.9\ \text{Ci}$$

Method 2: $R = \left(\frac{1}{2}\right)^{n} R_0$

Here $n = \frac{120\,\text{min}}{10\,\text{min}} = 12$

$$R = \left(\frac{1}{2}\right)^{12}\times3.75\times10^{3} \approx 0.9\,\text{Ci}$$

d. mean life

$$\tau = \frac{T_{1/2}}{0.6931} = \frac{10\times60}{0.6931}$$
$$= 865.67\,s$$

**Carbon dating**

The interesting application of beta decay is radioactive dating or carbon dating. Using this technique, the age of an ancient object can be calculated. All living organisms absorb carbon dioxide ($CO_2$) from air to synthesize organic molecules. In this absorbed $CO_2$, the major part is ${}^{12}_{6}C$ and very small fraction ($1.3\times10^{-12}$) is radioactive ${}^{14}_{6}C$ whose half-life is 5730 years.

Carbon-14 in the atmosphere is always decaying but at the same time, cosmic rays from outer space are continuously bombarding the atoms in the atmosphere which produces ${}^{14}_{6}C$. So the continuous production and decay of ${}^{14}_{6}C$ in the atmosphere keep the ratio of ${}^{14}_{6}C$ to ${}^{12}_{6}C$ always constant. Since our human body, tree or any living organism continuously absorb $CO_2$ from the atmosphere, the ratio of ${}^{14}_{6}C$ to ${}^{12}_{6}C$ in the living organism is also nearly constant. But when the organism dies, it stops absorbing $CO_2$. Since ${}^{14}_{6}C$ starts to decay, the ratio of ${}^{14}_{6}C$ to ${}^{12}_{6}C$ in a dead organism or specimen decreases over the years. Suppose the ratio of ${}^{14}_{6}C$ to ${}^{12}_{6}C$ in the ancient tree pieces excavated is known, then the age of the tree pieces can be calculated.

**EXAMPLE**

Keezhadi , a small hamlet, has become one of the very important archeological places of Tamilandu. It is located in Sivagangai district. A lot of artefacts (gold coins, pottery, beads, iron tools, jewellery and charcoal, etc.) have been unearthed in Keezhadi which have given substantial evidence that an ancient urban civilization had thrived on the banks of river Vaigai. To determine the age of those materials, the charcoal of 200 g sent for carbon dating is given in the following figure (b). The activity of 6 14C is found to be 38 decays/s. Calculate the age of charcoal.

**Solution**

To calculate the age, we need to know the initial activity ($R_0$) of the characol (when the sample was alive).

The activity R of the sample

$$R=R_0e^{-\lambda t}$$

To find the time t, rewriting the above equation (1),

$$e^{\lambda t}=\frac{R_0}{R}$$

By taking the logarithm on both sides, we get

$$t=\frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right)$$

Here R = 38 decays/s=38 Bq.

To find decay constant, we use the equation

$$\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{5730\,\text{yr} \times 3.156 \times 10^{7}\,\text{s/yr}}$$

$$[\because\ 1\text{yr} = 365.25 \times 24 \times 60 \times 60\,\text{s} = 3.156 \times 10^{7}\text{s}]$$

$$\lambda = 3.83 \times 10^{-12} s^{-1}$$

To find the initial activity R0 , we use the equation $R_0 = \lambda N_0$. Here $N_0$ is the number of carbon-14 atoms present in the sample when it was alive. The mass of the characol is 200 g. In 12 g of carbon, there are $6.02 \times 10^{23}$ carbon atoms. So 200 g contains,

$$\frac{6.02 \times 10^{23}\,\text{atoms}/mol}{12g/mol} \times 200 \approx 1 \times 10^{25}$$

atoms When the tree(sample) was alive, the ratio of ${}^{14}_{6}C$ to ${}^{12}_{6}C$ is $1.3 \times 10^{-12}$. So the total number of carbon-14 atoms is given by $N_0$=1×1025×1.3×10−12 =1.3×1013 atoms

The initial activity

$$R_0 = 3.83 \times 10^{-12} \times 1.3 \times 10^{13} \approx 50\,\text{decays}/s$$

$$= 50\,Bq$$

By substituting the value of $R_0$ and $\lambda$ in the equation (2), we get

$$t = \frac{1}{3.83 \times 10^{-12}} \times \ln\left[\frac{50}{38}\right]$$

$$t = \frac{0.27}{3.83} \times 10^{12} \approx 7 \times 10^{10}\ \text{sec}$$

In years

$$t = \frac{7 \times 10^{10} s}{3.156 \times 10^{7} s/yr} \approx 2200\,\text{years}$$

In fact, the excavated materials were sent for carbon dating to USA by Archeological Department of Tamilnadu and the report confirmed that the age of Keezhadi artefacts lies between 2200 years to 2500 years (Sangam era- 400 BC to 200 BC). The Keezhadi excavations experimentally proved that urban civilization existed in Tamil Nadu even 2000 years ago!

**Discovery of Neutrons**

In 1930, two German physicists Bothe and Becker found that when beryllium was bombarded with α particles, highly penetrating radiation was emitted. This radiation was capable of penetrating the thick layer of lead and was unaffected by the electric and magnetic fields. Initially, it was thought as γ radiation. But in the year 1932, James Chadwick discovered that those radiations are not EM waves but they are particles of mass little greater than the mass of the proton and had no charge. He called them as neutrons. The above reaction can be written as

$$ {}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + {}^{1}_{0}n $$

where ${}^{1}_{0}n$ denotes neutron.

Neutrons are stable inside the nucleus. But outside the nucleus they are unstable. If the neutron comes out of the nucleus (free neutron), it decays with emission of proton, electron, and antineutrino with the half life of 13 minutes.

Neutrons are classified according to their kinetic energy as (i) slow neutrons (0 to 1000 eV) (ii) fast neutrons (0.5 MeV to 10 MeV). The neutrons with average energy of about 0.025 eV in thermal equilibrium are called thermal neutron, because at 298K, the thermal energy $kT \quad 0.025\text{eV}$ . Slow and fast neutrons play a vital role in nuclear reactors.

**NUCLEAR FISSION**

In 1939, German scientists Otto Hahn and F. Strassman discovered that when uranium nucleus is bombarded with a neutron, it breaks up into two smaller nuclei of comparable masses with the release of energy. The process of breaking up of the nucleus of a heavier atom into two smaller nuclei with the release of a large amount of energy is called nuclear fission. The fission is accompanied by the release of neutrons. The energy that is released in the nuclear fission is of many orders of magnitude greater than the energy released in chemical reactions.

Uranium undergoes fission reaction in 90 different ways. The most common fission reactions of ${}^{235}_{92}U$ nuclei are shown here.

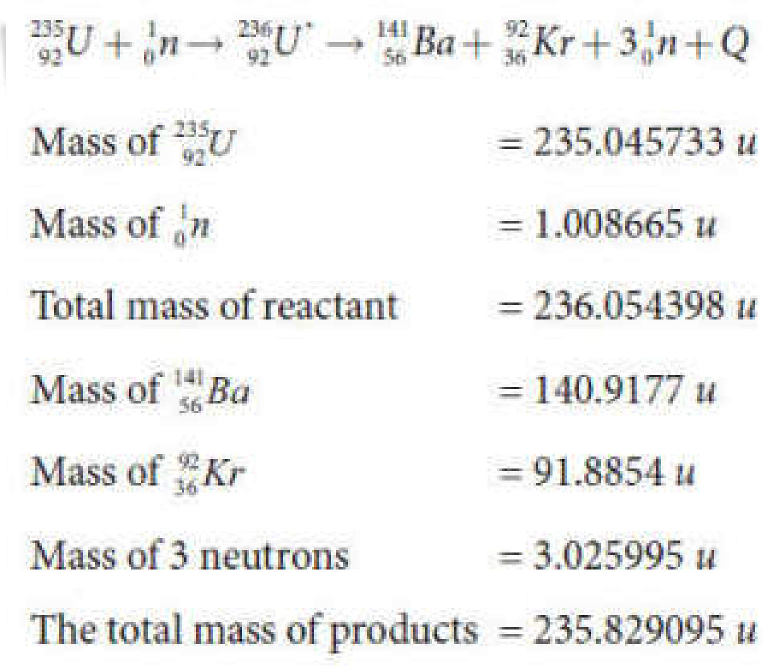
$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U^{*} \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q $$

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U^{*} \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n + Q $$

Here Q is energy released during the decay of each uranium nuclei. When the slow neutron is absorbed by the uranium nuclei, the mass number increases by one and goes to an excited state ${}^{236}_{92}U$ . But this excited state does not last longer than $10^{-12}$s and decay into two daughter nuclei along with 2 or 3 neutrons. From each reaction, on an average, 2.5 neutrons are emitted.

**Energy released in fission:**

We can calculate the energy (Q) released in each uranium fission reaction. We choose the most favorable fission which is given in the equation



$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U^{*} \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q $$

Mass of ${}^{235}_{92}U$ = 235.045733 $u$

Mass of ${}^{1}_{0}n$ = 1.008665 $u$

Total mass of reactant = 236.054398 $u$

Mass of ${}^{141}_{56}Ba$ = 140.9177 $u$

Mass of ${}^{92}_{36}Kr$ = 91.8854 $u$

Mass of 3 neutrons = 3.025995 $u$

The total mass of products = 235.829095 $u$

Mass defect $\Delta m$ = 236.054398 u – 235.829095 u = 0.225303 u

So the energy released in each fission = 0.225303× 931MeV ≈ 200MeV

This energy first appears as kinetic energy of daughter nuclei and neutrons. But later, this kinetic energy is transferred to the surrounding matter as heat.

**Chain reaction:**

When one $^{235}_{92}U$ nucleus undergoes fission, the energy released might be small. But from each fission reaction, three neutrons are released. These three neutrons cause further fission in another three $^{235}_{92}U$ nuclei which in turn produce nine neutrons. These nine neutrons initiate fission in another 27 $^{235}_{92}U$ nuclei and so on. This is called a chain reaction and the number of neutrons goes on increasing almost in geometric progression.

There are two kinds of chain reactions: (i) uncontrolled chain reaction (ii) controlled chain reaction. In an uncontrolled chain reaction, the number of neutrons multiply indefinitely and the entire amount of energy released in a fraction of second.

The atom bomb is an example of nuclear fission in which uncontrolled chain reaction occurs. Atom bombs produce massive destruction for mankind. During World War II, in the year 1946 August 6 and 9, USA dropped two atom bombs in two places of Japan, Hiroshima and Nagasaki. As a result, lakhs of people were killed and the two cities were completely destroyed. Even now the people who are living in those places have side effects caused by the explosion of atom bombs.

It is possible to calculate the typical energy released in a chain reaction. In the first step, one neutron initiates the fission of one nucleus by producing three neutron and energy of about 200 MeV. In the second step, three nuclei undergo fission, in third step nine nuclei undergo fission, in fourth step 27 nucleus undergo fission and so on. In the 100th step, the number of nuclei which undergoes fission is around $2.5\times10^{40}$. The total energy released after 100th step is $2.5\times10^{40}\times200\text{MeV}=8\times10^{29}\text{J}$. It is really an enormous amount of energy which is equivalent to electrical energy required in Tamilnadu for several years.

If the chain reaction is controllable, then we can harvest an enormous amount of energy for our needs. It is achieved in a controlled chain reaction. In the controlled chain reaction, the average number of neutron released in each stage is kept as one such that it is possible to store the released energy. In nuclear reactors, the controlled chain reaction is achieved and the produced energy is used for power generation or for research purpose.

## EXAMPLE

Calculate the amount of energy released when 1 kg of ${}^{235}_{92}U$ undergoes fission reaction.

**Solution**

235 g of ${}^{235}_{92}U$ has $6.02\times10^{23}$ atoms. In one gram of ${}^{235}_{92}U$ the number of atoms is equal to $\frac{6.02\times10^{23}}{235}=2.56\times10^{21}$.

So the number of atoms in 1 kg of ${}^{235}_{92}U$ $=2.56\times10^{21}\times1000=2.56\times10^{24}$

Each ${}^{235}_{92}U$ nucleus releases 200 MeV of energy during the fission. The total energy released by 1kg of ${}^{235}_{92}U$ is

$$Q=2.56\times10^{24}\times200MeV=5.12\times10^{26}\ MeV$$

By converting in terms of joules,

$$Q=5.12\times10^{26}\times1.6\times10^{-13}J=8.192\times10^{13}J\ .$$

In terms of Kilowatt hour,

$$Q=\frac{8.192\times10^{13}}{3.6\times10^{6}}=2.27\times10^{7}\ kWh$$

This is enormously large energy which is enough to keep 100 W light bulb operating for 30,000 years. To produce this much energy through chemical reaction, around 20,000 tons of TNT(tri nitro toluene) has to be exploded.

**Nuclear reactor:**

Nuclear reactor is a system in which the nuclear fission takes place in a self-sustained controlled manner and the energy produced is used either for research purpose or for power generation. The first nuclear reactor was built in the year

1942 at Chicago, USA by physicist Enrico Fermi. The main parts of a nuclear reactor are fuel, moderator and control rods. In addition to this, there is a cooling system which is connected with power generation set up.

**Fuel:**

The fuel is fissionable material, usually uranium or plutonium. Naturally occurring uranium contains only 0.7% of ${}^{235}_{92}U$ and 99.3% are only ${}^{238}_{92}U$. So the ${}^{238}_{92}U$ must be enriched such that it contains at least 2 to 4% of . In addition to this, a neutron source is required to initiate the chain reaction for the first time. A mixture of beryllium with plutonium or polonium is used as the neutron source. During fission of ${}^{235}_{92}U$ only fast neutrons are emitted but the probability of initiating fission by it in another nucleus is very low. Therefore, slow neutrons are preferred for sustained nuclear reactions.

**Moderators:**

The moderator is a material used to convert fast neutrons into slow neutrons. Usually the moderators are chosen in such a way that it must be very light nucleus having mass comparable to that of neutrons. Hence, these light nuclei undergo collision with fast neutrons and the speed of the neutron is reduced (Note that a billiard ball striking a stationary billiard ball of equal mass would itself be stopped but the same billiard ball bounces off almost with same speed when it strikes a heavier mass. This is the reason for using lighter nuclei as moderators). Most of the reactors use water, heavy water ($D_2O$) and graphite as moderators. The blocks of uranium stacked together with blocks of graphite (the moderator) to form a large pile is shown in the Figure 8.29 (a) & (b).

**Control rods:**

The control rods are used to adjust the reaction rate. During each fission, on an average 2.5 neutrons are emitted and in order to have the controlled chain reactions, only one neutron is allowed to cause another fission and the remaining neutrons are absorbed by the control rods.

Usually cadmium or boron acts as control rod material and these rods are inserted into the uranium blocks as shown in the Figure 8.29 (a) and (b). Depending on the insertion depth of control rod into the uranium, the average number of neutrons produced per fission is set to be equal to one or greater than one. If the average number of neutrons produced per fission is equal to one, then reactor is said to be in critical state. In fact, all the nuclear reactors are maintained in critical state by suitable adjustment of control rods. If it is greater than one, then reactor is said to be in super-critical and it may explode sooner or may cause massive destruction.

**Shielding:**

For a protection against harmful radiations, the nuclear reactor is surrounded by a concrete wall of thickness of about 2 to 2.5 m.

**Cooling system:**

The cooling system removes the heat generated in the reactor core. Ordinary water, heavy water and liquid sodium are used as coolant since they have very high specific heat capacity and have large boiling point under high pressure. This coolant passes through the fuel block and carries away the heat to the steam generator through heat exchanger as shown in Figure 8.29(a) and (b). The steam runs the turbines which produces electricity in power reactors.

## NUCLEAR FUSION

When two or more light nuclei (A<20) combine to form a heavier nucleus, then it is called nuclear fusion. In the nuclear fusion, the mass of the resultant nucleus is less than the sum of the masses of original light nuclei. The mass difference appears as energy. The nuclear fusion never occurs at room temperature unlike nuclear fission. It is because when two light nuclei come closer to combine, it is strongly repelled by the coulomb repulsive force.

To overcome this repulsion, the two light nuclei must have enough kinetic energy to move closer to each other such that the nuclear force becomes effective. This can be achieved if the temperature is very much greater than the value 107 K. When the surrounding temperature reaches around 107K, lighter nuclei start fusing to form heavier nuclei and this resulting reaction is called thermonuclear fusion reaction.

**Energy generation in stars:**

The natural place where nuclear fusion occurs is the core of the stars, since its temperature is of the order of $10^7$K. In fact, the energy generation in every star is only through thermonuclear fusion. Most of the stars including our Sun fuse hydrogen into helium and some stars even fuse helium into heavier elements.

The early stage of a star is in the form of cloud and dust. Due to their own gravitational pull, these clouds fall inward. As a result, its gravitational potential energy is converted to kinetic energy and finally into heat. When the temperature is high enough to initiate the thermonuclear fusion, they start to release enormous energy which tends to stabilize the star and prevents it from further collapse.

The sun's interior temperature is around $1.5\times10^7$ K. The sun is converting $6\times10^{11}$kg hydrogen into helium every second and it has enough hydrogen such that these fusion lasts for another 5 billion years. When the hydrogen is burnt out, the sun will enter into new phase called red giant where helium will fuse to become carbon. During this stage, sun will expand greatly in size and all its planets will be engulfed in it.

According to Hans Bethe, the sun is powered by proton-proton cycle of fusion reaction. This cycle consists of three steps and the first two steps are as follows:

$$^1_1H + ^1_1H \rightarrow ^2_1H + e^+ + \nu$$

$$^1_1H + ^2_1H \rightarrow ^3_2He + \gamma$$

A number of reactions are possible in the third step. But the dominant one is

$$^3_2He + ^3_2He \rightarrow ^4_2He + ^1_1H + ^1_1H$$

The overall energy production in the above reactions is about 27 MeV. The radiation energy we received from the sun is due to these fusion reactions.

**Elementary particles:**

An atom has a nucleus surrounded by electrons and nuclei is made up of protons and neutrons. Till 1960s, it was thought that protons, neutrons and electrons are fundamental building blocks of matter. In 1964, physicist Murray Gellman and George Zweig theoretically proposed that protons and neutrons are not fundamental particles; in fact they are made up of quarks. These quarks are now considered elementary particles of nature. Electrons are fundamental or elementary particles because they are not made up of anything. In the year 1968, the quarks were discovered experimentally by Stanford Linear Accelerator Center (SLAC), USA. There are six quarks namely, up, down, charm, strange, top and bottom and their antiparticles. All these quarks have fractional charges. For example, charge of up quark is $+\frac{2}{3}e$ and that of down quark is $-\frac{1}{3}e$

According to quark model, proton is made up of two up quarks and one down quark and neutron is made up of one up quark and two down quarks.

The study of elementary particles is called particle physics and it is an active area of research even now. Till date, more than 20 Nobel prizes have been awarded in the field of particle physics.

**Fundamental forces of nature:**

It is known that there exists gravitational force between two masses and it is universal in nature. Our planets are bound to the sun through gravitational force of the sun. In +2 volume 1, we have learnt that between two charges there exists electromagnetic force and it plays major role in most of our day-to-day events. In this unit, we have learnt that between two nucleons, there exists a strong nuclear force and this force is responsible for stability of the nucleus. In addition to these three forces, there exists another fundamental force of nature called the weak force. This weak force is even shorter in range than nuclear force. This force plays an important role in beta decay and energy production of stars. During the fusion of hydrogen into helium in sun, neutrinos and enormous radiations are produced through weak force. The detailed mechanism of weak force is beyond the scope of this book and for further reading, appropriate books can be referred.

Gravitational, electromagnetic, strong and weak forces are called fundamental forces of nature. It is very interesting to realize that, even for our day-to-day life, we require these four fundamental forces. To put it in simple words: We are in the Earth because of Earth's gravitational attraction on our body. We are standing on the surface of the earth because of the electromagnetic force between atoms of the surface of the earth with atoms in our foot. The atoms in our body are stable because of strong nuclear force. Finally, the lives of species in the earth depend on the solar energy from the sun and it is due to weak force which plays vital role during nuclear fusion reactions going on in the core of the sun.

# Unit – 9

# SEMICONDUCTOR ELECTRONICS

## INTRODUCTION

Electronics has become a part of our daily life. All gadgets like mobile phones, computers, televisions, music systems etc work on the electronic principles. Electronic circuits are used to perform various operations in devices like air conditioners, microwave oven, dish washers and washing machines. Besides this, its applications are widespread in all fields like communication systems, medical diagnosis and treatments and even handling money through ATMs.

## Evolution of Electronics:

The history of electronics began with the invention of vacuum diode by J.A. Fleming in 1897. This was followed by a vacuum triode implemented by Lee De Forest to control electrical signals. This led to the introduction of tetrode and pentode tubes. Subsequently, the transistor era began with the invention of bipolar junction transistor by Bardeen, Brattain and Shockley in 1948 for which Nobel prize was awarded in 1956. The emergence of Germanium and Silicon semiconductor materials made this transistor gain popularity, in turn its application in different electronic circuits.

The following years witnessed the invention of the integrated circuits (ICs) that helped to integrate the entire electronic circuit on a single chip which is small in size and cost-effective.

Since 1958 ICs capableof holding several thousand electronic components on a single chip such as small- scale, medium-scale, large-scale, and very- large scale integration started coming into existence. Digital integrated circuits became another robust IC development that enhanced the architecture of computers.All these radical changes led to the introduction of microprocessor in 1969 by Intel.

The electronics revolution, in due course of time, accelerated the computer revolution. Now the world is on its way towards small particles of nano-size, far too small to see. This helps in the miniaturization to an unimaginable size. A room-size computer during its invention has now emerged as a laptop, palmtop, iPad, etc. In the recent past, IBM revealed the smallest computer whose size is comparable with the tip of the rice grain, measuring just 0.33 mm on each side.

Electronics is the branch of physics incorporated with technology towards the design of circuits using transistors and microchips. It depicts the behaviour and movement of electrons and holes in a semiconductor, electrons and ions in vacuum, or gas. Electronics deals with electrical circuits that involve active components such as transistors, diodes, integrated circuits, and sensors, associated with the passive components like resistors, inductors, capacitors, and transformers.

This chapter deals with semiconductor devices like p-n junction diodes, bipolar junction transistors and logic circuits.

The world's first computer 'ENIAC' was invented by J. Presper Eckert and John Mauchly at the University of Pennsylvania. The construction work started in 1943 and got over in 1946. It occupied an area of around 1800 square feet. It had 18,000 vacuum tubes and it weighed around 50 tons.

**Energy band diagram of solids**

In an isolated atom, the electronic energy levels are widely separated and are far apart and the energy of the electron is decided by the orbit in which it revolves around the nucleus. However, in the case of a solid, the atoms are closely spaced and hence the electrons in the outermost energy levels of nearby atoms influence each other. This changes the nature of the electron motion in a solid from that of an isolated atom to a large extent.

The valence electrons in an atom are responsible for the bonding nature. Let us consider an atom with one electron in the outermost orbit. It means that the number of valence electrons is one. When two such atoms are brought close to each other, the valence orbitals are split into two. Similarly the unoccupied orbitals of each atom will also split into two. The electrons have the choice of choosing any one of the orbitals as the energy of both the orbitals is the same. When the third atom of the same element is brought to this system, the valence orbitals of all the three atoms are split into three.

The unoccupied orbitals also will split into three. In reality, a solid is made up of millions of atoms. When millions of atoms are brought close to each other, the valence orbitals and the unoccupied orbitals are split according to the number of atoms. In this case, the energy levels will be closely spaced and will be difficult to differentiate the orbitals of one atom from the other and they look like a band as shown in Figure 9.2. This band **of very large number of closely spaced energy levels in a very small energy range is known as energy band. The energy band formed due to the valence orbitals is called valence band and that formed due to the unoccupied orbitals to which electrons can jump when energised is called the conduction band. The energy gap between the valence band and the conduction band is called forbidden energy gap.** Electrons cannot exist in the forbidden energy gap.

The representation of the valence band and conduction band is shown in Figure 9.2(a). EV represents the maximum energy of the valence band and EC represents minimum energy of the conduction band. The forbidden energy gap, Eg = EC– Ev. The kinetic energy of the electron increases from bottom to top (near the nucleus to the farthest) and the potential energy decreases indicating that the electrons in the orbitals closer to the nucleus are bound with large potential energy. Hence, the electrons closer to nucleus require a lot of energy to be excited. The electrons in the valence band are less bound to the nucleus and can be easily excited.

## Classification of materials

The classification of solids into insulators, metals, and semiconductors can be explained with the help of the energy band diagram.

## Insulators

The energy band structure of insulators is shown in Figure 9.2(b). The valence band and the conduction band are separated by a large energy gap. The forbidden energy gap is approximately 6 eV in insulators. The gap is very large that electrons from valence band cannot move into conduction band even on the application of strong external electric field or the increase in temperature. Therefore, the electrical conduction is not possible as the free electrons are almost nil and hence these materials are called insulators. Its resistivity is in the range of 1011–1019 Ωm.

## Conductors

In condutors, the valence band and conduction band overlap as shown in Figure 9.2(c). Hence, electrons can move freely into the conduction band which results in a large number of free electrons in the conduction band. Therefore, conduction becomes possible even at low temperatures. The application of electric field provides sufficient energy to the electrons to drift in a particular direction to constitute a current. For condutors, the resistivity value lies between 10–2 and 10–8 Ωm.

**Semiconductors**

In semiconductors, there exists a narrow forbidden energy gap (Eg <3eV ) between the valence band and the conduction band. At a finite temperature, thermal agitations in the solid can break the covalent bond between the atoms (covalent bond is formed due to the sharing of electrons to attain stable electronic configuration). This releases some electrons from valence band to conduction band. Since free electrons are small in number, the conductivity of the semiconductors is not as high as that of the conductors. The resistivity value of semiconductors is from 10–5 to 106 Ωm.

When the temperature is increased further, more number of electrons is promoted to the conduction band and increases the conduction. Thus, we can say that the electrical conduction increases with the increase in temperature. In other words, resistance decreases with increase in temperature. Hence, semiconductors are said to have negative temperature coefficient of resistance. The most important elemental semiconductor materials are Silicon (Si) and Germanium (Ge). The forbidden energy gaps for Si and Ge at at room temperature are 1.1 eV and 0.7 eV respectively.

## TYPES OF SEMICONDUCTRORS

### Intrinsic semiconductors

A semiconductor in its pure form without impurity is called an intrinsic semiconductor. Here, impurity means any other atom in the crystal lattice. The Silicon lattice is shown in Figure 9.3(a). Each Silicon atom has four electrons in the outermost orbit and is covalently bonded with the neighbouring atoms to form the lattice. The band diagram for this case is show in Figure 9.4(a).

A small increase in temperature is sufficient enough to break some of the covalent bonds and release the electrons free from the lattice as shown in Figure 9.3(b). As a result, some states in the valence band become empty and the same number of states in the conduction band will be occupied as shown in Figure 9.4(b). The vacancies produced in the valence band are called holes. As the holes are deficiency of electrons, they are treated to possess positive charges. Hence, electrons and holes are the two charge carriers in semiconductors.

In intrinsic semiconductors, the number of electrons in the conduction band is equal to the number of holes in the valence band. The conduction is due to the electrons in the conduction band and holes in the valence band. These currents are represented as $I_e$ and $I_h$ respectively.

The total current (I) is always the sum of the electron current (Ie) and the hole
Current ($I_n$).$I= I_e+I_h$

An intrinsic semiconductor behaves like an insulator at 0 K. The increase in temperature increases the number of charge carriers (electrons and holes). The schematic diagram of the intrinsic semiconductor in band diagram is shown in Figure 9.4(b). The intrinsic carrier concentration is the number electron in the conduction band or the number of holes in the valence band in an intrinsic semiconductor.

**Extrinsic semiconductors**

The carrier concentration in an intrinsic semiconductor is not sufficient enough to develop efficient electronic devices. Another way of increasing the carrier concentration in an intrinsic semiconductor is by adding impurity atoms. The process of adding impurities to the intrinsic semiconductor is called doping. It increases the concentration of charge carriers (electrons and holes) in the semiconductor and in turn, its electrical conductivity. The impurity atoms are called dopants and its order is approximately 100 ppm (parts per million).

**n-type semiconductor**

A n-type semiconductor is obtained by doping a pure Germanium (or Silicon) crystal with a dopant from group V pentavalent elements like Phosphorus, Arsenic, and Antimony as shown in Figure 9.5(a). The dopant has five valence electrons while the Germanium atom has four valence electrons. During the process of doping, a few of the Germanium atoms are replaced by the group V dopants. Four of the five valence electrons of the impurity atom are bound with the

4 valence electrons of the neighbouring replaced Germanium atom. The fifth valence electron of the impurity atom will be loosely attached with the nucleus as it has not formed the covalent bond.

The energy level of the loosely attached fifth electron from the dopant is found just below the conduction band edge and is called the donor energy level as shown in Figure9.5(b). At room temperature, these electrons can easily move to the conduction band with the absorption of thermal energy. Besides, an external electric field also can set free the loosely bound electrons and lead to conduction.

It is important to note that the energy required for an electron to jump from the valence bandto the conduction band($E_g$) in an intrinsic semiconductor is 0.7 eV for Ge and 1.1 eV for Si, while the energy required to set free a donor electron is only 0.01 eV for Ge and 0.05 eV for Si.

The group V pentavalent impurity atoms donate electrons to the conduction band and are called donor impurities. Therefore, each impurity atom provides one extra electron to the conduction band in addition to the thermally generated electrons. These thermally generated electrons leave holes in valence band. Hence, the majority carriers of current in an n-type semiconductor are electrons and the minority carriers are holes. Such a semiconductor doped with a pentavalent impurity is called an n-type semiconductor.

**p-type semiconductor**

Here, a trivalent atom from group III elements such as Boron, Aluminium, Gallium and Indium is added to the Germanium or Silicon substrate. The dopant with three valence electrons are bound with the neighbouring Germanium atom as shown in Figure 9.7(a). As Germanium atom has four valence electrons, one electron position of the dopant in the Germanium crystal lattice will remain vacant. The missing electron position in the covalent bond is denoted as a hole.

To make complete covalent bonding with all four neighbouring atoms, the dopant is in need of one more electron. These dopants can accept electrons from the neighbouring atoms. Therefore, this impurity is called an acceptor impurity. The energy level of the hole created by each impurity atom is just above the valence band and is called the acceptor energy level,

For each acceptor atom, there will be a hole in the valence band in addition to the thermally generated holes. In such an extrinsic semiconductor, holes are the majority carriers and thermally generated electrons are minority carriers as shown in Figure 9.8. The semiconductor thus formed is called a p-type semiconductor.

The n-type and p-type semiconductor are neutral as we are topping neutral atoms to the intrinsic semiconductors.

## DIODES

### P-N Junction formation

### Formation of depletion layer

A p-n junction is formed by joining n-type and p-type semiconductor materials as Since the n-region has a high electron concentration and the p-region a high hole concentration, electrons diffuse from the n-side to the p-side. This causes diffusion current which exists due to the concentration difference of electrons. The electrons diffusing into the p-region may occupy holes in that region and make it negative. The holes left behind by these electrons in the n-side are equivalent to the diffusion of holes from the p-side to the n-side. If the electrons and holes were not charged, this diffusion process would continue until the concentration of electrons as happens if two gasses come into contact with each other.

But, in a p-n junction, when the electrons and holes move to the other side of the junction, they leave behind exposed charges on dopant atom sites, which are fixed in the crystal lattice and are unable to move. On the n-side, positive ion cores are exposed and on the p- side, negative ion cores are exposed as shown in Figure 9.9(b). An electric field E forms between the positive ion cores in the n-type material and negative ion cores in the p-type material. The electric field sweeps free carriers out of this region and hence it is called depletion region as it is depleted of free carriers. A barrier potential Vb due to the electric field E is formed at the junction .

As this diffusion of charge carriers from both sides continues, the negative ions form a layer of negative space charge region along the p-side. Similarly, a positive space charge region is formed by positive ions on the n-side. The positive space charge region attracts electrons from p-side to n-side and the negative space charge region attracts holes from n-side to p-side. This moment of carriers happen in this region due to the formed electric field and it constitutes a current called drift current. The diffusion current and drift current flow in the opposite direction and at one instant they both become equal. Thus, a p-n junction is formed.

## Junction potential or barrier potential

The recombination of charge carriers takes place only to a certain point beyond which the depletion layer acts like a barrier to further diffusion of free charges across the junction.

This is due to the fact that the immobile ions on both sides establish an electric potential difference across the junction. Therefore, an electron trying to diffuse into the interior of the depletion region encounters a negative wall of ions repelling it backwards. If the free electron has enough energy, it can break through the wall and enter into the p-region, where it can recombine with a hole and create another negative ion.

The strength of the electric potential difference across the depletion region keeps increasing with the crossing of each electron until equilibrium is reached; at this point, the internal repulsion of the depletion layer stops further diffusion of free electrons across the junction. This difference in potential across the depletion layer is called the barrier potential as shown in Figure 9.10. At 25 0C, this barrier potential approximately equals 0.7 V for Silicon and 0.3 V for Germanium.

## P-N Junction diode

A p-n junction diode is formed when a p-type semiconductor is fused with an-type semiconductor. It is a device with single p-n junction as shown in Figure 9.11(a). The circuit symbol is shown in Figure 9.11(b).

## Biasing a diode

Biasing means providing external energy to charge carriers to overcome the barrier potential and make them move in a particular direction. The charge carriers can either move towards the junction or away from the junction. The external voltage applied to the p-n junction is called bias voltage. Depending on the polarity of the external source to the p-n junction we have two types of biasing

1. Forward bias
2. Reverse bias

## Forward Bias

If the positive terminal of the external voltage source is connected to the p-side and the negative terminal to the n-side, it is called forward biased The application of a forward bias potential makes the electrons move into the n-side and the holes into the p-side. This initiates the recombination with the ions near the junction which in turn reduces the width of the depletion region and hence the barrier potential.

The electron from the n-side is now accelerated towards the p-side as it experiences a reduced barrier potential at the junction. In addition, the accelerated electrons experience a strong attraction by the positive potential applied to the p-side. This results in the movement of electrons towards the p-side and in turn, holes towards the n-side. When the applied voltage is increased, the width of the depletion region and hence the barrier potential are further reduced. This results in a large number of electrons passing through the junction resulting in an exponential rise in current through the junction.

## *Reverse Bias*

f the positive terminal of the battery is connected to the n-side and the negative potential to the p-side, the junction is said to be reverse biased

As the positive potential is connected to the n-type material, the electrons in the n-type material are attracted towards thepositive terminal in turn, the holes in the p-type material move towards the negative terminal (both away from the junction). It increases the immobile ions at the junction. The net effect is the widening of the depletion region. This leads to an increase in the barrier potential. Consequently, the majority charge carriers from both sides experience a great barrier to cross the junction. This reduces the diffusion current across the junction effectively.

Yet, a small current flows across the junction due to the minority charge carriers in both regions. The reverse bias for majority charge carriers serves as the forward bias for minority charge carriers. The current that flows under a reverse bias is called the reverse saturation current**.** It is represented as $I_s$ .

The reverse saturation current is independent of the applied voltage and it depends only on the thermally generated minority charge carriers. Even a small voltage is sufficient enough to drive the minority charge carriers across the junction.

The reverse saturation current of a silicon diode doubles for every 10 °C rise in temperature.

## Characteristics of a junction diode

### Forward characteristics

It is the study of the variation in current through the diode with respect to the applied voltage across the diode when it is forward biased. The p-n junction diode is forward biased An external resistance (R) is used to limit the flow of current through the diode. The voltage across the diode is varied by varying the biasing voltage across the dc power supply. The forward bias voltage and the corresponding forward bias current are noted. A graph is plotted by taking the forward bias voltage (*V*) along the x-axis and the current (*I*) through the diode along the y-axis. This graph is called the **forward V-I characteristics** of the p-n junction diode and is shown in Figure 9.14(b). Three inferences can be brought out from the graph:

i. At room temperature, a potential difference equal to the barrier potential is required before a reasonable forward current starts flowing across the diode. This voltage is known as **threshold voltage or cut-in voltage or knee voltage** (Vth). It is approximately 0.3 V for Germanium and 0.7 V for Silicon. The current flow is negligible when the applied voltage is less than the threshold voltage. Beyond the threshold voltage, increase in current is significant even for a small increase in voltage.

ii. (ii) The graph clearly infers that the current flow is not linear and is exponential. Hence it does not obey Ohm's law.

iii. (iii) The forward resistance (*rf*) of the diode is the ratio of the small change in voltage ($\Delta V$)to the small change in current ($\Delta I$), $r\ VI\ f = \frac{\Delta V}{\Delta I}$ .

iv. (iv) Thus the diode behaves as a conductor when it is forward biased.

However, if the applied voltage is increased beyond a rated value, it will produce an extremely large current which may destroy the junction due to overheating. This is called as the breakdown of the diode and the voltage at which the diode breaks down is called the breakdown voltage. Thus, it is safe to operate a diode well within the threshold voltage and the breakdown voltage.

**Reverse characteristics**

The circuit to study the reverse characteristics is shown in Figure 9.15(a). In the reverse bias, the p-region of the diode is connected to the negative terminal and n-region to the positive terminal of the dc power supply.

A graph is drawn between the reverse bias voltage and the current across the junction, which is called the reverse characteristics of a p-n junction diode. It is shown in Figure 9.15(b). Under this bias, a very small current in $\mu A$, flows across the junction. This is due to the flow of the minority charge carriers called the leakage current or reversesaturation current. Besides, the current is almost independent of the voltage. The reverse bias voltage can be increased only up to the rated value otherwise the diode will enter into the breakdown region.

**EXAMPLE**

An ideal diode and a 5 Ω resistor are connected in series with a 15 V power supply as shown in figure below. Calculate the current that flows through the diode

**Solution**

The diode is forward biased and it is an ideal one. Hence, it acts like a closed switch with no barrier voltage. Therefore, current that flows through the diode can be calculated using Ohm's law. V = IR

$$I = \frac{V}{R} = \frac{15}{5} = 3A$$

**EXAMPLE**

Consider an ideal junction diode. Find the value of current flowing through AB is

**Solution**

The barrier potential of the diode is neglected as it is an ideal diode. The value of current flowing through AB can be obtained by using Ohm's law

$$I = \frac{V}{R} = \frac{3-(-7)}{1\times 10^{3}} = \frac{10}{10^{3}} = 10^{-2}A = 10mA$$

**Rectification**

**The process of converting alternating current into direct current is called rectification.**In this section, we will discuss two types of rectifiers namely, half wave rectifier and full wave rectifier.

**Half wave rectifier circuit**

The half wave rectifier circuit is shown in Figure 9.17(a). The circuit consists of a transformer, a p-n junction diode and a resistor. In a half wave rectifier circuit, either a positive half or the negative half of the AC input is passed through while the other half is blocked. Only one half of the input wave reaches the output. Therefore, it is called half wave rectifier. Here, a p-n junction diode acts as a rectifier diode.

**During the positive half cycle**

When the positive half cycle of the ac input signal passes through the circuit, terminal A becomes positive with respect to terminal B. The diode is forward biased and hence it conducts. The current flows through the load resistor RL and the ACvoltage developed across RLconstitutes the output voltage V0 and the waveform of the diode current

**During the negative half cycle**

When the negative half cycle of the ac input signal passes through the circuit, terminal A is negative with respect to terminal B. Now the diode is reverse biased and does not conduct and hence no current passes through RL. The reverse saturation current in a diode is negligible. Since there is no voltage drop across RL, the negative half cycle of ac supply is suppressed at the output. The output waveform is for electronic equipments. A constant or a steady voltage is required which can be obtained with the help of filter circuits and voltage regulator circuits. **Efficiency** ($h$ ) **is the ratio of the output dc power to the ac input power supplied to the circuit**. Its value for half wave rectifier is 40.6 %

## Full wave rectifier

The positive and negative half cycles of the AC input signal pass through the full wave rectifier circuit and hence it is called the full wave rectifier. The circuit is shown in Figure 9.18(a). It consists of two p-n junction diodes, a center tapped transformer, and a load resistor (RL). The centre is usually taken as the ground or zero voltage reference point. Due to the centre tap transformer, the output voltage rectified by each diode is only one half of the total secondary voltage.

### During positive half cycle

When the positive half cycle of the ac input signal passes through the circuit, terminal M is positive, G is at zero potential and N is at negative potential. This forward biases diode D1 and reverse biases diode D2. Hence, being forward biased, diode D1 conducts and current flows along the path MD AGC 1 . As a result, positive half cycle of the voltage appears across RL in the direction G to C

### During negative half cycle

When the negative half cycle of the ac input signal passes through the circuit, terminal N is positive, G is at zero potential and M is at negative potential. This forward biases diode D2 and reverse biases diode D1. Hence, being forward biased, diode D2 conducts and current flows along the pathND BGC 2 . As a result, negative half cycle of the voltage appears across RL in the same direction from G to C

Hence in a full wave rectifier both positive and negative half cycles of the input signal pass through the load in the same direction as shown in Figure 9.18(b). Though both positive and negative half cycles of ac input are rectified, the output is still pulsating in nature.

The efficiency (η) of full wave rectifier is twice that of a half wave rectifier and is found to be 81.2 %. It is because both the positive and negative half cycles of the ac input source are rectified.

### Breakdown mechanism

The reverse current or the reverse saturation current due to the minority charge carriers is small. If the reverse bias applied to a p-n junction is increased beyond a point, the junction breaks down and the reverse current rises sharply. The voltage at which this happens is called the breakdown voltage and it depends on the width of the depletion region, which in turn depends on the doping level.

A normal p-n junction diode gets damaged at this point. Specially designed diodes like Zener diode can be operated at this region and can be used for the purpose of voltage regulation in circuits. There are two mechanisms that are responsible for breakdown under increasing reverse voltage.

**Zener breakdown**

Heavily doped p-n junctions have narrow depletion layers of the order of <10-6 m. When a reverse voltage across this junction is increased to the breakdown limit, a very strong electric field of strength 3 × 107 V m–1 is set up across the narrow layer. This electric field is strong enough to break or rupture the covalent bonds in the lattice and thereby generating electron-hole pairs. This effect is called **Zener effect**.

Even a small further increase in reverse voltage produces a large number of charge carriers. Hence the junction has very low resistance in the breakdown region. This process of emission of electrons due to the rupture of bands in from the lattice due to strong electric field is known as **internal field emission or field ionization**. The electric field required for this is of the order of $10_{6}$ V m$_{-1}$.

**Avalanche breakdown**

Avalanche breakdown occurs in lightly doped junctions which have wide depletion layers. Here, in this case, the electric field is not strong enough to produce breakdown. Alternatively, the thermally generated minority charge carriers accelerated by the electric field gains sufficient kinetic energy, collide with the semiconductor atoms while passing through the depletion region. This leads to the breaking of covalent bonds and in turn generates electron-hole pairs.

The newly generated charge carriers are also accelerated by the electric field resulting in more collisions and further production of charge carriers. This cumulative process leads to an avalanche of charge carriers across the junction and consequently reduces the reverse resistance. The diode current increases sharply.

## Zener diode

Zener diode is a heavily doped silicon diode used in reverse biased condition and is named after its inventor C. Zener. It is specially designed to be operated in the breakdown region. The doping level of the Silicon diode can be varied to have a wide range of breakdown voltages from 2 V to over 1000 V.

As explained in the previous section, Zener breakdown occurs due to the breaking of covalent bonds by the strong electric field set up in the depletion region by the reverse voltage. It produces an extremely large number of electrons and holes which constitute the reverse saturation current. The current is limited by both external resistance and power dissipation of the diode. A Zener diodes is shown in Figure 9.19(a) and its circuit symbol of Zener diode is shown in Figure 9.19(b).

It looks like an ordinary p-n junction diode except the cathode lead approximating the shape of a 'z' letter. The arrow head points the direction of conventional current. In Figure 9.19(a), black ring indicates the cathode lead.

## V-I Characteristics of Zener diode

The circuit to study the forward and reverse characteristics of a Zener diode is shown in Figure 9.20(a) and Figure 9.20 (b). The V-I characteristics of a Zener diode is shown in Figure 9.20(c). The forward characteristic of a Zener diode is similar to that of an ordinary p-n junction diode. It starts conducting approximately around 0.7 V. However, the reverse characteristics is highly significant in Zener diode. The increase in reverse voltage normally generates very small reverse current. While in Zener diode, when the reverse voltage is increased to the breakdown voltage (VZ), the increase in current is very sharp. The voltage remains almost constant throughout the breakdown region. In Figure 9.20(c), IZ(max) represents the maximum reverse current. If the reverse current is increased further, the diode will be damaged. The important parameters on the reverse characteristics are

$V_Z$→Zener breakdown voltage

$I_{Z(min)}$→minimum current to sustain breakdown *IZ*max()→maximum current limited by maximum power dissipation.

The Zener diode is operated in the reverse bais having the voltage greater than VZ and current less than IZ(max). The reverse characteristic is not exactly vertical which means that the diode possesses some small resistance called Zener dynamic impedance. Zener resistance is the inverse of the slope in the breakdown region. It means an increase in the Zener current produces only a very small increase in the reverse voltage. However this can be neglected. The voltage of an ideal Zener diode does not change once it goes into breakdown. It means that *VZ* remains almost constant even when *IZ* increases considerably.

**Applications**

The zener diode can be used as

1. Voltage regulators
2. Calibrating voltages
3. Provide fixed reference voltage in a network for biasing
4. Protection of any gadget against damage from accidental application of excessive voltage.

**Zener diode as a voltage regulator**

A Zener diode working in the breakdown region can serve as a voltage regulator. It maintains a constant output voltage even when input voltage Vi or load current IL varies. The circuit used for the same . Here in this circuit, the input voltage Vi is regulated at a constant voltage, *Vz* (Zener voltage) at the output represented as $V_0$ using a Zener diode. The output voltage is maintained constant as long as the input voltage does not fall below *Vz.*

When the potential developed across the diode is greater than , the diode moves into the Zener breakdown region. It conducts and draws relatively large current through the series resistance *R*s. The total current I passing through *R*s equals the sum of diode current $I_Z$ and load current $I_L$ . (I= $I_Z + I_L$) It is to be noted that the total current is always less than the maximum Zener diode current.

Under all conditions $V_O = V_Z$ . Thus, output voltage is regulated.

**EXAMPLE:**

Find the current through the Zener diode when the load resistance is 1 KΩ. Use diode approximation.

**Solution**

Voltage across AB is VZ = 9V
Voltage drop across R = 15 - 9 = 6V
Therefore current through the resistor R,

$$I = \frac{6}{1 \times 10^3} = 6mA$$

Voltage across the load resistor = $V_{AB}$ = 9V Current through load resistor,

$$I_L = \frac{V_{AB}}{R_L} = \frac{9}{2 \times 10^3} \mathbf{=4.5mA}$$

The current through the Zener diode,

$$\mathbf{I_Z=I-I_L=6\ mA\ =1.5mA}$$

## Optoelectronic devices

Optoelectronics deals with devices which convert electrical energy into light and light into electrical energy through semiconductors. Optoelectronic device is an electronic device which utilizes light for useful applications. We will discuss some important optoelectronic devices namely, light emitting diodes, photo diodes and solar cells.

## Light Emitting Diode (LED)

LED is a p-n junction diode which emits visible or invisible light when it is forward biased. Since, electrical energy is converted into light energy, this process is also called electroluminescence. The circuit symbol of LED is shown in Figure 9.22(a).

The cross-sectional view of a commercial LED is shown in Figure 9.22(b). It consists of a p-layer, n-layer and a substrate. A transparent window is used to allow light to travel in the desired direction. An external resistance in series with the biasing source is required to limit the forward current through the LED. In addition, it has two leads; anode and cathode.

When the p-n junction is forward biased, the conduction band electrons on n-side and valence band holes on p-side diffuse across the junction. When they cross the junction, they become excess minority carriers (electrons in p-side and holes in n-side). These excess minority carriers recombine with oppositely charged majority carriers in the respective regions, i.e. the electrons in the conduction band recombine with holes in the valence band as shown in the Figure 9.22(c).
During recombination process, energy is released in the form of light (radiative) or heat (non-radiative). For radiative recombination, a photon of energy $hv$ is emitted. For non-radiative recombination, energy is liberated in the form of heat.

The colour of the light is determined by the energy band gap of the material. Therefore, LEDs are available in a wide range of colours such as blue (SiC), green (AlGaP) and red (GaAsP). Now a days, LED which emits white light (GaInN) is also available.

**Applications**

- Indicator lamps on the front panel of the scientific and laboratory equipments.

- Seven-segment displays.
- Traffic signals, emergency vehicle lighting etc.
- Remote control of television, airconditioner etc.

**EXAMPLE**

Determine the wavelength of light emitted from LED which is made up of GaAsP semiconductor whose forbidden energy gap is 1.875 eV. Mention the colour of the light emitted (Take $h = 6.6 \times 10^{-34}$ Js).

## Solution

$$E_g = \frac{hc}{\lambda}$$

Therefore,

$$\lambda = \frac{hc}{E_g} = \frac{6.6\times10^{-34}\times3\times10^{8}}{1.875\times1.6\times10^{-19}}$$

= 660 *nm*

The wavelength 660 nm corresponds to red colour light.

**Photodiodes**

A p-n junction diode which converts an optical signal into electric signal is known as photodiode. Thus, the operation of photodiode is exactly inverse to that of an LED. Photo diode works in reverse bias. Its circuit symbol is shown in Figure 9.23(a). The direction of arrows indicates that the light is incident on the photo diode.

The device consists of a p-n junction semiconductor made of photosensitive material kept safely inside a plastic case as shown in Figure 9.23(b). It has a small transparent window that allows light to be incident on the p-n junction. Photodiodes can generate current when the p-n junction is exposed to light and hence are called as light sensors.

When a photon of sufficient energy (*hu*) strikes the depletion region of the diode, some of the valence band electrons are elevated into conduction band, in turn holes are developed in the valence band. This creates electron-hole pairs. The amount of electron-hole pairs generated depends on the intensity of light incident on the p-n junction.

These electrons and holes are swept across the p-n junction by the electric field created by reverse voltage before recombination takes place. Thus, holes move towards the n-side and electrons towards the p-side. When the external circuit is made, the electrons flow through the external circuit and constitute the photocurrent.

When the incident light is zero, there exists a reverse current which is negligible. This reverse current in the absence of any incident light is called dark current and is due to the thermally generated minority carriers.

**Applications**

- Alarm system
- Count items on a conveyer belt
- Photoconductors
- Compact disc players, smoke detectors
- Medical applications such as detectors for computed tomography etc.

**Solar cell**

A solar cell, also known as photovoltaic cell, converts light energy directly into electricity or electric potential difference by **photovoltaic effect**. It is basically a p-n junction which generates emf when solar radiation falls on the p-n junction. A solar cell is of two types: p-type and n-type.

Both types use a combination of p-type and n-type Silicon which together forms the p-n junction of the solar cell. The difference is that p-type solar cells use p-type Silicon as the base with an ultra-thin layer of n-type Silicon as shown in Figure 9.24, while n-type solar cell uses the opposite combination. The other side of the p-Silicon is coated with metal which forms the back electrical contact. On top of the n-type Silicon, metal grid is deposited which acts as the front electrical contact. The top of the solar cell is coated with anti-reflection coating and toughened glass.

In a solar cell, electron–hole pairs are generated due to the absorption of light

near the junction. Then the charge carriers are separated due to the electric field of the depletion region. Electrons move towards n–type Silicon and holes move towards p-type Silicon layer. The electrons reaching the n-side are collected by the front contact and holes reaching p-side are collected by the back electrical contact. Thus a potential difference is developed across solar cell.When an external load is connected to the solar cell, photocurrent flows through the load.
Many solar cells are connected together either in series or in parallel combination to form solar panel or module. Many solar panels are connected with each other to form solar arrays. For high power applications, solar panels and solar arrays are used.

**Applications:**

- Solar cells are widely used in calculators, watches, toys, portable power supplies, etc.

- Solar cells are used in satellites and space applications

- Solar panels are used to generate electricity.

## THE BIPOLAR JUNCTION TRANSISTOR [BJT]

### Introduction

In 1951, William Schockley invented the modern version of transistor. It is a semiconductor device that led to a technological revolution in the twentieth century. The heat loss in transistor is very less. This has laid the foundation of integrated chips which contain thousands of miniaturized transistors. The emergence of the integrated chips led to increasing applications in the fast developing electronics industry.

### Bipolar Junction Transistor

The BJT consists of a semiconductor (Silicon or Germanium) crystal in which an n-type material is sandwiched between two p-type materials (PNP transistor) or a p-type material sandwiched between two n-type materials (NPN transistor). To protect it against moisture, it is sealed inside a metal or a plastic case. The two types of transistors with their circuit symbols are shown in Figure 9.25.

The three regions formed are called as emitter, base and collector which are provided with terminals or ohmic contacts labeled as E, B, and C. As a BJT has two p-n junctions, two depletion layers are formed across the emitter-base junction (JEB) and collector-base junction (JCB) respectively. The circuit symbol carries an arrowhead at the emitter lead pointing from p to n indicating the direction of conventional current.

*Emitter*:

The main function of the emitter is to supply majority charge carriers to the collector region through the base region. Hence, emitter is more heavily doped than the other two regions.

**Base:**

Base is very thin (10-6 m) and very lightly doped compared to the other two regions.

**Collector:**

The main function of collector is to collect the majority charge carriers supplied by the emitter through the base. Hence, collector is made physically larger than the other two as it has to dissipate more power. Its is modarately dopped.

use of he dif ering siz and the amo nt of op ing, th emi ter and col ector can ot be nt rchanged.**Tran**

**Transistor Biasing**

The application of suitable dc voltages across the transistor terminals is called biasing.

**Different modes of transistor biasing**
**Forward Active**:

In this bias the emitter-base junction is forward biased and the collector-base junction is reverse biased. The transistor is in the active mode of operation. In this mode, the transistor functions as an amplifier. Saturation: Here, the emitter-base junction and collector-base junction are forward biased. The transistor has a very large flow of currents across the junctions. In this mode, transistor is used as a closed switch.

**Cut-off:**

In this bias, the emitter-base junction and collector-base junction are reverse biased. Transistor in this mode is an open switch.

## Transistor circuit configurations

There are three types of circuit connections for operating a transistor based on the terminal that is used in common to both input and output circuits.

### Common-Base (CB) configuration

The base is common to both the input and output circuits. The schematic and circuit symbol are shown in Figure 9.26(a) and 9.26(b). The input current is the emitter current IE and the output current is the collector current IC. The input signal is applied between emitter and base, the output is measured between collector and base.

### Common-Emitter (CE) configuration

In this configuration, the emitter is common to both the input and output loops as shown in Figure 9.27. Base current, IB is the input current and the collector current, IC is the output current. The input signal is applied between the emitter and base and the output is measured between the collector and the emitter.

### Common-Collector (CC)configuration

Here, the collector is common to both the input and output circuits as shown in Figure 9.28. The base current IB is the input current, the emitter current IE is the output current. The input signal is applied between the base and the collector, the output is measured between the emitter and collector.

## Transistor action in the common base mode

The operation of an NPN transistor in the common base mode is explained below. The current flow in a common base NPN transistor in the forward active mode is

Basically, a BJT can be considered as two p-n junction diodes connected back-to-back. In the forward active bias of the transistor, the emitter-base junction is forward biased by a dc power supply VEB and the collector-base junction is reverse biased by the bias power supply $V_{CB}$. The forward bias decreases the depletion region across the emitter-base junction and the reverse bias increases the depletion region across the collector-base junction. Hence, the barrier potential across the emitter-base junction is decreased and the collector-base junction is increased. The voltage across the emitter-base junction is represented as $V_{EB}$ and the collector-base junction as $V_{CB}$.

In an NPN transistor, the majority charge carriers in the emitter are electrons. As it is heavily doped, it has a large number of electrons. The forward bias across the emitter- base junction causes the electrons in the emitter region to flow towards the base region and constitutes the emitter current ($I_E$ ). The electrons after reaching the base region recombine with the holes in the base region. Since the base region is very narrow and lightly doped, all the electrons will not have sufficient holes to recombine and hence most of the electrons reach the collector region.

Eventually, the electrons that reach the collector region will be attracted by the collector terminal as it has positive potential and flows through the external circuit. This constitutes the collector current ($I_C$). The holes that are lost due to recombination in the base region are replaced by the positive potential of the bias voltage VEE and constitute the base current ($I_B$ ). The magnitude of the base current will be in micoamperes as against milliamperes for emitter and collector currents. It is to be noted that if the emitter current is zero, then the collector current is almost zero. It is therefore imperative that a BJT is called a current controlled device. Applying Kirchoff's law, we can write the emitter current as the sum of the collector current and the base current.

$$\mathbf{I_E = I_B + I_C}$$

Since the base current is very small, we can write, $I_C \approx I_E$ . There is another component of collector current due to the thermally generated electrons called reverse saturation current, denoted as $(I_{CO})$. This factor is temperature sensitive. Therefore, care must be taken towards the stability of the system at high temperatures. The ratio of the collector current to the emitter current is called the forward current gain $(\alpha_{dc})$ of a transistor.

$$\alpha_{dc} = \frac{I_C}{I_E}$$

The α of a transistor is a measure of the quality of a transistor. Higher the value of α better is the transistor. It means that the collector current is closer to the emitter current. The value of α is less than unity and ranges from 0.95 to 0.99. This indicates that the collector current is 95% to 99% of the emitter current.

**Working of a PNP transistor**

The working of a PNP transistor is similar to the NPN transistor except for the fact that the emitter current IE is due to holes and the base current IB is due to electrons. However, the current through the external circuit is due to the flow of electrons.

**Static Characteristics of Transistor in Common Emitter Mode**

The know-how of certain parameters like the input resistance, output resistance, and current gain of a transistor are very important for the effective use of transistors in circuits. The circuit to study the static characteristics of an NPN transistor in the common emitter mode is given in Figure 9.30. The bias supply voltages $V_{BB}$ and $V_{CC}$ bias the base-emitter junction and collector-emitter junction respectively. The junction potential at the base-emitter is represented as VBE and the collector-emitter as $V_{CE}$. The rheostats $R_1$and $R_2$ are used to vary the base and collector currents respectively.

The static characteristics of the BJT are

1. Input characteristics
2. Output characteristics
3. Transfer characteristics

**Input Characteristics**

Input Characteristics curves give the relationship between the base current($I_B$) and base to emitter voltage($V_{BE)}$ at constant collector to emitter voltage ($V_{CE}$) and are

Initially, the collector to emitter voltage ($V_{CE}$) is set to a particular voltage (above 0.7 V to reverse bias the junction). Then the base-emitter voltage ($V_{BE}$) is increased in suitable steps and the corresponding base-current (IB) is recorded. A graph is plotted with VBE along the x-axis and IB along the y-axis. The procedure is repeated for different values of $V_{CE}$.

***The following observations are made from the graph.***

The curve looks like the forward characteristics of an ordinary p-n junction diode.

There exists a threshold voltage ($Vk$) or knee voltage below which the base current is very small. The value is 0.7 V for Silicon and 0.3 V for Germanium transistors. Beyond the knee voltage, the base current increases with the increase in base-emitter voltage.

It is also noted that the increase in the collector-emitter voltage decreases the base current. This shifts the curve outward. This is because the increase in collector-emitter voltage increases the width of the depletion region in turn, reduces the effective base width and thereby the base current.

**Input impedance**

The ratio of the change in base-emitter voltage($\Delta V_{BE}$) to the change in base current ($\Delta I_B$ ) at a constant collector-emitter voltage($V_{CE}$ ) is called the input impedance ($r_i$). The input impedance is not linear in the lower region of the curve.

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$$

The output impedance for transistor in common emitter configuration is very low.

**Current transfer characteristics**

This gives the variation of collector current($I_C$) with changes in base current at ($I_B$) constant collector-emitter voltage ($V_{CE}$ .It is seen that a small $I_C$ flows even when $I_B$ is zero. This current is called the common emitter leakage $I_{CEO}$ current, which is due to the flow of minority charge carriers.

**Forward current gain**

The ratio of the change in collector current $\Delta I_C$ to the change in base current $\Delta I_B$ at constant collector-emitter voltage $V_{CE}$ is called forward current gain (β).

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}}$$

Its value is very high and it generally ranges from 50 to 200. It depends on the construction of the transistor and will be provided by the manufacturer. There are transistors with b as high as 1000 as well.

**Relation between α and β**

here is a relation between current gain in the common base configuration α and current gain in the common emitter configuration β which is given below.

$$\alpha = \frac{\beta}{1+\beta} \text{ (or) } \beta = \frac{\alpha}{1-\alpha}$$

**EXAMPLE**

The output characteristics of a transistor connected in common emitter mode is shown in the figure. Determine the value of $I_C$ when $V_{CE}$ = 15 V. Also determine the value of $I_C$ when $V_{CE}$ is changed to 10 V

When $V_{CE} = 15\ V$, $I_C = 1.5\ \mu$A
When $V_{CE}$ is changed to 10 $V$, $I_C = 1.4\ \mu$A

**EXAMPLE**

In the circuit shown in the figure, the input voltage $V_i$ is 20 V, $V_{BE}$ =0 V and VCE = 0 V. What are the values of $I_B$, $I_C$, $\beta$?

$$I_B = \frac{V_i}{R_B} = \frac{20V}{500k\Omega} = 40\,\mu A \left[\because V_{BE} = 0V\right]$$

$$I_C = \frac{V_{CC}}{R_C} = \frac{20V}{4k\Omega} = 5\,mA \left[\because V_{CE} = 0V\right]$$

$$\beta = \frac{I_C}{I_B} = \frac{5\,mA}{40\,\mu A} = 125$$

## Transistors as a switch

The transistor in Saturation and cut-off regions functions like an electronic switch that helps to ON or OFF a given circuit by a small control signal.

### Presence of dc source at the input (saturation region):

When a high input voltage ($V\ in$=+5) is applied, the base current (IB) increases and in turn increases the collector current. The transistor will move into the saturation region (turned ON). The increase in collector current (IC) increases the voltage drop across $RC$ , thereby lowering the output voltage, close to zero. The transistor acts like a closed switch and is equivalent to ON condition.

### Absence of dc source at the input (cut- off region):

A low input voltage ($V_{in}$ =0V ), decreases the base current ($I_B$) and in turn decreases the collector current ($I_C$). The transistor will move into the cut-off region (turned OFF). The decrease in collector current ($I_C$) decreases the drop across $R_C$ , thereby increasing the output voltage, close to +5 V. The transistor acts as an open switch which is considered as the OFF condition. It is manifested that, a high input gives a low output and a low input gives a high output. In addition, we can say that the output voltage is opposite to the applied input voltage. Therefore, a transistor can be used as an inverter (NOT gate) in computer logic circuitry.

## Operating Point

The operating point is a point where the transistor can be operated efficiently. A line that is drawn with the values $V_{CC}$ (when $I_C$ = 0) and $I_C$ (when $V_{CE}$ = 0) is called the dc load line. The dc load line superimposed on the output characteristics of a transistor is used to learn the operating point of the transistor.Points P, Q, R in Figure 9.35 are called Q points or quiescent points which determine the operating point or the working point of a transistor. If the operating point is chosen at the middle of the dc load line (point Q), the transistor can effectively work as an amplifier. The operating point determines the maximum signal that can be obtained without being distorted.

For a transistor to work as a open switch, the Q point can be chosen at the cut-off region and to work as a closed switch, the Q point can be chosen in the saturation region.

**EXAMPLE**

The current gain of a common emitter transistor circuit shown in figure is 120. Draw the dc load line and mark the Q point on it ($V_{BE}$ to be ignored).

**Solution**

$$\beta = 120$$

$$\text{Base current } I_B = 25V/1M\Omega = 25/1X10^6 = 25\ \mu A$$

$$\beta = I_C/I_B$$

$$I_C = \beta I_B$$

$$I_C = 120X25\ \mu A$$

$$I_C = 3mA$$

$$V_{CE} = V_{CC} - I_C R_C$$

$$V_{CE} = 25\text{-}3\ mA \times 5k$$

$$V_{CE} = 10V$$

**Transistor as an amplifier**

A transistor operating in the active region has the capability to amplify weak signals. **Amplification is the process of increasing the signal strength (increase in the amplitude).** If a large amplification is required, the transistors are cascaded with coupling elements like resistors, capacitors, and transformers which is called as multistage amplifiers.

Here, the amplification of an electrical signal is explained with a single stage transistor amplifier as shown in Figure 9.36(a). Single stage indicates that the circuit consists of one transistor with the allied components. An NPN transistor is connected in the common emitter configuration.

To start with, the Q point or the operating point of the transistor is fixed so as to get the maximum signal swing at the output (neither towards saturation point nor towards cut-off). A load resistance,$R_C$ is connected in series with the collector circuit to measure the output voltage. The capacitor $C_1$ allows only the ac signal to pass through. The emitter bypass capacitor $C_E$ provides a low reactance path to the amplified $V_S$ ac signal. The coupling capacitor $C_C$ is used to couple one stage of the amplifier with the next stage while constructing multistage amplifiers. is the sinusoidal input signal source applied across the base-emitter. The output is taken across the collector-emitter.

Collector current, $I_C = \beta I_B \left[ \because \beta = \frac{I_C}{I_B} \right]$

Applying Kirchhoff 's voltage law in the output loop, the collector-emitter voltage is given by

$V_{CE} = V_{CC} - I_C R_C$

**Working of the amplifier**

- **During the positive half cycle**

Input signal ( $V_S$ ) increases the forward voltage across the emitter-base. As a result, the base current ( $I_B$ ) increases. Consequently, the collector current ( $I_C$ ) increases β times. This increases the voltage drop across $R_C$ ($I_c R_c$) which in turn decreases the collector-emitter voltage ( $V_{CE}$ ). Therefore, the input signal in the positive direction produces an amplified signal in the negative direction at the output. Hence, the output signal is reversed by 180°

- **During the negative half cycle**

Input signal ( $V_s$ ) decreases the forward voltage across the emitter-base. As a result, base current ( $I_B$) decreases and in turn increases the collector current ( $I_C$ ) . The increase in collector current ( $I_C$ ) decreases the potential drop across $R_C$ and increases the collector-emitter voltage ( $V_{CE}$ ). Thus, the input signal in the negative direction produces an amplified signal in the positive direction at the output. Therefore, 1800 phase reversal is observed during the negative half cycle of the input signal as well as shown in Figure 9.36(b).

## Transistor as an oscillator

**An electronic oscillator basically converts dc energy into ac energy of high frequency ranging from a few Hz to several MHz.** Hence, it is a source of alternating current or voltage. Unlike an amplifier, oscillator does not require any external signal source.

Basically, there are two types of oscillators: **Sinusoidal and non-sinusoidal.** Sinusoidal oscillators generate oscillations in the form of sine waves at constant amplitude and frequency as shown in Figure 9.37(a). Whereas non-sinusoidal oscillators generate complex non-sinusoidal waveforms like Square-wave, Triangular-wave or Sawtooth-wave as shown in Figure 9.36(b).

Sinusoidal oscillations can be of two types: **Damped and undamped.** If the amplitude of the electrical oscillations decreases with time due to energy loss, it is called damped oscillations as shown in Figure 9.38(a). On the other hand, the amplitude of the electrical oscillations remains constant with time in undamped oscillations as shown in Figure 9.38(b).

## Transistor Oscillator

An oscillator circuit consists of a tank circuit, an amplifier and a feedback circuit as shown in Figure 9.39. The tank circuit generates electrical oscillations and acts as the ac input source to the transistor amplifier. Amplifier amplifies the input ac signal. The feedback circuit provides a portion of the output to the tank circuit to sustain the oscillations without energy loss. Hence, an oscillator does not require an external input signal. The output is said to be self-sustained.

## Amplifier

The transistor amplifier circuit is already explained in section {9.4.5}.

## Feedback network

The circuit used to feedback a portion of the output to the input is called the feedback network. If the portion of the output fed to the input is in phase with the input, then the magnitude of the input signal increases. It is necessary for sustained oscillations.

## Tank circuit

The LC tank circuit consists of an inductance and a capacitor connected in parallel as shown in Figure 9.39. Whenever energy is supplied to the tank circuit from a DC source, the energy is stored in inductor and capacitor alternatively. This produces electrical oscillations of definite frequency. (Refer section 4.9.1, Volume 1 of XII std. Physics text book)

But in practical oscillator circuits there will be loss of energy across resistors, inductor coils and capacitors. A small amount of energy is used up in overcoming these losses during every cycle of charging and discharging of the capacitor. Due to this, the amplitude of the oscillations decreases gradually. Hence, the tank circuit produces damped electrical oscillations. Therefore, in order to produce undamped oscillations, a positive feedback is provided from the output circuit to the input circuit.

The frequency of oscillations is determined by the values of L and C using the equation.

$$f=\frac{1}{2\pi\sqrt{LC}}$$

## Barkhausen conditions for sustained oscillations

The following condition called Barkhausen conditions should be satisfied for sustained oscillations in the oscillator.

- The loop phase shift must be $0^0$ or integral multiples of $2\pi$.
- The loop gain must be unity. $|A\beta| = 1$

Here, $A$→Voltage gain of the amplifier,$\beta$→feedback ratio; (fraction of the output that is fed back to the input)

There are different types of oscillator circuits based on the different types of tank circuits. Examples: Hartley oscillator, Colpitt's oscillator, Phase shift oscillator, and Crystal oscillator.

## Applications of oscillators

- ❖ to generate a periodic sinusoidal or non sinusoidal wave forms
- ❖ to generate RF carriers
- ❖ to generate audio tones
- ❖ to generate clock signal in digital circuits
- ❖ as sweep circuits in TV sets and CRO

## EXAMPLE

Calculate the range of the variable capacitor that is to be used in a tuned-collector oscillator which has a fixed inductance of 150 μH. The frequency band is from 500 kHz to 1500 kHz.

Resonant frequency, $f_o = \frac{1}{2\pi\sqrt{LC}}$

On simplifying we get $C = \frac{1}{4\pi^2 f_o^2 L}$

When frequency is equal to 500 kHz

$$C = \frac{1}{4 \times 3.14^2 \times (500 \times 10^3)^2 \times 150 \times 10^{-6}}$$
$$= 676 \text{ pF}$$

When frequency is equal to 1500 kHz

$$C = \frac{1}{4 \times 3.14^2 \times (1500 \times 10^3)^2 \times 150 \times 10^{-6}}$$
$$= 75 \text{ pF}$$

Therefore, the capacitor range is 75 – 676 Pf

## DIGITAL ELECTRONICS

Digital Electronics is the sub-branch of electronics which deals with digital signals. It is increasingly used in numerous applications ranging from high end processor circuits to miniature circuits for signal processing, communication etc. Digital signals are preferred than analog signals due to their better performance, accuracy, speed, flexibility, and immunity to noise.

### Analog and Digital Signals

There are 2 different types of signals used in Electronics. They are (i) Analog signals and (ii) Digital signals. An analog signal is a continuously varying voltage or current with respect to time. Such signals have been employed in rectifying circuits and transistor amplifier circuits.

Digital signals are signals which contain only discreate values of voltages. Digital signals need two states: switch ON and OFF. ON is considered as one state and OFF is considered as the other state. It can also be defined as high (ON) or low (OFF) state, closed (ON) or open (OFF). These high and low states are defined using binary numbers 1or 0 in Boolean Algebra. The state 1 represents the terms: circuit on, high voltage, a closed switch. Similarly a 0 state represents circuit off, low voltage or an open switch.

### Positive and Negative Logic

In digital systems, there exists two voltage levels: 5V (high) and 0V (low). In a positive logic system; a binary 1 stands for 5V and 0 stands for 0V while in negative logic system, 1 state for 0V and 0 state for 5V as shown in Figure 9.40.

### Logic gates

A logic gate is an electronic circuit which functions based on digital signals. The logic gates are considered as the basic building blocks of most of the digital systems. It has one output with one or more inputs. There are three types of basic logic gates: AND, OR, and NOT. The other logic gates are Ex-OR, NAND, and NOR. They can be constructed from the basic logic gates.

Digital electronics deals with logical operations. The variables are called logical variables. The operators like logical addition (+) and logical multiplication

(.) are called logical operators. When the logical operators (+, .) operate on logical variables (A, B), it gives logical constant (Y). The equation that represents this operation is called logical statement.

For example,

Logical operator: +
Logical variable: A, B
Logical constant: Y
Logical Statement: Y = A + B

The possible combinations of inputs and the corresponding output is given in table called truth table. The circuits which perform the basic logical operations such as logical addition, multiplication and inversion are discussed below.

**AND gate**

**Circuit symbol**

The circuit symbol of a two input AND gate is shown in Figure 9.41(a). A and B are inputs and Y is the output. It is a logic gate and hence A, B, and Y can have the value of either 1 or 0.

**Boolean equation:**

$$Y = A.B$$

It performs logical multiplication and is different from arithmetic multiplication.

**Logic operation**

The output of AND gate is high (1) only when all the inputs are high (1). The rest of the cases the output is low. Hence the output of AND gate is high (1) only when all the inputs are high. It is represented in the truth table (Figure 9.41(b)).

**OR gate**

**Circuit Symbol**

The circuit symbol of a two input OR gate is shown in Figure 9.42(a). A and B are inputs and Y is the output.

**Boolean equation:**

$Y = A + B$

It performs logical addition and is different from arithmetic addition.

**Logic operation**

The output of OR gate is high (logic 1 state) when either of the inputs or both are high. The truth table of OR gate is shown in Figure 9.42(b).

**NOT gate**

**Circuit symbol**

The circuit symbol of NOT gate is shown in Figure 9.43(a). A is the input and Y is the output.

**Boolean equation**

$$Y = \overline{A}$$

**Logic operation**

The output is the complement of the input. It is represented with an overbar. It is also called as inverter. The truth table infers that the output Y is 1 when input A is 0 and vice versa. The truth table of NOT is shown in Figure 9.43(b).

**NAND gate**

The circuit symbol of NAND gate is shown in Figure 9.44(a). A and B are inputs and Y is the output.

**Boolean equation**

$Y = \overline{A.B}$

### Logic operation

The output Y equals the complement of AND operation. The circuit is an AND gate followed by a NOT gate. Therefore, it is summarized as NAND. The output is at logic zero only when all the inputs are high. The rest of the cases, the output is high (Logic 1 state). The truth table of NAND gate is shown in Figure 9.44(b).

## NOR gate

### Circuit symbol

The circuit symbol of NOR gate is shown in Figure 9.45(a). A and B are inputs and Y is the output.

### Boolean equation

$$Y = \overline{A + B}$$

### Logic operation

Y equals the complement of OR operation (A OR B). The circuit is an OR gate followed by a NOT gate and is summarized as NOR.

The output is high when all the inputs are low. The output is low for all other combinations of inputs. The truth table of NOR gate is shown in Figure 9.45(b).

## Ex-OR gate

### Circuit symbol

The circuit symbol of Ex-OR gate is shown in Figure 9.46(a). A and B are inputs and Y is the output. The Ex-OR operation is denoted as $\oplus$.

### Boolean equation

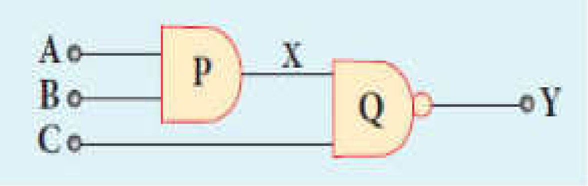
$$Y = A.\overline{B} + \overline{A}.B$$
$$Y = A \oplus B$$

## Logic operation

The output is high only when either of the two inputs is high. In the case of an Ex-OR gate with more than two inputs, the output will be high when odd number of inputs are high. The truth table of Ex-OR gate is shown in Figure 9.46(b).

## EXAMPLE

What is the output Y in the following circuit, when all the three inputs A, B, and C are first 0 and then 1?



## Solution

| A | B | C | X = A.B | $Y = \overline{X \cdot C}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

## EXAMPLE

In the combination of the following gates, write the Boolean equation for output Y in terms of inputs A and B.

## Solution

The output at the 1st AND gate: $A\overline{B}$

The output at the 2nd AND gate: $\overline{A}B$

The output at the OR gate: $Y = A.\overline{B} + \overline{A}.B$

## BOOLEAN ALGEBRA

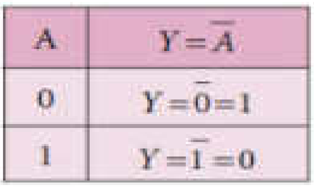
Boolean Algebra is basically a choice between two options (i) yes or no (ii) high or low. These two options in Boolean algebra are represented by binary numbers 0 or 1. It is a concept that relates logic and mathematics which is a century old, made up by George Boole in 1854. Later the importance of Boolean algebra was realized in the design of computer circuits. Today we are in a digital world and most of the comforts that we experience is due to digitization with the foundation based on Boolean algebra.

**Do You Know:** The concept of high (1) and low (0) is not a new one. In fact, it was applied in telephone switching circuits by Shannon in 1938.

### Laws of Boolean Algebra

The NOT, OR and AND operations discussed in 9.5.2 are the Boolean operations. The results of these operations can be summarised as:

### Complement law



| A | $Y=\overline{A}$ |
|---|---|
| 0 | $Y=\overline{0}=1$ |
| 1 | $Y=\overline{1}=0$ |

The complement law can be realised as $\overline{\overline{A}}=A$

### OR laws

| A | B | Y = A+B |
|---|---|---|
| 0 | 0 | Y = 0+0= 0 |
| 0 | 1 | Y = 0+1= 1 |
| 1 | 0 | Y = 1+0 =1 |
| 1 | 1 | Y = 1+1 =1 |

The OR laws can be realised as

| | |
|---|---|
| 1ST Law | A+0= A |
| 2nd Law | A+1=1 |
| 3rd Law | A+A=A |
| 4th Law | A+$\overline{A}$ =1 |

**AND laws**

| A | B | Y = A.B |
|---|---|---|
| 0 | 0 | Y = 0.0= 0 |
| 0 | 1 | Y = 0.1= 1 |
| 1 | 0 | Y = 1.0 =1 |
| 1 | 1 | Y = 1.1 =1 |

The AND laws can be realised as

| | |
|---|---|
| 1ST Law | A.0= A |
| 2nd Law | A.1=A |
| 3rd Law | A.A=A |
| 4th Law | A+$\overline{A}$ =1 |

The Boolean operations obey the following laws.

**Commutative laws**

$$A + B = B + A$$
$$A . B = B . A$$

**Associative laws**

$$A + (B + C) = (A + B) + C$$
$$A . (B . C) = (A .B) . C$$

**Distributive laws**

$$A(B + C) = AB + AC$$
$$A + BC = (A + B)(A + C)$$

The above laws are used to simplify complicated expressions and to simplify the logic circuitry.

## DE MORGAN'S THEOREM

### De Morgan's First Theorem

The first theorem states that the complement of the sum of two logical inputs is equal to the product of its complements.

**Proof**

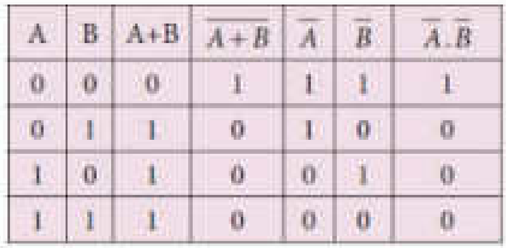
The Boolean equation for NOR gate is

$Y = \overline{A + B}$

The Boolean equation for a bubbled

AND gate is $Y = \overline{A} . \overline{B}$

Both cases generate same outputs for same inputs. It can be verified using the following truth table.



| A | B | A+B | $\overline{A+B}$ | $\overline{A}$ | $\overline{B}$ | $\overline{A}.\overline{B}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

From the above truth table, we can conclude

$\overline{A + B} = \overline{A} \cdot \overline{B}$.

Thus De Morgan's First Theorem is proved. It also says that a NOR gate is equal to a bubbled AND gate.

The corresponding logic circuit diagram is shown in Figure 9.47.

## De Morgan’s Second Theorem

The second theorem states that the complement of the product of two inputs is equal to the sum of its complements.

**Proof**

The Boolean equation for NAND gate is

$$Y = \overline{AB}$$

The Boolean equation for bubbled OR gate is

$$Y = \overline{A} + \overline{B}$$

A and B are the inputs and Y is the output. The above two equations produces the same output for the same inputs. It can be verified by using the truth table

| A | B | A.B | $\overline{A.B}$ | $\overline{A}$ | $\overline{B}$ | $\overline{A}+\overline{B}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

From the above truth table we can conclude

$$\overline{A.B} = \overline{A} + \overline{B}$$

Thus De Morgan’s First Theorem is proved. It also says, a NAND gate is equal to a bubbled OR gate.

The corresponding logic circuit diagram is shown in Figure 9.48

## EXAMPLE

Simplify the Boolean identity

AC + ABC = AC

**Solution**

Step 1: AC (1 + B) = AC.1 [OR law-2]
Step 2: AC . 1 = AC [AND law – 2]

Therefore, AC + ABC = AC

Thus the given statement is proved.

## Integrated Chips

An integrated circuit is also referred as an IC or a chip or a microchip (Figure 9.49). It consists of thousands to millions of transistors, resistors, capacitors, etc. integrated on a small flat piece of semiconductor material that is normally Silicon.

Integrated circuits (ICs) are the keystone of modern electronics. With the advancement in technology and the emergence of Very Large Scale Integration (VLSI) era it is possible to fit more and more transistors on chips of same piece.
ICs have two main advantages over ordinary circuits: cost and performance. The size, speed, and capacity of chips have progressed enormously with the advancement in technology. Computers, mobile phones, and other digital home appliances are now made possible by the small size and low cost of ICs. ICs can function as an amplifier, oscillator, timer, microprocessor and computer memory.

These extremely small ICs can perform calculations and store data using either digital or analog technology. Digital ICs use logic gates, which work only with values of ones and zeros. A low signal sent to a component on a digital IC will result in a value of 0, while a high signal creates a value of 1.

**Digital ICs** usually find their applications in computers, networking equipment, and most consumer electronics. **Analog ICs** or linear ICs work with continuous values. This means a component on a linear IC can take any value and output another value. Linear ICs are typically used in audio and radio frequency amplification.

## SUMMARY

- Energy bands in solids are used to classify them into metals, insulators, and semiconductors

- In a N-type semiconductor, electrons are the majority charge carriers and holes are the minority carriers

- In P-type semiconductor, holes are the majority charge carriers and electrons are the minority charge carriers

- A depletion region is formed in an unbiased PN junction. It is devoid of mobile charge carriers. Instead, it has immobile ions

- When a PN junction diode is forward biased, the depletion region decreases and the diode conducts once after the barrier potential is crossed. It acts like a closed switch.

- A PN junction diode in reverse biased condition functions as a open switch as it does not conduct. The depletion region increases.

- A forward biased PN junction diode functions as a rectifier. Rectification is the process of converting an AC current into DC current

- The half wave rectifier rectifies one half of the input signal and produces a pulsating output.

- Full wave rectifier rectifies both halves of the input signal.

- The efficiency of the full wave rectifier is two times the efficiency of the half wave rectifier

- The two mechanisms that is responsible for breakdown under increasing reverse voltage: Zener and Avalanche breakdown

- Zener breakdown happens in a heavily doped PN junction diode when a strong electric field is applied.

- Avalanche breakdown occurs in lightly doped junctions which have wide depletion layers. It is due to the breaking of covalent bonds by the thermally generated minority charge carriers.

- Zener diode is a heavily doped PN junction diode works in the reverse biased direction

- Light emitting diode is a forward biased semiconductor device that emits emits visible or invisible light when energized. The recombination of minority charge carriers with the majority charge carriers in the respective regions release energy in the form of Photons.

- A PN junction diode made of photosensitive material converts an optical signal into electric signal is called a photodiode.

- When a photon of sufficient energy strikes the diode, it creates an electron-hole pair. These electrons and holes are swept across the p-n junction by the electric field created by reverse voltage before recombination takes place and in turn generates photo current.

- A solar cell is an electrical device that converts the energy of light directly into electricity by the photovoltaic effect.
- A bipolar junction transistor is a semiconductor device is of two types: NPN and PNP.

- BJT has three regions: emitter, base, and collector

- To operate the transistor in the active region, emitter base must be forward biased and collector base must be reverse biased.

- A BJT can be operated in three different configurations: Common base, common emitter, common collector.

- The forward current gain in common base configuration α gives the ratio of the collector current to emitter current.

- The forward current gain in common emitter configuration β gives the ratio of the collector current to the base current

- The BJT connected in common emitter configuration functions as a switch

- The BJT connected in common emitter configuration can be used as an amplifier. There exists a phase reversal of 1800 between the input signal and the amplified output signal.

- A transistor amplifier combined with a tank circuit and positive feedback acts as an oscillator

- The logic gates are logical circuits provides output only for a combination of inputs.

- The basic logic gates are AND, OR, and NOT gates.

- Boolean algebra is used to simplify complicated expressions and hence to simplify the logic circuit.

- De Morgan's First theorem states that the complement of the sum of two inputs is equal to the product of its complements.

- The second theorem states that the complement of the product of two inputs is equal to the sum of its complements.

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