Electricity
TEST -8

12th Vol- I	Unit - 1	Electristatics
	Unit - 2	Current Electricity

12TH PHYSICS
UNIT – 1 -Electrostatics

Electricity is really just organized lightning
– George Carlin

In this unit, student is exposed to

- Historical background of electricity and magnetism
- The role of electrostatic force in day – to-day life
- Coulomb's law and superposition principle
- The concept of electric field
- Calculation of electric field for various charge configurations
- Electrostatic potential and electrostatic potential energy
- Electric dipole and dipole moment
- Electric field and electrostatic potential for a dipole
- Electric flux
- Gauss law and its various applications
- Electrostatic properties of conductors and dielectrics
- Polarisation
- Capacitors in series and parallel combinations
- Effect of a dielectric in a capacitor
- Distribution of charges in conductors, corona discharge
- Working of a Van de Graff generator

INTRODUCTION

Electromagnetism is one of the most important branches of physics. The technological developments of the modern 21st century are primarily due to our understanding of electromagnetism. The forces we experience in everyday life are electromagnetic in nature except gravity.

In standard XI, we studied about the gravitational force, tension, friction, normal force etc. Newton treated them to be independent of each other with each force being a separate natural force. But what is the origin of all these forces? It is now understood that except gravity, all forces which we experience in every day life (tension in the string, normal force from the surface, friction etc.) arise from electromagnetic forces with in the atoms. Some examples are

(i) When an object is pushed, the atoms in our hand interact with the atoms in the object and this interaction is basically electromagnetic in nature.

(ii) When we stand on Earth's surface, the gravitational force on us acts downwards and the normal force acts upward to counter balance the gravitational force. What is the origin of this normal force?

It arises due to the electromagnetic interaction of atoms on the surface of the Earth with the atoms present in the feet of the person. Though, we are attracted by the gravitational force of the Earth, we stand on Earth only because of electromagnetic force of atoms.

(iii) When an object is moved on a surface, static friction resists the motion of the object. This static friction arises due to electromagnetic interaction between the atoms present in the object and atoms on the surface. Kinetic friction also has similar origin.

From these examples, it is clear that understanding electromagnetism is very essential to understand the universe in a holistic manner. The basic principles of electromagnetism are dealt with in volume 1 at XII standard physics. This unit deals with the behaviour and other related phenomena of charges at rest. This branch of electricity which deals with stationary charges is called Electrostatics

Historical background of electric charges

Two millenniums ago, Greeks noticed that amber (a solid, translucent material formed from the resin of a fossilized tree)after rubbing with animal fur attracted small pieces of leaves and dust. The amber possessing this property is said to be 'charged'. It was initially thought that amber has this special property. Later people found that not only amber but even a glass rod rubbed with silk cloth, attracts pieces of papers. So glass rod also becomes 'charged' when rubbed with a suitable material.

Consider a charged rubber rod hanging from a thread as shown in Figure 1.1. Suppose another charged rubber rod is brought near the first rubber rod; the rods repel each other. Now if we bring a charged glass rod close to the charged rubber rod, they attract each other. At the same time, if a charged glass rod is brought near another charged glass rod, both the rods repel each other.

From these observations, the following inferences are made

(i) The charging of rubber rod and that of glass rod are different from one another.

(ii) The charged rubber rod repels another charged rubber rod, which implies that 'like charges repel each other'. We can also arrive at the same inference by observing that a charged glass rod repels another charged glass rod.

(iii) The charged rubber rod attracts the charged glass rod, implying that the charge in the glass rod is not the same kind of charge present in the rubber. Thus unlike charges attract each other.

Therefore, two kinds of charges exist in the universe. In the 18^{th} century, Benjamin Franklin called one type of charge as positive (+) and another type of charge as negative (–). Based on Franklin's convention, rubber and amber rods are negatively charged while the glass rod is positively charged. If the net charge is

zero in the object, it is said to be electrically neutral.

Following the pioneering work of J. J. Thomson and E. Rutherford, in the late 19^{th} century and in the beginning of 20^{th} century, we now understand that the atom is electrically neutral and is made up of the negatively charged electrons, positively charged protons, and neutrons which have zero charge. The material objects made up of atoms are neutral in general. When an object is rubbed with another object (for example rubber with silk cloth), some amount of charge is transferred from one object to another due to the friction between them and the object is then said to be electrically charged. Charging the objects through rubbing is called triboelectric charging.

Basic properties of charges

(i) Electric charge

Most objects in the universe are made up of atoms, which in turn are made up of protons, neutrons and electrons. These particles have mass, an inherent property of particles. Similarly, the electric chargeis another intrinsic and fundamental property of particles. The nature of charges is understood through various experiments performed in the 19^{th} and 20^{th} century. The SI unit of charge is coulomb.

(ii) Conservation of charges

Benjamin Franklin argued that when one object is rubbed with another object, charges get transferred from one to the other. Before rubbing, both objects are electrically neutral and rubbing simply transfers the charges from one object to the other. (For example, when a glass rod is rubbed against silk cloth, some negative charge are transferred from glass to silk. As a result, the glass rod is positively charged and silk cloth becomes negatively charged).

From these observations, he concluded that charges are neither created or nor destroyed but can only be transferred from one object to other. This is called conservation of total charges and is one of the fundamental conservation laws in physics. It is stated more generally in the following way.

The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physicalprocess, the net change in charge will always be zero.

(iii) Quantisation of charges

What is the smallest amount of charge that can be found in nature? Experiments show that the charge on an electron is - e and the charge on the proton is + e. Here, e denotes the fundamental unit of charge. The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.

$$q = ne$$

Here n is any integer (0, ±1, ±2, ±3, ±4…). This is called quantisation of electric charge.

Robert Millikan in his famous experiment found that the value of $e = 1.6 \times 10^{-19}$ C. The charge of an electron is -1.6×10^{-19} C and the charge of the proton is $+1.6 \times 10^{-19}$ C.When a glass rod is rubbed with silk cloth, the number of charges transferred is usually very large, typically of the order of 1010. So the charge quantisation is not appreciable at the macroscopic level. Hence the charges are treated to be continuous (not discrete). But at the microscopic level, quantisation of charge plays a vital role.

EXAMPLE

Calculate the number of electrons in one coulomb of negative charge.

Solution

According to the quantisation of charge,

$$q = ne$$

Here q = 1C. So the number of electrons in 1 coulomb of charge is

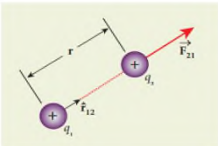
$$n = \frac{q}{e} = \frac{1C}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \text{ electrons}$$

COULOMB'S LAW

In the year 1786, Coulomb deduced the expression for the force between two stationary point charges in vacuum or free space. Consider two point charges *q*1 and *q*2 at rest in vacuum, and separated by a distance of *r* as shown in Figure 1.2. According to Coulomb, the force on the point charge *q*2 exerted by another point charge *q*1 is

$$\vec{F}_{21} = K\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

where $\hat{r}_{12}$ is the unit vector directed from charge *q*1 to charge *q*2 and *k* is the proportionality constant.



Coulomb force between two positive point charges

Important aspects of Coulomb's law

(i) Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.

(ii) The force on the charge q2 exerted by the charge q1 always lies along the line joining the two charges. $\hat{r}_{12}$ is the unit vector pointing from charge q1 to q2 .It is shown in the Figure 1.2. Likewise, the force on the charge q1 exerted by q2 is along $-\hat{r}_{12}$ (i.e., in the direction opposite to $\hat{r}_{12}$).

(iii) In SI units, $K = \frac{1}{4\pi\epsilon_0}$ and its value is 9×10^9 N m^2 C^{-2}. Here ϵ_0 is the permittivity of free space or vacuum and its value is

$$\epsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$$

(iv) The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:

$$|F| = \frac{9 \times 10^9 \times 1 \times 1}{1^2} = 9 \times 10^9 N$$

This is a huge quantity, almost equivalent to the weight of one million ton. We never come across 1 coulomb of charge in practice. Most of the electrical phenomena in day-to-day life involve electrical charges of the order of μC (micro coulomb) or nC (nano coulomb).

(v) In SI units, Coulomb's law in vacuum takes the form $\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12}$

In a medium of permittivity e, the force between two point charges is given by $\vec{F}_{21} = \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}\hat{r}_{12}$ Since $\epsilon > \epsilon_0$, the force between two point charges in a medium other than vacuum is always less than that in vacuum. We define the relative permittivity for a given medium as $\epsilon_r = \frac{\epsilon}{\epsilon_0}$ For vacuum or air, $\epsilon_r = 1$ and for all other media $\epsilon_r > 1$.

(vi) Coulomb's law has same structure as Newton's law of gravitation. Both are inversely proportional to the square of the distance between the particles. The electrostatic force is directly proportional to the product of the magnitude of two point charges and gravitational force is directly proportional to the product of two masses. But there are some important differences between these two laws.]

- The gravitational force between two masses is always attractive but Coulomb force between two charges can be attractive or repulsive, depending on the nature of charges.

- The value of the gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}. The value of the constant k in Coulomb law is $k = 9 \times 10^9$ N m^2 C^{-2}. Since k is much more greater than G, the electrostatic force is always greater in magnitude than gravitational force for smaller size objects.

- The gravitational force between two masses is independent of the medium. For example, if 1 kg of two masses are kept in air or inside water, the gravitational force between two masses remains the same. But the electrostatic force between the two charges depends on nature of the medium in which the two charges are kept at rest.

(vii) The force on a charge q_1 exerted by a point charge q_2 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{21}$$

Here $\hat{r}_{21}$ is the unit vector from charge q_2 to q_1. But $\hat{r}_{21} = -\hat{r}_{12}$

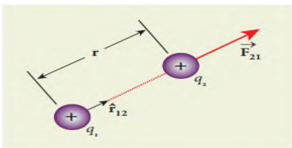
$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r^2}(-\hat{r}_{12}) = -\frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r^2}(\hat{r}_{12}) \quad \text{(or)} \quad \vec{F}_{12} = -\vec{F}_{21}$$

Therefore, the electrostatic force obeys Newton's third law.

(viii) The expression for Coulomb force is true only for point charges. But the point charge is an ideal concept. However we can apply Coulomb's law for two charged objects whose sizes are very much smaller than the distance between them. In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges. The distance between the two charged spheres is much greater than the radii of the spheres.

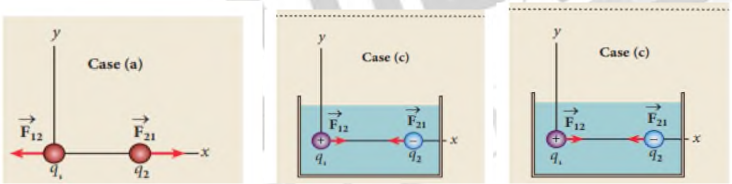
EXAMPLE

Consider two point charges q_1 and q_2 at rest as shown in the figure.



They are separated by a distance of 1m. Calculate the force experienced by the two charges for the following cases:

(a) $q_1 = +2\ \mu C$ and $q_2 = +3\ \mu C$
(b) $q_1 = +2\ \mu C$ and $q_2 = -3\ \mu C$
(c) $q_1 = +2\ \mu C$ and $q_2 = -3\ \mu C$ kept in water ($\varepsilon_r = 80$)



(a) $q_1 = +2\ \mu C$, $q_2 = +3\ \mu C$, and r = 1m. Both are positive charges. so the force will be repulsive.
Force experienced by the charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Here $\hat{r}_{12}$ is the unit vector from q_1 to q_2. Since q_2 is located on the right of q_1, we have

$$\hat{r}_{12} = \hat{i} \text{ and } \frac{1}{4\pi\varepsilon_o} = 9\times 10^9$$

$$\vec{F}_{21} = \frac{9\times 10^9 \times 2\times 10^{-6}\times 3\times 10^{-6}}{1^2}\hat{i}$$

$$= 54\times 10^{-3}\,\hat{i}N$$

According to Newton’s third law, the force experienced by the charge q_1 due to q_2 is $\vec{F}_{12} = -\vec{F}_{21}$ Therefore,

$$\vec{F}_{12} = -54 \times 10^{-3}\,\hat{i}N$$

The directions of $\vec{F}_{21}$ and $\vec{F}_{12}$ are shown in the above figure in case (a)

(b) q_1 = +2 μC, q_2 = –3 μC, and r = 1m. They are unlike charges. So the force will be attractive.
Force experienced by the charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{9 \times 10^{9} \times \left(2 \times 10^{-6}\right) \times \left(-3 \times 10^{-6}\right)}{1^2}\hat{r}_{12}$$

$$= -54 \times 10^{-3} N\,\hat{i} \text{ (Using } \hat{r}_{12} = \hat{i}\text{)}$$

The charge q_2 will experience an attractive force towards q_1 which is in the negative x direction.
According to Newton’s third law, the force experienced by the charge q_1 due to q_2 is $\vec{F}_{12} = -\vec{F}_{21}$ Therefore,

$$\vec{F}_{12} = -54 \times 10^{-3}\,\hat{i}N$$

The directions of $\vec{F}_{21}$ and $\vec{F}_{12}$ are shown in the figure (case (b)).
(c) If these two charges are kept inside the water, then the force experienced by q_2 due to q_1

$$\vec{F}_{21}^{\,W} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Since $\varepsilon = \varepsilon_r \varepsilon_o$

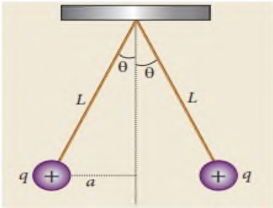
We have $\vec{F}_{21}^{\,W} = \dfrac{1}{4\pi\varepsilon_r\varepsilon_o}\dfrac{q_1 q_2}{r^2}\hat{r}_{12} = \dfrac{\vec{F}_{21}}{\varepsilon_r}$

Therefore,

$$\vec{F}_{21}^{\,W} = -\frac{54 \times 10^{-3} N}{80}\hat{i} = -0.675 \times 10^{-3} N\hat{i}$$

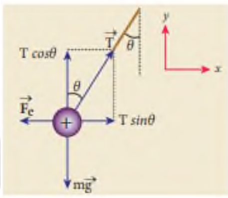
EXAMPLE

Two small-sized identical equally charged spheres, each having mass 1 g are hanging in equilibrium as shown in the figure. The length of each string is 10 cm and the angle θ is 30° with the vertical. Calculate the magnitude of the charge in each sphere.



Solution

If the two spheres are neutral, the angle between them will be 0^{o} when hanged vertically. Since they are positively charged spheres, there will be a repulsive force between them and they will be at equilibrium with each other at an angle of 30° with the vertical. At equilibrium, each charge experiences zero net force in each direction. We can draw a free body diagram for one of the charged spheres and apply Newton's second law for both vertical and horizontal directions.



The free body diagram

In the *x*-direction, the acceleration of the charged sphere is zero.

Using Newton's second law $\left(\vec{F}_{tot} = m\vec{a}\right)$ we have

$$T\sin\theta\,\hat{i} - F_e\,\hat{i} = 0$$
$$T\sin\theta = F_e$$

Here *T* is the tension acting on the charge due to the string and F_e is the electrostatic force between the two charges.

In the *y*-direction also, the net acceleration experienced by the charge is zero.

$$T\cos\theta\,\hat{j} - mg\,\hat{j} = 0$$
$$T\cos\theta = mg$$

By dividing equation (1) by equation

$$\tan\theta = \frac{F_e}{mg}$$

Since they are equally charged, the magnitude of the electrostatic force is

$$F_e = k\frac{q^2}{r^2} \; where \; k = \frac{1}{4\pi\varepsilon_0}$$

Here r = 2a = 2Lsinθ. By substituting these values in equation (3),

$$\tan\theta = k\frac{q^2}{mg\left(2L\sin\theta\right)^2}$$

Rearranging the equation (4) to get q

$$q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2\times 0.1\times \sin 30^o \times \sqrt{\frac{10^{-3}\times 10\times \tan 30^o}{9\times 10^9}}$$

$$q = 8.01\times 10^{-8} C = 80.1\, nC$$

EXAMPLE

Calculate the electrostatic force and gravitational force between the proton and the electron in a hydrogen atom. They are separated by a distance of 5.3×10^{-11} m. The magnitude of charges on the electron and proton are 1.6×10^{-19} C. Mass of the electron is $m_e = 9.1 \times 10^{-31}$ kg and mass of proton is $m_p = 1.6 \times 10^{-27}$ kg.

Solution

The proton and the electron attract each other. The magnitude of the electrostatic force between these two particles is given by

$$F_e = \frac{ke^2}{r^2} = \frac{9\times 10^9 \times \left(1.6\times 10^{-19}\right)^2}{\left(5.3\times 10^{-11}\right)^2}$$

$$= \frac{9\times 2.56}{28.09}\times 10^{-7} = 8.2\times 10^{-8} N$$

The gravitational force between the proton and the electron is attractive. The magnitude of the gravitational force between these particles is

$$F_G = \frac{G m_e m_p}{r^2}$$
$$= \frac{6.67\times 10^{-11}\times 9.1\times 10^{-31}\times 1.6\times 10^{-27}}{\left(5.3\times 10^{-11}\right)}$$
$$= \frac{97.11}{28.09}\times 10^{-47} = 3.4\times 10^{-47}\,N$$

The ratio of the two forces

$$\frac{F_e}{F_G} = \frac{8.2\times 10^{-8}}{3.4\times 10^{-47}} = 2.41\times 10^{39}$$

Note that $F_e \approx 10^{39} F_G$

The electrostatic force between a proton and an electron is enormously greater than the gravitational force between them. Thus the gravitational force is negligible when compared with the electrostatic force in many situations such as for small size objects and in the atomic domain. This is the reason why a charged comb attracts an uncharged piece of paper with greater force even though the piece of paper is attracted downward by the Earth.

Superposition principle

Coulomb’s law explains the interaction between two point charges. If there are more than two charges, the force on one charge due to all the other charges needs to be calculated. Coulomb’s law alone does not give the answer. The superposition principle explains the interaction between multiple charges.

According to this superposition principle, the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

Consider a system of *n* charges, namely q_1, q_2, q_3 ….q_n. The force on $q1$ exerted by the charge q_2

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

where $\hat{r}_{21}$ is the unit vector from q_2 to q_1 along the line joining the two charges and $r_{21}$is the distance between the charges $q_1$and q_2. The electrostatic force between two charges is not affected by the presence of other charges in the neighbourhood.

The force on q_1 exerted by the charge q_3 is

$$\vec{F}_{12} = k\frac{q_1 q_3}{r_{31}^2}\hat{r}_{31}$$

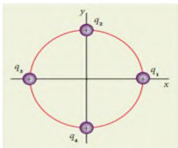
By continuing this, the total force acting on the charge q_1 due to all other charges is given by

$$\vec{F}_1^{\,tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots\dots\dots \vec{F}_{18}$$

$$\vec{F}_1^{\,tot} = k\left\{\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \frac{q_1 q_4}{r_{41}^2}\hat{r}_{41}\dots\dots + \frac{q_1 q_n}{r_{n1}^2}\hat{F}_{n1}\right\}$$

EXAMPLE

Consider four equal charges q_1, q_2, q_3 and q_4 = q= +1 µC located at four different points on a circle of radius 1m, as shown in the figure. Calculate the total force acting on the charge q_1 due to all the other charges.

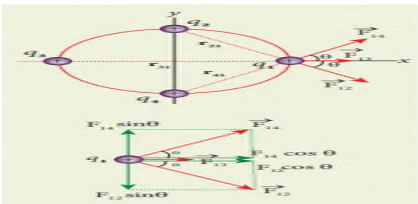


Solution

According to the superposition principle, the total electrostatic force on charge q_1 is the vector sum of the forces due to the other charges,

$$\vec{F}_1^{\,tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

The following diagram shows the direction of each force on the charge q_1.



The charges q_2 and q_4 are equi-distant from q_1. As a result the strengths (magnitude) of the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are the same even though their directions are different. Therefore the vectors representing these two forces are drawn with equal lengths. But the charge q_3 is located farther compared to q_2 and q_4. Since the strength of the electrostatic force decreases as distance increases, the strength of the force $\vec{F}_{13}$ is lesser than that of forces $\vec{F}_{12}$ and $\vec{F}_{14}$. Hence the vector representing the force $\vec{F}_{13}$ is drawn with smaller length compared to that for forces $\vec{F}_{12}$ and $\vec{F}_{14}$.

From the figure, $r_{21} = \sqrt{2}m = r_{41} and\ r_{31} = 2m$

The magnitudes of the forces are given by

$$F_{12} = \frac{kq^2}{{r_{31}}^2} = \frac{9\times 10^9 \times 10^{-12}}{4}$$

$$F_{13} = 2.25\times 10^{-13} N$$

$$F_{13} = \frac{kq^2}{{r_{21}}^2} = F_{14} = \frac{9\times 10^9\times 10^{-12}}{2}$$

$$= 4.5\times 10^{-3} N$$

From the figure, the angle $\theta = 45^{o}$. In terms of the components, we have

$$\vec{F}_{12} = F_{12}\cos\theta\hat{i} - F_{12}\sin\theta\hat{j}$$

$$= 4.5\times 10^{-3}\times \frac{1}{\sqrt{2}}\hat{i} - 4.5\times 10^{-3}\times \frac{1}{\sqrt{2}}\hat{j}$$

$$\vec{F}_{13} = F_{13}\hat{i} = 2.25\times 10^{-3} N\hat{i}$$

$$\vec{F}_{14} = F_{14}\cos\theta\hat{i} + F_{14}\sin\theta\hat{j}$$

$$= 4.5\times 10^{-3}\times \frac{1}{\sqrt{2}}\hat{i} + 4.5\times 10^{3}\times \frac{1}{\sqrt{2}}\hat{j}$$

Then the total force on q_1 is

$$\vec{F}_1^{\,tot} = \left(F_{12}\cos\theta\hat{i} - F_{12}\sin\theta\hat{j}\right) + F_{13}\hat{i}$$
$$+ \left(F_{14}\cos\theta\hat{i} + F_{14}\sin\theta\hat{j}\right)$$

$$\vec{F}_1^{\,tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i}$$
$$+ \left(-F_{12}\sin\theta + F_{14}\sin\theta\right)\hat{j}$$

Since $F_{12} = F_{14}$, the j^{th} component is zero. Hence we have

$$\vec{F}_1^{\,tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i}$$

Substituting the values in the above equation,

$$= \left(\frac{4.5}{\sqrt{2}} + 2.25\frac{4.5}{\sqrt{2}}\right)\times 10^{-3}\hat{i}$$

$$= \left(4.5\sqrt{2} + 2.25\right)\times 10^{-3}\hat{i}$$

$$\vec{F}_1^{\,tot} = 8.61\times 10^{-3}\hat{i} N$$

The resultant force is along the positive *x* axis.

ELECTRIC FIELD AND ELECTRIC FIELD LINES

Electric Field

The interaction between two charges is determined by Coulomb's law. How does the interaction itself occur? Consider a point charge kept at a point in space. If another point charge is placed at some distance from the first point charge, it experiences either an attractive force or repulsive force. This is called 'action at a distance'. But how does the second charge know about existence of the first charge which is located at some distance away from it? To answer this question, Michael Faraday introduced the concept of field.

According to Faraday, every charge in the universe creates an electric field in the surrounding space, and if another charge is brought into its field, it will interact with the electric field at that point and will experience a force. It may be recalled that the interaction of two masses is similarly explained using the concept of gravitational field (Refer unit 6, volume 2, XI physics). Both the electric and gravitational forces are non-contact forces, hence the field concept is required to explain action at a distance.

Consider a source point charge *q* located at a point in space. Another point charge q_o (test charge) is placed at some point P which is at a distance *r* from the charge q. The electrostatic force experienced by the charge q_o due to qis given by Coulomb's law.

$$\vec{F} = \frac{kqp_0}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\hat{r} \quad where\ k = \frac{1}{4\pi\epsilon_0}$$

The charge q creates an electric field in the surrounding space within which its effect can be felt by another charge. It is measured in
terms of a quantity called electric field intensity or simply called electric field $\vec{E}$.The electric field at the point P at a distance *r* from the point charge *q* is defined as the force that would be experienced by a unit positive charge placed at that point P and is given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Here $\hat{r}$ is the unit vector pointing from *q* to the point of interest P. The electric field is a vector quantity and its SI unit is newton per coulomb (NC^{-1}).

Important aspects of Electric field

(i) If the charge *q* is positive then the electric field points away from the source charge and if *q* is negative, the electric field points towards the source charge *q*.

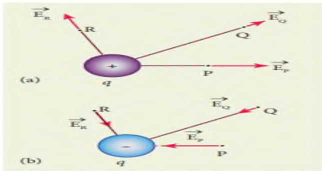
(ii) If the electric field at a point P is $\vec{E}$,then the force experienced by the test charge q_o placed at the point P is

$$\vec{F} = q_0 \vec{E}$$

(iii) The equation (1.4) implies that the electric field is independent of the test charge q_0 and it depends only on the source charge q.

(iv) Since the electric field is a vector quantity, at every point in space, this field has unique direction and magnitude as shown in Figures 1.6(a) and (b). From equation (1.4), we can infer that as distance increases, the electric field decreases in magnitude.

Note that in Figures 1.6 (a) and (b) the length of the electric field vector is shown for three different points. The strength or magnitude of the electric field at point P is stronger than at the points Q and R because the point P is closer to the source charge.



(a) Electric field due to positive charge (b) Electric field due to negative charge

(v) In the definition of electric field, it is assumed that the test charge q_0 is taken sufficiently small, so that bringing this test charge will not move the source charge. In other words, the test charge is made sufficiently small such that it will not modify the electric field of the source charge.

(vi) The expression (1.4) is valid only for point charges. For continuous and finite size charge distributions, integration techniques must be used (Refer Appendix A1.1). However, this expression can be used as an approximation for a finite-sized charge if the test point is very far away from the finite sized source charge. Note that we similarly treat the Earth as a point mass when we calculate the gravitational field of the Sun on the Earth (Refer unit 6, volume 2, XI physics).

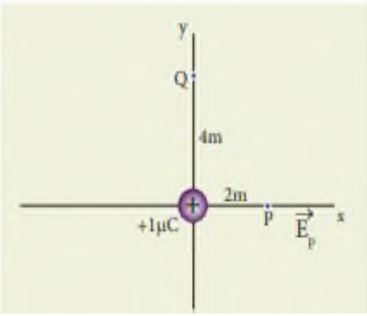
(vii) There are two kinds of the electric field: uniform (constant) electric field and non-uniform electric field. Uniform electric field will have the same direction and constant magnitude at all points in space. Non-uniform electric field will have different directions or different magnitudes or both at different points in space. The electric field created by a point charge is basically a non uniform electric field. This non-uniformity arises, both in direction and magnitude, with the direction being radially outward (or inward) and the magnitude changes as distance increases. These are shown in Figure 1.7.

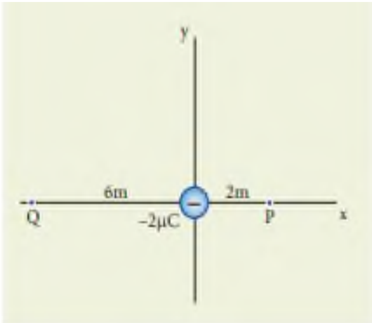
EXAMPLE

Calculate the electric field at points P, Q for the following two cases, as shown in the figure.

(a) A positive point charge +1 μC is placed at the origin

(b) A negative point charge –2 μC is placed at the origin





Solution

Case (a)

The magnitude of the electric field at point P is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{9\times 10^9 \times 1\times 10^{-6}}{4}$$
$$= 2.25\times 10^3 \, NC^{-1}$$

Since the source charge is positive, the electric field points away from the charge. So the electric field at the point P is given by

$$\vec{E}_p = 2.25\times 10^3 \hat{i} \, NC^{-1}$$

For the point Q

$$\left|\vec{E}_Q\right| = \frac{9\times 10^9 \times 1\times 10^{-6}}{16} = 0.56\times 10^3 \, NC^{-1}$$

Hence $\vec{E}_Q = 0.56\times 10^3 \hat{j} \, NC^{-1}$

Case (b)

The magnitude of the electric field at point P

$$\left|\vec{E}_P\right| = \frac{kq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times2\times10^{-6}}{4}$$
$$= 4.5\times10^3\,NC^{-1}$$

Since the source charge is negative, the electric field points towards the charge. So the electric field at the point P is given by

$$\vec{E}_P = -4.5\times10^3\,\hat{i}\,NC^{-1}$$

For the point Q, $$\left|\vec{E}_Q\right| = \frac{9\times10^9\times2\times10^{-6}}{36}$$
$$= 0.5\times10^3\,\hat{i}\,NC^{-1}$$

$$\vec{E}_Q = 0.5\times10^3\,\hat{i}\,NC^{-1}$$

At the point Q the electric field is directed along the positive *x*-axis.

Electric field due to the system of point charges

Suppose a number of point charges are distributed in space. To find the electric field at some point P due to this collection of point charges, superposition principle is used. The electric field at an arbitrary point due to a collection of point charges is simply equal to the vector sum of the electric fields created by the individual point charges. This is called superposition of electric fields.

Consider a collection of point charges $q_1, q_2, q_3 \ldots\ldots q_n$ located at various points in space. The total electric field at some point P due to all these *n* charges is given by

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots + \vec{E}_n$$

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}\left\{\frac{q_1}{r_{1p}^2}\hat{r}_{1p} + \frac{q_2}{r_{2p}^2}\hat{r}_{2p} + \frac{q_2}{r_{3p}^2}\hat{r}_{3p} + \ldots\ldots + \frac{q_n}{r_{np}^2}\hat{r}_{np}\right\}$$

Where $r_{1P}, r_{2P}, r_{3P}, \ldots\ldots\ldots\ldots r_{nP}$ are the distance of the charges $q_1, q_2, q_3, \ldots\ldots\ldots\ldots q_n$ from the point P respectively. Also $\hat{r}_{1P}, \hat{r}_{2P}, \hat{r}_{3P}, \ldots\ldots\ldots\ldots \hat{r}_{nP}$ are the corresponding unit vectors directed from $q_1, q_2, q_3, \ldots\ldots\ldots\ldots q_n$ to P. Equation (1.7) can be re-written as,

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\left(\frac{q_i}{r_{iP}^2}\hat{r}_{iP}\right)$$

For example in Figure 1.8, the resultant electric field due to three point charges q_1, q_2, q_3 at point P is shown.

Note that the relative lengths of the electric field vectors for the charges depend on relative distances of the charges to the point P.

EXAMPLE

Consider the charge configuration as shown in the figure. Calculate the electric field at point A. If an electron is placed at points A, what is the acceleration experienced by this electron? (mass of the electron = 9.1×10^{-31} kg and charge of electron = -1.6×10^{-19} C)

Solution

By using superposition principle, the net electric field at point A is

$$\vec{E}_A = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{1A}^2}\hat{r}_{1A} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{2A}^2}\hat{r}_{2A}$$

where r_{1A} and r_{2A} are the distances of point A from the two charges respectively.

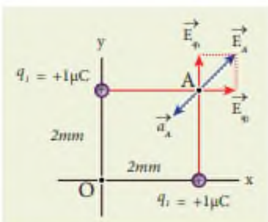
$$\vec{E}_A = \frac{9\times 10^9 \times 2\times 10^{-6}}{\left(2\times 10^{-1}\right)^2}\left(\hat{j}\right) + \frac{9\times 10^9 \times 2\times 10^{-6}}{\left(2\times 10^{-3}\right)^2}\left(\hat{i}\right)$$

$$= 2.25\times 10^9\,\hat{j} + 2.25\times 10^9\,\hat{i} = 2.25\times 10^9\left(\hat{i}+\hat{j}\right)$$

The magnitude of electric field

$$\left|\vec{E}_A\right| = \sqrt{\left(2.25\times 10^9\right)^2 + \left(2.25\times 10^9\right)^2}$$
$$= 2.25\times\sqrt{2}\times 10^9\,NC^{-1}$$

The direction of $\vec{E}_A$ is given by $\frac{\vec{E}_A}{\left|\vec{E}_A\right|} = \frac{2.25\times 10^9\left(\hat{i}+\hat{j}\right)}{2.25\times\sqrt{2}\times 10^9} = \frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$ which is the unit vector along OA as shown in the figure.



The acceleration experienced by an electron placed at point A is

$$\vec{a}_A = \frac{\vec{F}}{m} = \frac{q\vec{E}_A}{m}$$

$$= \frac{\left(-1.6\times 10^{-19}\right)\times\left(2.25\times 10^{9}\right)\left(\hat{i}+\hat{j}\right)}{9.1\times 10^{-31}}$$

$$= -3.95\times 10^{20}\left(\hat{i}+\hat{j}\right) N kg^{-1}$$

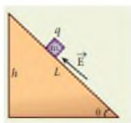
The electron is accelerated in a direction exactly opposite to $\vec{E}_A$.

Electric field due to continuous charge distribution

The electric charge is quantized microscopically. The expressions (1.2), (1.3), (1.4) are applicable to only point charges. While dealing with the electricfield due to a charged sphere or a charged wire etc., it is very difficult to look at individual charges in these charged bodies. Therefore, it is assumed that charge is distributed continuously on the charged bodies and the discrete nature of charges is not considered here. The electric field due to such continuous charge distributions is found by invoking the method of calculus. (For further reading, refer Appendix A1.1).

EXAMPLE 1.8

A block of mass *m* carrying a positive charge *q* is placed on an insulated frictionless inclined plane as shown in the figure. A uniform electric field *E* is applied parallel to the inclined surface such that the block is at rest. Calculate the magnitude of the electric field *E*.



Solution

Note: A similar problem is solved in XIth Physics volume I, unit 3 section 3.3.2.

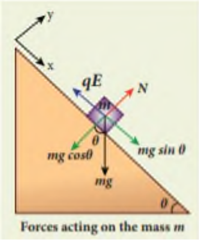
There are three forces that acts on the mass *m*:

(i) The downward gravitational force exerted by the Earth (*mg*)

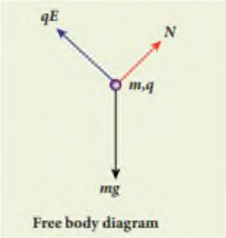
(ii) The normal force exerted by the inclined surface (*N*)

(iii) The Coulomb force given by uniform electric field (*qE*)

The free body diagram for the mass *m* is drawn below.



Forces acting on the mass m



Free body diagram

A convenient inertial coordinate system is located in the inclined surface as shown in the figure. The mass *m* has zero net acceleration both in *x* and *y*-direction.

Along *x*-direction, applying Newton's second law, we have

$$mg\sin\theta\,\hat{i} - qE\,\hat{i} = 0$$

$$mg\sin\theta - qE = 0$$

$$E = \frac{mg\sin\theta}{q}$$

Note that the magnitude of the electric field is directly proportional to the mass *m* and inversely proportional to the charge *q*. It implies that, if the mass is increased by keeping the charge constant, then a strong electric field is required to stop the object from sliding. If the charge is increased bykeeping the mass constant, then a weak electric field is sufficient to stop the mass from sliding down the plane.

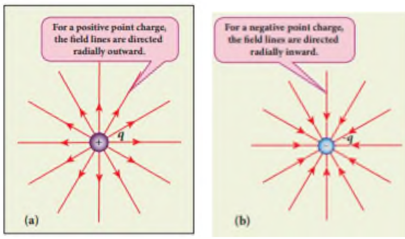
The electric field also can be expressed in terms of height and the length of the inclined surface of the plane.

$$E = \frac{mgh}{qL}$$

Electric field lines

Electric field vectors are visualized by the concept of electric field lines. They form a set of continuous lines which are the visual representation of the electric field in some region of space. The following rules are followed while drawing electric field lines for charges.

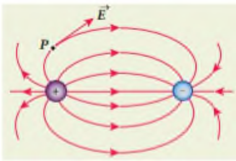
- The electric field lines start from a positive charge and end at negative charges or at infinity. For a positive point charge the electric field lines point radially outward and for a negative point charge, the electric field lines point radially inward. These are shown in Figure 1.9 (a) and (b).



Electric field lines for isolated positive and negative charges

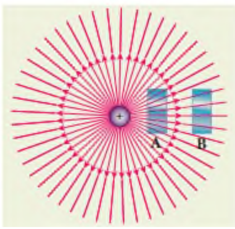
Note that for an isolated positive point charge the electric field line starts from the charge and ends only at infinity. For an isolated negative point charge the electric field lines start at infinity and end at the negative charge.

- The electric field vector at a point in space is tangential to the electric field line at that point. This is shown in Figure 1.10



Electric field at a point P

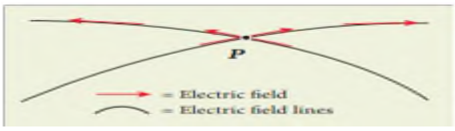
- The electric field lines are denser (more closer) in a region where the electric field has larger magnitude and less dense in a region where the electric field is of smaller magnitude. In other words, the number of lines passing through a given surface area perpendicular to the lines is proportional to the magnitude of



Electric field has larger magnitude at surface A than B

Figure 1.11 shows electric field lines from a positive point charge. The magnitude of the electric field for a point charge decreases as the distance increases $\left(|\vec{E}| \propto \frac{1}{r^2}\right)$. So the electric field has greater magnitude at the surface A than at B. Therefore, the number of lines crossing the surface A is greater than the number of lines crossing the surface B. Note that at surface B the electric field lines are farther apart compared to the electric field lines at the surface A.

· No two electric field lines intersect each other. If two lines cross at a point, then there will be two different electric field vectors at the same point, as shown in Figure 1.12.

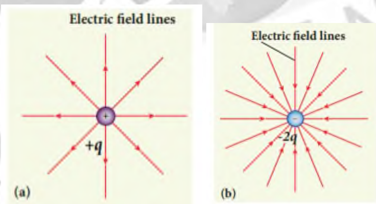


Two electric field lines never intersect each other

As a consequence, if some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible. Hence, electric field lines do not intersect.

· The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.

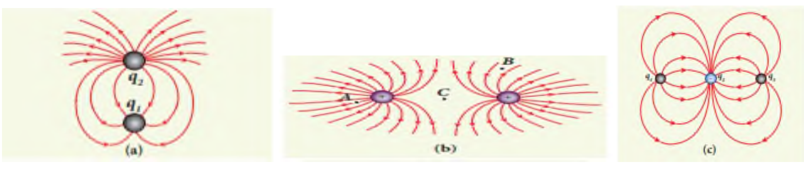
For example in the Figure 1.13, the electric field lines are drawn for charges $+q$ and $-2q$. Note that the number of field lines emanating from $+q$ is 8 and the number of field lines ending at $-2q$ is 16. Since the magnitude of the second charge is twice that of the first charge,the number of field lines drawn for $-2q$ is twice in number than that for charge $+q$.



Electric field lines and magnitude of the charge

EXAMPLE

The following pictures depict electric field lines for various charge configurations.



(i) In figure (a) identify the signs of two charges and find the ratio $\left|\frac{q_1}{q_2}\right|$

(ii) In figure (b), calculate the ratio of two positive charges and identify the strength of the electric field at three points A, B, and C

(iii) Figure (c) represents the electric field lines for three charges. If $q_2 = -20$ nC, then calculate the values of q_1 and q_3

Solution

(i) The electric field lines start at q_2 and end at $q1$. In figure (a), q_2 is positive and q_1 is negative. The number of lines starting from $q2$ is 18 and number of the lines ending at $q1$ is 6. So q_2 has greater magnitude. The ratio of $\left|\frac{q_1}{q_2}\right| = \frac{N_1}{N_2} = \frac{6}{18} = \frac{1}{3}$ .It implies that $|q_2| = 3|q_1|$

(ii) In figure (b), the number of field lines emanating from both positive charges are equal (N=18). So the charges are equal. At point A, the electric field lines are denser compared to the lines at point B. So the electric field at point A is greater in magnitude compared to the field at point B. Further, no electric field line passes through C, which implies that the resultant electric field at C due to these two charges is zero.

(iii) In the figure (c), the electric field lines start at $q1$ and $q3$ and end at $q2$. This implies that $q1$ and $q3$ are positive charges. The ratio of the number of field lines is *qqqq*123281612===,implying that $q1$and $q3$ are half of the magnitude of $q2$. So $q1 = q3 = +10$ nC.

ELECTRIC DIPOLE AND ITS PROPERTIES

Electric dipole

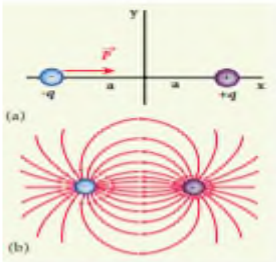
Two equal and opposite charges separated by a small distance constitute an electric dipole. In many molecules, the centres of positive and negative charge do not coincide. Such molecules behave as permanent dipoles. Examples: CO, water, ammonia, HCl etc.

Consider two equal and opposite point charges ($+q$, $-q$) that are separated by a distance $2a$ as shown in Figure 1.14(a).

The electric dipole moment is defined as

$$\vec{p} = q\vec{r}_+ + (-q)\vec{r}_-$$

Where $\vec{r}_+$ is the position vector of $+q$ from the origin and $\vec{r}_-$ is the position vector of $-q$ from the origin. Then, from Figure 1.14 (a),



(a) Electric dipole (b) Electric field lines for the electric dipole

$$\vec{p} = qa\hat{i} - qa\left(-\hat{i}\right) = 2qa\hat{i}$$

The electric dipole moment vector lies along the line joining two charges and is directed from –*q* to +*q*. The SI unit of dipole moment is coulomb metre (Cm). The electric field lines for an electric dipole are shown in Figure 1.14 (b).

- For simplicity, the two charges are placed on the *x*-axis. Even if the two charges are placed on *y* or *z*-axis, dipole moment will point from –*q* to +*q*.

- The magnitude of the electric dipole moment is equal to the product of the magnitude of one of the charges and the distance between them,

$$\left|\vec{p}\right| = 2qa$$

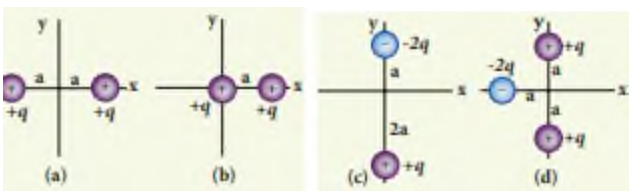
- Though the electric dipole moment for two equal and opposite charges is defined, it is possible to define and calculate the electric dipole moment for a collection of point charges. The electric dipole moment for a collection of *n* point charges is given by

$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i$$

Where $\vec{r}_i$ is the position vector of charge q_i from the origin.

EXAMPLE

Calculate the electric dipole moment for the following charge configurations.



Solution

Case (a) The position vector for the $+q$ on the positive x-axis is $a\hat{i}$ and position vector for the $+q$ charge the negative x axis is $-a\hat{i}$. So the dipole moment is,

$$\vec{p} = (+q)\left(a\hat{i}\right) + (+q)\left(-a\hat{i}\right) = 0$$

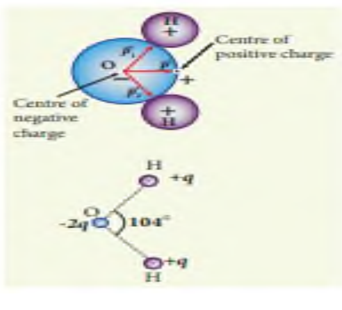
Case (b) In this case one charge is placed at the origin, so its position vector is zero. Hence only the second charge $+q$ with position vector $a\hat{i}$ $a\hat{i}$ contributes to the dipole moment, which is $\vec{p} = qa\hat{i}$.

From both cases (a) and (b), we can infer that in general the electric dipole moment depends on the choice of the origin and charge configuration. But for one special case, the electric dipole moment is independent of the origin. If the total charge is zero, then the electric dipole moment will be the same irrespective of the choice of the origin. It is because of this reason that the electric dipole moment of an electric dipole (total charge is zero) is always directed from $-q$ to $+q$, independent of the choice of the origin.

Case (c) $\vec{p} = (-2q)a\hat{j} + q(2a)\left(-\hat{j}\right) = -4qa\hat{j}$ .Note that in this case $\vec{p}$ is directed from $-2q$ to $+q$.

Case (d)
$$\vec{p} = -2qa\left(-\hat{i}\right) + qa\hat{j} + qa\left(-\hat{j}\right)$$
$$= 2qa\hat{i}$$

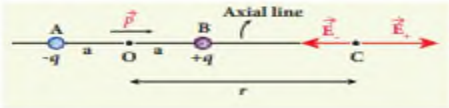
The water molecule (H_2O) has this charge configuration. The water molecule has three atoms (two H atom and one O atom). The centres of positive (H) and negative (O) charges of a water molecule lie at different points, hence it possess permanent dipole moment. The electric dipole moment $\vec{p}$ is directed from centre of negative charge to the centre of positive charge, as shown in the figure.



Electric field due to a dipole

Case (i) Electric field due to an electric dipole at points on the axial line

Consider an electric dipole placed on the x-axis as shown in Figure 1.15. A point C is located at a distance of r from the midpoint O of the dipole on the axial line.



Electric field of the dipole along the axial line

The electric field at a point C due to +q is

$$\vec{E}_{+} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} \text{ along BC}$$

Since the electric dipole moment vector $\vec{P}$ is from –q to +q and is directed along BC, the above equation is rewritten as

$$\vec{E}_{+} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\hat{P}$$

Where $\hat{P}$ is the electric dipole moment unit vector from –q to +q.
The electric field at a point C due to –q is

$$\vec{E}_{+} = -\frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}\hat{P}$$

Since +q is located closer to the point C than –q, $\vec{E}_{+}$ is stronger than $\vec{E}_{-}$.Therefore, the length of the $\vec{E}_{+}$ vector is drawn larger than that of $\vec{E}_{-}$ vector.

The total electric field at point C is calculated using the superposition principle of the electric field.

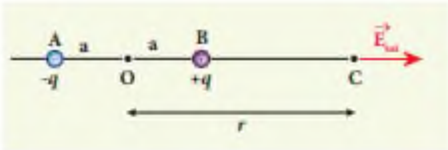
$$\vec{E}_{tot} = \vec{E}_{+} + \vec{E}_{-}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\hat{P} - \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}\hat{P}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)\hat{P}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}q\left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{P}$$

Note that the total electric field is along $\vec{E}_{+}$, since +q is closer to C than –q.The direction of $\vec{E}_{tot}$ is shown



Total electric field of the dipole on the axial line

If the point C is very far away from the dipole ($r >> a$). Then under this limit the term $\left(r^2 - a^2\right)^2 \approx r^4$. Substituting this into equation (1.16), we get

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0} q \left(\frac{4aq}{r^3}\right)\hat{P} \left(r >> a\right)$$

$$\text{since } 2aq\hat{p} = \vec{P}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0} q \frac{2\vec{P}}{r^3} \left(r >> a\right)$$

If the point C is chosen on the left side of the dipole, the total electric field is still in the direction of r p. We infer this result by examining the electric field lines of the dipole shown in Figure 1.14(b).

Case (ii) Electric field due to an electric dipole at a point on the equatorial plane

Consider a point C at a distance r from the midpoint O of the dipole on the equatorial plane as shown in Figure 1.17.
Since the point C is equi-distant from $+q$ and $-q$, the magnitude of the electric fields at C due to $+q$ and $-q$ are the same. The directionof $\vec{E}_+$ is along BC and the direction $\vec{E}_-$ of is along CA. $\vec{E}_+$ and $\vec{E}_-$ can be resolved into two components; one component parallel to the dipole axis and the other perpendicular to it. Since perpendicular components $\left|\vec{E}_+\right| \sin\theta$ and $\left|\vec{E}_-\right| \sin\theta$ are eqaul in magnitude and oppositely directed, they cancel each other. The magnitude of the total electric field at point C is the sum of the parallel components of $\vec{E}_+$ and $\vec{E}_-$ and its direction is along $-\hat{p}$ as shown in the Figure 1.17.

$$\vec{E}_{tot} = -\left|\vec{E}_+\right| \cos\theta\,\hat{p} - \left|\vec{E}_-\right| \cos\theta\,\hat{p}$$

The magnitudes $\vec{E}_+$ and $\vec{E}_-$ are the same and are given by

$$\left|\vec{E}_+\right| = \left|\vec{E}_-\right| = \frac{1}{4\pi\epsilon_0} \frac{q}{\left(r^2 + a^2\right)}$$

By substituting equation (1.19) into equation (1.18), we get

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0} \frac{2q\cos\theta}{\left(r^2 + a^2\right)} \hat{p}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2qa}{\left(r^2 + a^2\right)^{\frac{1}{2}}} \hat{p}$$

$$\text{since } \cos\theta = \frac{a}{\sqrt{r^2 + a^2}}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0} \frac{2qa}{\left(r^2 + a^2\right)^{\frac{3}{2}}}$$

$$\text{since } \vec{p} = 2qa\hat{p}$$

At very large distances ($r >> a$), the equation (1.20) becomes

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_o} \frac{\vec{p}}{r^2} \qquad (r \gg a)$$

Important inferences

(i) From equations (1.17) and (1.21), it is inferred that for very large distances, the magnitude of the electric field at point on the dipole axis is twice the magnitude of the electric field at the point at the same distance on the equatorial plane. The direction of the electric field at points on the dipole axis is directed along the direction of dipole moment vector $\vec{p}$ but at points on the equatorial plane it is directed opposite to the dipole moment vector, that is along $-\vec{p}$.

(ii) At very large distances, the electric field due to a dipole varies as $\frac{1}{r^3}$. Note that for a point charge, the electric field $\frac{1}{r^2}$. varies as This implies that the electric field due to a dipole at very large distances goes to zero faster than the electric field due to a point charge. The reason for this behavior is that at very large distance, the two charges appear to be close to each other and neutralize each other.

(iii) The equations (1.17) and (1.21) are valid only at very large distances ($r>>a$). Suppose the distance $2a$ approaches zero and q approaches infinity such that the product of $2aq = p$ is finite, then the dipole is called a point dipole. For such point dipoles, equations (1.17) and (1.21) are exact and hold true for any r.

Torque experienced by an electric dipole in the uniform electric field

Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$ whose field lines are equally spaced and point in the same direction. The charge $+q$ will experience a force $q\vec{E}$ in the direction of the field and charge $-q$ will experience a force $-q$

$\vec{E}$ in a direction opposite to the field. Since the external field $\vec{E}$ is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque as shown in Figure 1.18. This torque tends to rotate the dipole. (Note that electric field lines of a uniform field are equally spaced and point in the same direction).

The total torque on the dipole about the point O

$$\vec{\tau} = \vec{OA} \times \left(-q\vec{E}\right) + \vec{OB} \times q\vec{E}$$

Using right-hand corkscrew rule (Refer XI, volume 1, unit 2), it is found that total torque is perpendicular to the plane of the paper and is directed into it.
The magnitude of the total torque $\tau = \left|\vec{OA}\right|\left(-q\vec{E}\right)\sin\theta + \left|\vec{Ob}\right|\left|q\vec{E}\right|\sin\theta$

$$\tau = qE.2a\sin\theta$$

where θ is the angle made by $\vec{p}$ with $\vec{E}$. Since $p = 2aq$, the torque is written in terms of the vector product as

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The magnitude of this torque is τ θ = *pE*sinand is maximum when θ=90°.

This torque tends to rotate the dipole and align it with the electric field $\vec{E}$. Once $\vec{p}$ is aligned with $\vec{E}$, the total torque on the dipole becomes zero.

If the electric field is not uniform, then the force experienced by +*q* is different from that experienced by –*q*. In addition to the torque, there will be net force acting on the dipole.

EXAMPLE

A sample of HCl gas is placed in a uniform electric field of magnitude 3 × 10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4 × 10^{-30} Cm. Calculate the maximum torque experienced by each HCl molecule.

Solution

The maximum torque experienced by the dipole is when it is aligned perpendicular to the applied field.

$$\tau_{max} = pE\sin 90^o = 3.4 \times 10^{-30} \times 3 \times 10^4$$
$$\tau_{max} = 10.2 \times 10^{26}\, Nm$$

ELECTROSTATIC POTENTIAL AND POTENTIAL ENERGY

Introduction

In mechanics, potential energy is defined for conservative forces. Since gravitational force is a conservative force, its gravitational potential energy is defined in XI standard physics (Unit 6). Since Coulomb force is an inverse-square-law force, its also a conservative force like gravitational force. Therefore, we can define potential energy for charge configurations.

Electrostatic Potential energy and Electrostatic potential

Consider a positive charge *q* kept fixed at the origin which produces an electric field $\vec{E}$ around it. A positive test charge *q′* is brought from point R to point P against the repulsive force between *q* and *q′* as shown in Figure 1.20. Work must be done to overcome the repulsion between the charges and this work done is stored as potential energy of the system.

Work done is equal to potential energy

The test charge *q′* is brought from R to P with constant velocity which means that external force used to bring the test charge *q′* from R to P must be equal and opposite to the coulomb force $\left(\vec{F}_{ext} = -\vec{F}_{coulomp}\right)$. The work done is

$$W = \int_R^P \vec{F}_{ext} \cdot d\vec{r}$$

Since coulomb force is conservative, work done is independent of the path and it depends only on the initial and final positions of the test charge. If potential energy associated with *q′* at P is U_P and that at R is U_R, then difference in potential energy is defined as the work done to bring a test charge *q′* from point R to P and is given as

$$U_P - U_R = W = \Delta U$$

$$\Delta U = \int_R^P \vec{F}_{ext} \cdot d\vec{r}$$

$$Since\ \vec{F}_{ext} = -\vec{F}_{coulomb} = -q'\vec{E}$$

$$\Delta U = \int_R^P \left(-q'\vec{E}\right) \cdot d\vec{r} = -q'\int_R^P \vec{E} \cdot d\vec{r}$$

The potential energy difference per unit charge is given by

$$\frac{\Delta U}{q'} = \frac{q' \int_R^P (-\vec{E}).d\vec{r}}{q'} = -\int_R^P \vec{E}.d\vec{r}$$

The above equation is independent of *q′*. The quantity $\frac{\Delta U}{q'} = -\int_R^P \vec{E}.d\vec{r}$ is called electric potential difference between P and R and is denoted as $V_P - V_R = \Delta V$.

In otherwords, the electric potential difference is defined as the work done by an external force to bring unit positive charge from point R to point P.

$$V_P - V_R = \Delta V = \int_R^P \vec{E}.d\vec{r}$$

The electric potential energy difference can be written as
$\Delta U = q' \Delta V$. Physically potential difference between two points is a meaningful quantity. The value of the potential itself at one point is not meaningful. Therefore the point R is taken to infinity and the potential at infinity is considered as zero $(V_\infty = 0)$.

Then the electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field $\vec{E}$. Mathematically this is written as

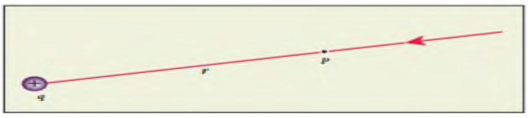
$$V_P = \int_\infty^P \vec{E}.d\vec{r}$$

Important points

1. Electric potential at point P depends only on the electric field which is due to the source charge *q* and not on the test charge *q′*. Unit positive charge is brought from infinity to the point P with constant velocity because external agency should not impart any kinetic energy to the test charge.
2. From equation (1.29), the unit of electric potential is Joule per coulomb. The practical unit is volt (V) named after Alessandro Volta (1745-1827) who invented the electrical battery. The potential difference between two points is expressed in terms of volt.

Electric potential due to a point charge

Consider a positive charge q kept fixed at the origin. Let P be a point at distance r from the charge q.



Electrostatic potential at a point P

The electric potential at the point P is

$$V = \int_{\infty}^{r} (-\vec{E}).d\vec{r} = -\int_{\infty}^{r} \vec{E}.d\vec{r}$$

Electric field due to positive point charge q is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$V = \frac{-1}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{q}{r^2}\hat{r}.d\vec{r}$$

The infinitesimal displacement vector, $d\vec{r} = dr\hat{r}$ and using $\hat{r}.\hat{r} = 1$, we have

$$V = \frac{1}{4\pi\epsilon_o}\int_{\infty}^{r}\frac{q}{r^2}\hat{r}.dr\hat{r} = -\frac{1}{4\pi\epsilon_o}\int_{\infty}^{r}\frac{q}{r^2}dr$$

After the integration,

$$V = \frac{1}{4\pi\epsilon_o}q\left\{-\frac{1}{r}\right\}_{\infty}^{r} = \frac{1}{4\pi\epsilon_o}\frac{q}{r}$$

Hence the electric potential due to a point charge *q* at a distance r is

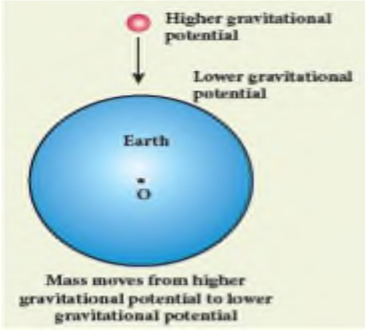
$$V = \frac{1}{4\pi\epsilon}\frac{q}{r}$$

Important points

(i) If the source charge *q* is positive, *V* > 0. If *q* is negative, then *V* is negative and equal to $V = \frac{1}{4\pi\epsilon}\frac{q}{r}$

(ii) From expression (1.33), it is clear that the potential due to positive charge decreases as the distance increases, but for a negative charge the potential increases as the distance is increased. At infinity (r=∞) electrostatic potential is zero (V = 0).

In the case of gravitational force, mass moves from a point of higher gravitational potential to a point of lower

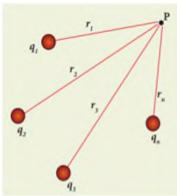


gravitational potential (Figure 1.22). Similarly a positive charge moves from a point of higher electrostatic potential to a point of lower electrostatic potential. However a negative charge moves from lower electrostatic potential to higher electrostatic potential. This comparison is shown in Figure 1.23.

(iii) The electric potential at a point P due to a collection of charges $q_1, q_2, q_3, \ldots q_n$ is equal to sum of the electric potentials due to individual charges.

$$V_{tot} = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \ldots\ldots + \frac{kq_n}{r_n} = \frac{1}{4\pi\epsilon_o} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

Where $r_1, r_2, r_3 \ldots\ldots r_n$ are the distances of $q_1, q_2, q_3 \ldots\ldots q_n$ respectively from P



Electrostatic potential due to collection of charges

EXAMPLE

(a) Calculate the electric potential at points P and Q as shown in the figure below.
(b) Suppose the charge +9 μC is replaced by –9 μC find the electrostatic potentials at points P and Q

(c) Calculate the work done to bring a test charge +2 μC from infinity to the point Q. Assume the charge +9 μC is held fixed at origin and +2 μC is brought from infinity to P.

Solution

(a) Electric potential at point P is given by

$$V_p = \frac{1}{4\pi \epsilon_o} \frac{q}{r_p} = \frac{9 \times 10^9 \times 9 \times 10^{-6}}{10} = 8.1 \times 10^3 V$$

Electric potential at point Q is given by

$$V_Q = \frac{1}{4\pi \epsilon_o} \frac{q}{r_Q} = \frac{9 \times 10^9 \times 9 \times 10^{-6}}{16} = 5.06 \times 10^3 V$$

Note that the electric potential at point Q is less than the electric potential at point P. If we put a positive charge at P, it moves from P to Q. However if we place a negative charge at P it will move towards the charge +9 μC.

The potential difference between the points P and Q is given by

$$\Delta V = V_p - V_Q = +3.04 \times 10^3 V$$

(b) Suppose we replace the charge +9 μC by –9 μC, then the corresponding potentials at the points P and Q are,

$$V_p = -8.1 \times 10^3 V, V_Q = -5.06 \times 10^3 V$$

Note that in this case electric potential at the point Q is higher than at point P.

The potential difference between the points P and Q is given by

$$\Delta V = V_p - V_Q = -3.04 \times 10^3 V$$

(c) The electric potential V at a point Q due to some charge is defined as the work done by an external force to bring a unit positive charge from infinity to Q. So to bring the *q* amount of charge from infinity to the point Q, work done is given as follows.

$$W = qV$$
$$W_Q = 2 \times 10^{-6} \times 5.06 \times 10^3 = 10.12 \times 10^{-3} J$$

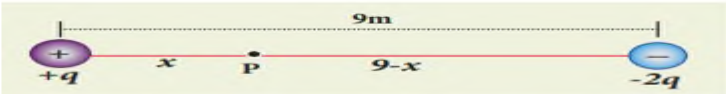
EXAMPLE

Consider a point charge +*q* placed at the origin and another point charge -2*q* placed at a distance of 9 m from the charge +*q*. Determine the point between the two charges at which electric potential is zero.

Solution

According to the superposition principle, the total electric potential at a point is equal to the sum of the potentials due to each charge at that point.

Consider the point at which the total potential zero is located at a distance x from the charge +q as shown in the figure.



Since the total electric potential at P is zero,

$$V_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{x} - \frac{2q}{(9-x)}\right) = 0 \,(or)$$

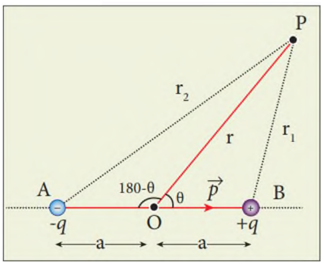
$$\frac{q}{x} = \frac{2q}{(9-x)} \,(or)$$

$$\frac{1}{x} = \frac{2}{(9-x)}$$

Hence, x = 3m

Electrostatic potential at a point due to an electric dipole

Consider two equal and opposite charges separated by a small distance 2a as shown in Figure 1.25. The point P is located at a distance r from the midpoint of the dipole. Let θ be the angle between the line OP and dipole axis AB.



Potential due to electric dipole

Let r_1 be the distance of point P from +q and r_2 be the distance of point P from –q.

Potential at P due to charge $+q = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1}$

Potential at P due to charge $-q = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}$

Total potential at the point P

$$V = \frac{1}{4\pi\varepsilon_0} q \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Suppose if the point P is far away from the dipole, such that r>>a, then equation can be expressed in terms of r.

By the cosine law for triangle BOP

$$r_1^{\ 2} = r^2 + a^2 - 2ra\cos\theta$$

$$r_1^{\ 2} = r^2 \left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta \right)$$

Since the point P is very far from the dipole (r>>a). As a result the term $\frac{a^2}{r^2}$ is very small and can be neglected. Therefore

$$r_1^{\ 2} = r^2 \left(1 - 2a\frac{\cos\theta}{r} \right)$$

$$(or)\ r_1 = r\left(1 - \frac{2a}{r}\cos\theta \right)^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta \right)^{-\frac{1}{2}}$$

Since $\frac{a}{r}$ <<1, we can use binomial theorem and retain the terms up to first order

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta \right)$$

Similarly applying the cosine law for triangle AOP,

$$r_2^{\ 2} = r^2 + a^2 - 2ra\cos(180 - \theta)$$

sincecosθ (180−θ) = − cosθ we get

$$r_2^{\,2} = r^2 + a^2 + 2ra\cos\theta$$

Neglecting the term $\frac{a^2}{r_2^{\,2}}$ (because r>>a)

$$r_2^{\,2} = r^2\left(1 + \frac{2a\cos\theta}{r}\right)$$

$$r_2 = r\left(1 + \frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$$

Using Binomial theorem, we get

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right)$$

Substituting equation

$$V - \frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r}\right) - \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r} - 1 + a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{2aq}{r^2}\cos\theta$$

But the electric dipole moment p = 2qa and we get,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

Now we can write p cosθ = $\vec{p}\,.\,\hat{r}$, where $\hat{r}$ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\hat{r}}{r^2} \quad \text{(r>>a)}$$

Equation is valid for distances very large compared to the size of the dipole. But for a point dipole, the equation is valid for any distance.

Special cases

Case (i) If the point P lies on the axial line of the dipole on the side of +q, then θ = 0. Then the electric potential becomes

$$V = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2}$$

Case (ii)If the point P lies on the axial line of the dipole on the side of –q, then θ = 180°. Then

$$V = -\frac{1}{4\pi\epsilon_0}\frac{p}{r^2}$$

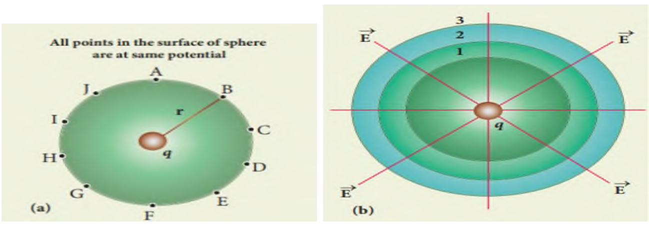
Case (iii)If the point P lies on the equatorial line of the dipole, then θ = 90°. Hence

$$V=0$$

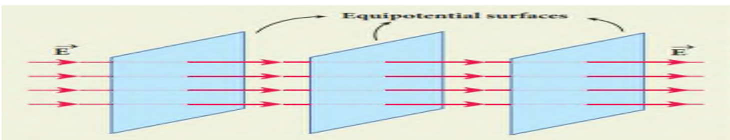
Equi-potential Surface

Consider a point charge q located at some point in space and an imaginary sphere of radius r is chosen by keeping the charge q at its centre. The electric potential at all points on the surface of the given sphere is the same. Such a surface is called an equipotential surface.

An equipotential surface is a surface on which all the points are at the same electric potential. For a point charge the equipotential surfaces are concentric spherical surfaces as shown. Each spherical surface is an equipotential surface but the value of the potential is different for different spherical surfaces.



Equipotential surface of point Charge



Equipotential surface for uniform electric field

For a uniform electric field, the equipotential surfaces form a set of planes normal to the electric field $\vec{E}$.

Properties of equipotential surfaces

(i) The work done to move a charge q between any two points A and B, $W = q\,(V_B - V_A)$. If the points A and B lie on the same equipotential surface, work done is zero because $V_A = V_B$.

(ii) The electric field is normal to an equipotential surface. If it is not normal, then there is a component of the field parallel to the surface. Then work must be done to move a charge between two points on the same surface. This is a contradiction. Therefore the electric field must always be normal to equipotential surface.

Relation between electric field and potential

Consider a positive charge q kept fixed at the origin. To move a unit positive charge by a small distance dx towards q in the electric field E, the work done is given by $dW = -E\,dx$. The minus sign implies that work is done against the electric field. This work done is equal to electric potential difference. Therefore,

$dW = dV$.

(or) $dV = -E\,dx$

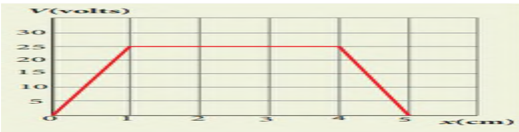
Hence $E = -\dfrac{dV}{dx}$

The electric field is the negative gradient of the electric potential. In vector form,

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

EXAMPLE

The following figure represents the electric potential as a function of x – coordinate. Plot the corresponding electric field as a function of x.



Solution In the given problem, since the potential depends only on x, we can use $\vec{E} = -\dfrac{dV}{dx}\hat{i}$ (the other two terms $\dfrac{\partial V}{\partial y}$ and $\dfrac{\partial V}{\partial z}$ are zero)

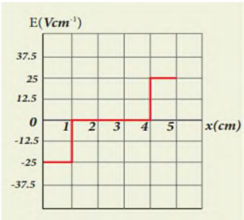
From 0 to 1 cm, the slope is constant and so $\dfrac{dV}{dx} = -\,25\ 1\ Vcm^{-1}$.

So $\vec{E} = -25\,Vcm^{-1}\hat{i}$

From 1 to 4 cm, the potential is constant, V = 25 V. It implies that $\frac{dV}{dx}=0$. So $\vec{E}=0$

From 4 to 5 cm, the slope $\frac{dV}{dx}=-25\ Vcm^{-1}$. So $\vec{E}=+25\ V\ cm^{-1}\hat{i}$.

The plot of electric field for the various points along the x axis is given below.



Electrostatic potential energy for collection of point charges

The electric potential at a point at a distance r from point charge q_1 is given by

$$V=\frac{1}{4\pi\epsilon_0}\frac{p}{r}$$

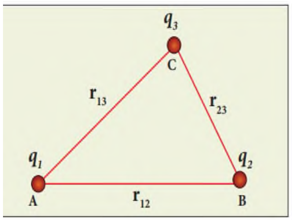
This potential V is the work done to bring a unit positive charge from infinity to the point. Now if the charge q_2 is brought from infinity to that point at a distance r from q_1, the work done is the product of q_2 and the electric potential at that point. Thus we have

W = q_2 V

This work done is stored as the electrostatic potential energy U of a system of charges q_1 and q_2 separated by a distance r. Thus we have

$$U=q_2V=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

The electrostatic potential energy depends only on the distance between the two point charges. In fact, the expression is derived by assuming that q_1 is fixed and q_2 is brought from infinity. The equation holds true when q_2 is fixed and q_1 is brought from infinity or both q_1 and q_2 are simultaneously brought from infinity to a distance r between them. Three charges are arranged in the following configuration as shown. To calculate the total electrostatic potential energy, we use the following procedure. We bring all the charges one by one and arrange them according to the configuration as shown.



Electrostatic potential energy for collection of point charges

(i) Bringing a charge q1 from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge q_1.

(ii) To bring the second charge q_2 to the point B, work must be done against the electric field created by the charge q_1. So the work done on the charge q_2 is $W = q_2 V_1B$. Here V1B is the electrostatic potential due to the charge q_1 at point B.

$$U_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$$

Note that the expression is same when q_2 is brought first and then q_1 later.

(iii) Similarly to bring the charge q_3 to the point C, work has to be done against the total electric field due to both charges q1 and q_2. So the work done to bring the charge q_3 is = q_3 (V1C + V2C). Here V1C is the electrostatic potential due to charge q_1 at point C and V_2C is the electrostatic potential due to charge q_2 at point C.

The electrostatic potential energy is

$$U_{II} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

(iv) Adding equations, the total electrostatic potential energy for the system of three charges q_1, q_2 and q_3 is $U = U_I + U_{II}$

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

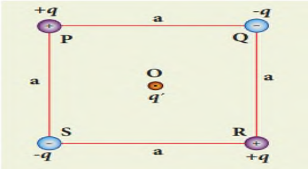
Note that this stored potential energy U is equal to the total external work done to assemble the three charges at the given locations. The expression is same if the charges are brought to their positions in any other order. Since the Coulomb force is a conservative force, the electrostatic potential energy is independent of the manner in which the configuration of charges is arrived at.

EXAMPLE

Four charges are arranged at the corners of the square PQRS of side a as shown in the figure.

(a) Find the work required to assemble these charges in the given configuration.

(b) Suppose a charge q′ is brought to the centre of the square, by keeping the four charges fixed at the corners, how much extra work is required for this?



Solution

(a) The work done to arrange the charges in the corners of the square is independent of the way they are arranged. We can follow any order.

(i) First, the charge +q is brought to the corner P. This requires no work since no charge is already present,
WP = 0

(ii) Work required to bring the charge –q to the corner
Q = (–q) × potential at a point Q due to +q located at a point P.

$$W_Q = -q \times \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$$

(iii) Work required to bring the charge +q to the corner R = q × potential at the point R due to charges at the point P and Q.

$$W_R = q \times \frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{a} + \frac{q}{\sqrt{2}a}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(-1 + \frac{1}{\sqrt{2}}\right)$$

(iv) Work required to bring the fourth charge –q at the position S = q × potential at the point S due the all the three charges at the point P, Q and R

$$W_S = q \times \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} + \frac{q}{a} - \frac{q}{\sqrt{2}a}\right)$$

$$W_S = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(2 - \frac{1}{\sqrt{2}}\right)$$

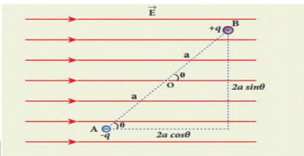
(b) Work required to bring the charge q′ to the centre of the square = q′ × potential at the centre point O due to all the four charges in the four corners

The potential created by the two +q charges are canceled by the potential created by the –q charges which are located in the opposite corners. Therefore the net electric potential at the centre O due to all the charges in the corners is zero.

Hence no work is required to bring any charge to the point O. Physically this implies that if any charge q′ when brought close to O, then it moves to the point O without any external force.

Electrostatic potential energy of a dipole in a uniform electric field

Consider a dipole placed in the uniform electric field $\vec{E}$ as shown. A dipole experiences a torque when kept in an uniform electric field $\vec{E}$. This torque rotates the dipole to align it with the direction of the electric field. To rotate the dipole (at constant angular velocity) from its initial angle θ′ to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.



The dipole in a uniform electric field

The work done by the external torque to rotate the dipole from angle θ′ to θ at constant angular velocity is

$$W = \int_{\theta'}^{\theta} \tau_{ext}\, d\theta$$

Since $\vec{\tau}_{ext}$ is equal and opposite to $\vec{\tau}_E = \vec{p} \times \vec{E}$, we have

$$\left|\vec{\tau}_{ext}\right| = \left|\vec{\tau}_E\right| = \left|\vec{p} \times \vec{E}\right|$$

Substituting equation, we get

$$W = \int_{\theta'}^{\theta} pE \sin\theta\, d\theta$$

$$W = pE\left(\cos\theta' - \cos\theta\right)$$

This work done is equal to the potential energy difference between the angular positions θ and θ′.

$$U(\theta) - U(\theta') = \Delta U = -pE\cos\theta + pE\cos\theta'$$

If the initial angle is $\theta' = 90^o$ and is taken as reference point, then
$U(\theta') = pE\cos 90^o$.

The potential energy stored in the system of dipole kept in the uniform electric field is given by

$$U = -pE\cos\theta = -\vec{p}.\vec{E}$$

In addition to p and E, the potential energy also depends on the orientation θ of the electric dipole with respect to the external electric field.

The potential energy is maximum when the dipole is aligned anti-parallel ($\theta = \pi$) to the external electric field and minimum when the dipole is aligned parallel ($\theta = 0$) to the external electric field.

EXAMPLE

A water molecule has an electric dipole moment of 6.3×10^{-30} Cm. A sample contains 10^{22} water molecules, with all the dipole moments aligned parallel to the external electric field of magnitude 3×10^5 NC^{-1}. How much work is required to rotate all the water molecules from
$\theta = 0^o$ to 90^o?

Solution

When the water molecules are aligned in the direction of the electric field, it has minimum potential energy. The work done to rotate the dipole from $\theta = 0^o$ to 90^o is equal to the potential energy difference between these two configurations.

$$W = \Delta U = U(90^0) - U(0^0)$$

From the equation, we write $U = -pE\cos\theta$, Next we calculate the work done to rotate one water molecule from $\theta = 0^o$ to 90^o.

For one water molecule

$$W = -pE\cos 90^0 + pE\cos 0^0 = pE$$
$$W = 6.3 \times 10^{-30} \times 3 \times 10^5 = 18.9 \times 10^{-25}\,J$$

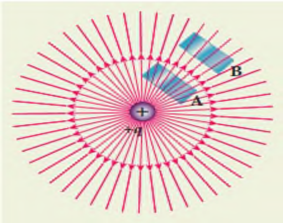
For 10^{22} water molecules, the total work done is

$$W_{tot} = 18.9 \times 10^{-25} \times 10^{22} = 18.9 \times 10^{-3}\,J$$

GAUSS LAW AND ITS APPLICATIONS

Electric Flux

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. It is usually denoted by the Greek letter Φ_E and its unit is N m^2 C^{-1}. Electric flux is a scalar quantity and it can be positive or negative. For a simpler understanding of electric flux, the following



Electric flux

The electric field of a point charge is drawn in this figure. Consider two small rectangular area elements placed normal to the field at regions A and B. Even though these elements have the same area, the number of electric field lines crossing the element in region A is more than that crossing the element in region B. Therfore the electric flux in region A is more than that in region B. Since electric field strength for a point charge decreases as the distance increases, electric flux also decreases as the distance increases. The above discussion gives a qualitative idea of electric flux. However a precise definition of electric flux is needed.

Electric flux for uniform Electric field

Consider a uniform electric field in a region of space. Let us choose an area A normal to the electric field lines as shown

(a) The electric flux for this case is $\Phi_E = EA$

Suppose the same area A is kept parallel to the uniform electric field, then no electric field lines pass through the area A, as shown.

(b) The electric flux for this case is zero. $\Phi_E = 0$

If the area is inclined at an angle θ with the field, then the component of the electric field perpendicular to the area alone contributes to the electric flux. The electric field component parallel to the surface area will not contribute to the electric flux. This is shown

(c) For this case, the electric flux

$$\Phi_E = (E \cos\theta)\ A$$

Further, θ is also the angle between the electric field and the direction normal to the area. Hence in general, for uniform electric field, the electric flux is defined as

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$

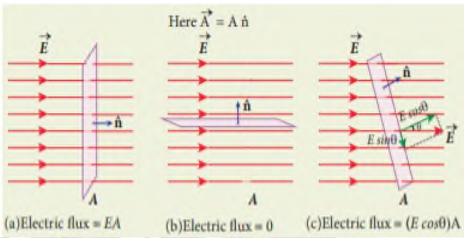
Here, note that $\vec{A}$ is the area vector $\vec{A} = A\hat{n}$. Its magnitude is simply the area A and its direction is along the unit vector $\hat{n}$ perpendicular to the area as shown. Using this definition for flux, $\Phi_E = \vec{E}\cdot\vec{A}$, equations can be obtained as special cases.

In (a), $\theta = 0^o$. Therefore,

$$\Phi_E = \vec{E}\cdot\vec{A} = EA$$

In (b), $\theta = 90^o$. Therefore,

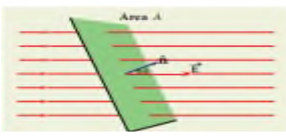
$$\Phi_E = \vec{E}\cdot\vec{A} = 0$$



The electric flux for Uniform electric field

EXAMPLE

Calculate the electric flux through the rectangle of sides 5 cm and 10 cm kept in the region of a uniform electric field 100 NC^{-1}. The angle θ is 60^o. If θ becomes zero, what is the electric flux?



Solution

The electric flux through the rectangular area

$$\Phi_E = \vec{E}\cdot\vec{A} = EA = \cos\theta$$

$$= 100 \times 5 \times 10 \times 10^{-4} \times \cos 60^0$$

$\Phi_E = 0.25\ Nm^2\ C^{-1}$

For $\theta = 0^o$,

$$\Phi_E = \vec{E}.\vec{A} = EA$$

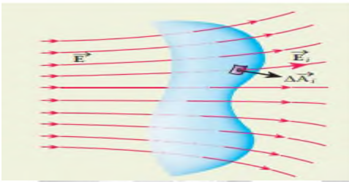
$$= 100 \times 5 \times 10 \times 10^{-4}$$

$= 0.5\ Nm^2\ C^{-1}$

Electric flux through an arbitrary area kept in a non uniform electric field

Suppose the electric field is not uniform and the area A is not flat surface. Then the entire area can be divided into n small area segments $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3, \ldots \Delta\vec{A}_n$ such that each area element is almost flat and the electric field over such area element can be considered uniform.

The electric flux for the entire area A is approximately written as



Electric flux for non - uniform electric Fi

$$\Phi_E = \vec{E}_1.\Delta\vec{A}_1 + \vec{E}_2.\Delta\vec{A}_2 + \vec{E}_3.\Delta\vec{A}_3 \ldots \vec{E}_n.\Delta\vec{A}_n$$

$$= \sum_{i=1}^{n} \vec{E}_i.\Delta\vec{A}_i$$

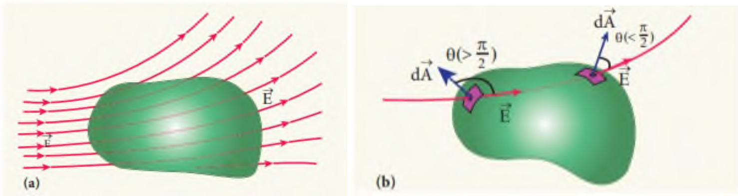
By taking the limit $\Delta\vec{A}_i \rightarrow 0$ (for all i) the summation in equation becomes integration. The total electric flux for the entire area is given by

$$\Phi_E = \int \vec{E}.d\vec{A}$$

From Equation, it is clear that the electric flux for a given surface depends on both the electric field pattern on the surface area and orientation of the surface with respect to the electric field.

Electric flux for closed surfaces

In the previous section, the electric flux for any arbitrary curved surface is discussed. Suppose a closed surface is present in the region of the non-uniform electric field as shown (a).



Electric flux over a closed surface

The total electric flux over this closed surface is written as

$$\Phi_E = \oint \vec{E}.d\vec{A}$$

Note the difference between equations. The integration in equation a closed surface integration and for each areal element, the outward normal is the direction of $d\vec{A}$ as shown in the Figure (b).

The total electric flux over a closed surface can be negative, positive or zero. In the Figure (b), it is shown that in one area element, the angle between $d\vec{A}$ and $\vec{E}$ is less than 90o , then the electric flux is positive and in another areal element, the angle between $d\vec{A}$ and $\vec{E}$ is greater than 90o , then the electric flux is negative.

In general, the electric flux is negative if the electric field lines enter the closed surface and positive if the electric field lines leave the closed surface.

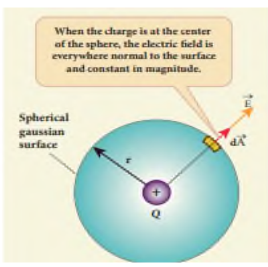
Gauss law

A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in Figure 1.34. We can calculate the total electric flux through the closed surface of the sphere using the equation.

$$\Phi_E = \oint \vec{E}.d\vec{A} = \oint E dA \cos\theta$$

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and θ = 0°.

$$\Phi_E = \oint E dA \qquad \text{since } \cos 0^0$$



Total electric flux of point charge

E is uniform on the surface of the sphere,

$$\Phi_E = E\oint dA$$

Substituting for $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ in equation

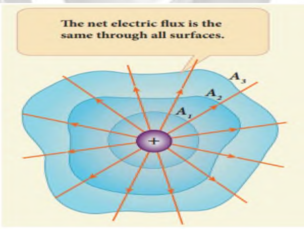
We get

$$\Phi_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \times 4\pi r^2 = 4\pi\frac{1}{4\pi\varepsilon_0}Q$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

The equation is called as Gauss's law.

The remarkable point about this result is that the equation is equally true for any arbitrary shaped surface which encloses the charge Q and as shown in the Figured below. It is seen that the total electric flux is the same for closed surfaces A_1, A_2 and A_3 as shown.



Gauss law for arbitrarily shaped surface

Gauss's law states that if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ_E through the closed surface is

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Where Q_{encl} denotes the charges within the closed surface

Discussion of Gauss law

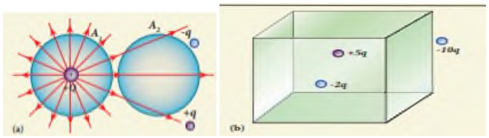
(i) The total electric flux through the closed surface depends only on the charges enclosed by the surface and the charges present outside the surface will not contribute to the flux and the shape of the closed surface which can be chosen arbitrarily.

(ii) The total electric flux is independent of the location of the charges inside the closed surface.

(iii) To arrive at equation, we have chosen a spherical surface. This imaginary surface is called a Gaussian surface. The shape of the Gaussian surface to be chosen depends on the type of charge configuration and the kind of symmetry existing in that charge configuration. The electric field is spherically symmetric for a point charge, therefore spherical Gaussian surface is chosen. Cylindrical and planar Gaussian surfaces can be chosen for other kinds of charge configurations.

(iv) In the LHS of equation, the electric field $\vec{E}$ is due to charges present inside and outside the Gaussian surface but the charge Q_{encl} denotes the charges which lie only inside the Gaussian surface.

EXAMPLE



(i) In figure (a), calculate the electric flux through the closed areas A_1 and A_2.

(ii) In figure (b), calculate the electric flux through the cube

Solution

(i) In figure (a), the area A_1 encloses the charge Q. So electric flux through this closed surface A_1 is $\frac{Q}{e_0}$. But the closed surface A_2 contains no charges inside, so electric flux through A_2 is zero.

(ii) In figure (b), the net charge inside the cube is 3q and the total electric flux in the cube is therefore $F_E = \frac{3q}{e_0}$. Note that the charge -10q lies outside the cube and it will not contribute the total flux through the surface of the cube.

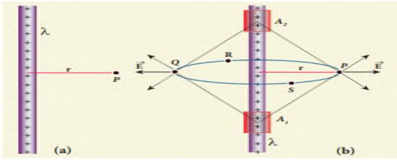
Applications of Gauss law

Electric field due to any arbitrary charge configuration can be calculated using Coulomb's law or Gauss law. If the charge configuration possesses some kind of symmetry, then Gauss law is a very efficient way to calculate the electric field. It is illustrated in the following cases.

(i) Electric field due to an infinitely long charged wire

Consider an infinitely long straight wire having uniform linear charge density λ(charge per unit length). Let P be a point located at a perpendicular distance r from the wire (a). The electric field at the point P can be found using Gauss law.

We choose two small charge elements A1 and A2 on the wire which are at equal distances from the point P. The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r. This is shown in the (b). Since the charged wire possesses



Electric field due to infinite long charged wire

a cylindrical symmetry, let us choose a cylindrical Gaussian surface of radius r and length L as shown.

The total electric flux through this closed surface is calculated as follows.

$$\Phi_E = \oint \vec{E}.d\vec{A}$$

$$= \int_{\substack{Curved\\surface}} \vec{E}.d\vec{A} + \int_{\substack{top\\surface}} \vec{E}.d\vec{A} + \int_{\substack{bottom\\surface}} \vec{E}.d\vec{A}$$

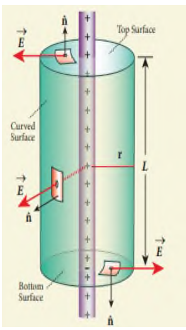
It is seen from Figure shown below that for the curved surface, $\vec{E}$ is parallel to $\vec{A}$ and $\vec{E}.d\vec{A} = EdA$. For the top and bottom surfaces, $\vec{E}$ is perpendicular to $\vec{A}$ and $\vec{E}.d\vec{A} = 0$

Substituting these values in the equation and applying Gauss law to the cylindrical surface, we have

$$\Phi_E = \int_{\substack{Curved\\surface}} EdA = \frac{Q_{encl}}{\varepsilon_0}$$

Since the magnitude of the electric field for the entire curved surface is constant, E is taken out of the integration and Q_{encl} is given by Q_{encl} = L λ, where λ is the linear charge density (charge present per unit length).

$$E \int_{\substack{Curved\\surface}} dA = \frac{\lambda L}{\varepsilon_0}$$



Cylindrical Gaussian surface

Here $\oint_{\substack{Curved\\surface}} dA =$ total area of the curved surface = 2πrL. Substituting this in equation, we get

$$E.2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

In vector form,

$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}$$

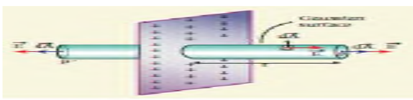
The electric field due to the infinite charged wire depends on $\frac{1}{r}$ rather than $\frac{1}{r^2}$ which is for a point charge.

Equation indicates that the electric field is always along the perpendicular direction $(\hat{r})$ to wire. In fact, if $\lambda > 0$ then $\vec{E}$ points perpendicularly outward $(\hat{r})$ from the wire and if $\lambda < 0$, then $\vec{E}$ points perpendicularly inward $(-\hat{r})$.

The equation is true only for an infinitely long charged wire. For a charged wire of finite length, the electric field need not be radial at all points. However, equation for such a wire is taken approximately true around the mid-point of the wire and far away from the both ends of the wire

(ii) Electric field due to charged infinite plane sheet

Consider an infinite plane sheet of charges with uniform surface charge density σ (charge present per unit area). Let P be a point at a distance of r from the sheet as shown



Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed outward at all points. A cylindrical Gaussian surface of length 2*r* and two flats surfaces each of area *A* is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.

Total electric flux linked with the cylindrical surface,

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_{\substack{Curved\\surplus}} \vec{E} \cdot d\vec{A} + \int_{p} \vec{E} \cdot d\vec{A} + \int_{p'} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and *P′* (Figure 1.38). Then, applying Gauss' law,

$$\phi_E = \int_{P} E\,dA + \int_{P'} E\,dA = \frac{Q_{encl}}{\varepsilon_0}$$

Since the magnitude of the electric field at these two equal flat surfaces is uniform, E is taken out of the integration and Q_{encl} is given by $Q_{encl} = \sigma A$, we get

$$2E \int_{p} dA = \frac{\sigma A}{\varepsilon_0}$$

The total area of surface either at P or P′

$$\int_{p} dA = A$$

$$Hence\ 2EA = \frac{\sigma A}{\varepsilon_0} \quad or \quad E = \frac{\sigma}{2\varepsilon_0}$$

$$In\ vector\ from,\ \vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

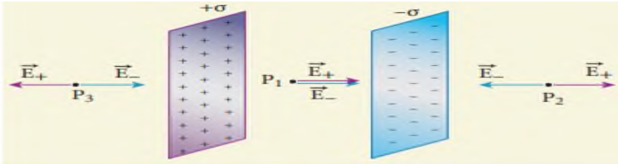
Here $\hat{n}$ is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance *r*.

The electric field will be the same at any point farther away from the charged plane. Equation implies that if $\sigma > 0$ the electric field at any point P is along outward perpendicular $\hat{n}$ drawn to the plane and if
$\sigma < 0$, the electric field points inward perpendicularly to the plane ($-\hat{n}$).

For a finite charged plane sheet, equation is approximately true only in the middle region of the plane and at points far away from both ends.

(iii) Electric field due to two parallel charged infinite sheets

Consider two infinitely large charged plane sheets with equal and opposite charge densities $+\sigma$ and $-\sigma$ which are placed parallel to each other as shown.



Electric field due to two parallel charged sheets

The electric field between the plates and outside the plates is found using Gauss law. The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\varepsilon_0}$ and it points perpendicularly outward if $\sigma > 0$ and points inward if $\sigma < 0$.

At the points P_2 and P_3, the electric field due to both plates are equal in magnitude and opposite in direction. As a result, electric field at a point outside the plates is zero. But between the plates, electric fields are in the same direction i.e., towards the right and the total electric field at a point P_1 is

$$E_{inside} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

The direction of the electric field between the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere between the plates.

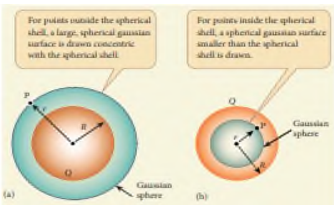
(iv) Electric field due to a uniformly charged spherical shell

Consider a uniformly charged spherical shell of radius *R* carrying total charge *Q* as shown. The electric field at points outside and inside the sphere can be found using Gauss law.

Case (a) At a point outside the shell ($r > R$)

Let us choose a point P outside the shell at a distance r from the centre as shown (a). The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if $Q > 0$ and point radially inward if $Q < 0$. So a spherical Gaussian surface of radius *r* is chosen andthe total charge enclosed by this Gaussian surface is *Q*. Applying Gauss law

$$\oint_{\text{Gaussian surface}} \vec{E}.d\vec{A} = \frac{Q}{\varepsilon_0}$$



The electric field due to a charged spherical shell

The electric field $\vec{E}$ and $d\vec{A}$ point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of $\vec{E}$ is also the same at all points due to the spherical symmetry of the charge distribution.

$$\text{Hence } E \oint_{\substack{Gaussian\\surface}} dA = \frac{Q}{\epsilon_0}$$

But $\oint_{\substack{Gaussian\\surface}} dA$ = total area of Gaussian surface = $4\pi r^2$. Substituting this value in equation (1.74)

$$E.4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E.4\pi r^2 = \frac{Q}{\epsilon_0} (or) E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

In vector form,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

The electric field is radially outward if $Q > 0$ and radially inward if $Q < 0$. From equation, we infer that the electric field at a point outside the shell will be the same as if the entire charge Q is concentrated at the centre of the spherical shell. (A similar result is observed in gravitation, for gravitational force due to a spherical shell with mass M)

Case (b): At a point on the surface of the spherical shell ($r = R$)

The electrical field at points on the spherical shell ($r = R$) is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{r}$$

Case (c): At a point inside the spherical shell ($r < R$)

Consider a point P inside the shell at a distance r from the centre. A Gaussiansphere of radius r is constructed as shown (b). Applying Gauss law

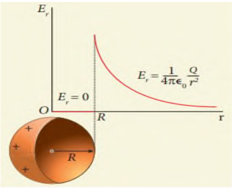
$$\oint_{\substack{Gaussian\\surface}} \vec{E}.d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E.4\pi r^2 = \frac{Q}{\epsilon_0}$$

Since Gaussian surface encloses no charge, Q = 0. The equation becomes

$E = 0 \qquad (r < R)$

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell. A graph is plotted between the electric field and radial distance. This is shown



Electric field versus distance for a spherical shell of radius R

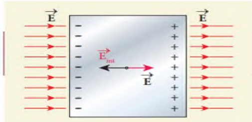
ELECTROSTATICS OF CONDUCTORS AND DIELECTRICS

Conductors at electrostatic equilibrium

An electrical conductor has a large number of mobile charges which are free to move in the material. In a metallic conductor, these mobile charges are free electrons which are not bound to any atom and therefore are free to move on the surface of the conductor. When there is no external electric field, the free electrons are in continuous random motion in all directions. As a result, there is no net motion of electrons along any particular direction which implies that the conductor is in electrostatic equilibrium. Thus at electrostatic equilibrium, there is no net current in the conductor. A conductor at electrostatic equilibrium has the following properties.

(i) The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.

This is an experimental fact. Suppose the electric field is not zero inside the metal, then there will be a force on the mobile charge carriers due to this electric field. As a result, there will be a net motion of the mobile charges, which contradicts the conductors being in electrostatic equilibrium. Thus the electric field is zero everywhere inside the conductor. We can also understand this fact by applying an external uniform electric field on the conductor. This is shown



Electric field of conductors

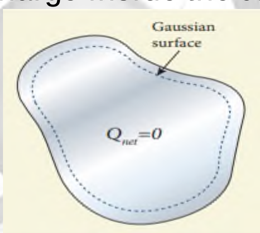
Before applying the external electric field, the free electrons in the conductor are uniformly distributed in the conductor. When an electric field is applied, the free electrons accelerate to the left causing the left plate to be negatively charged and the right plate to be positively charged as shown.

Due to this realignment of free electrons, there will be an internal electric field created inside the conductor which increases until it nullifies the external electric field. Once the external electric field is nullified the conductor is said to be in electrostatic equilibrium. The time taken by a conductor to reach electrostatic equilibrium is in the order of 10–16s, which can be taken as almost instantaneous.

(ii) There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.

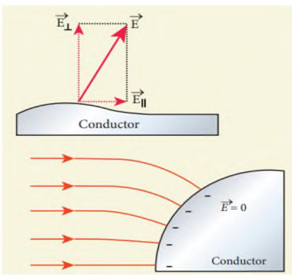
We can prove this property using Gauss law. Consider an arbitrarily shaped conductor as shown in Figure 1.43. A Gaussian surface is drawn inside the conductor such that it is very close to the surface of the conductor. Since the electric field is zero everywhere inside the conductor, the net electric flux is also zero over this Gaussian surface. From Gauss's law, this implies that there is no net charge inside the conductor. Even if some charge is introduced inside the conductor, it immediately reaches the surface of the conductor.

No net charge inside the conductor



(iii) The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{s}{e_0}$ where σ is the surface charge density at that point.

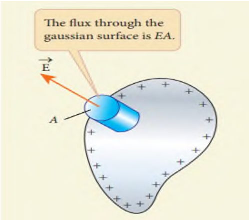
If the electric field has components parallel to the surface of the conductor, then free electrons on the surface of the conductor would experience acceleration (Figure (a)). This means that the conductor is not in equilibrium.



(a) Electric field is along the surface (b)Electric field is perpendicular to the surface of the conductor

Therefore at electrostatic equilibrium, the electric field must be perpendicular to the surface of the conductor. This is shown in Figure (b).

We now prove that the electric field has magnitude $\frac{\sigma}{\epsilon_0}$ just outside the conductor's surface. Consider a small cylindrical Gaussian surface, as shown. One half of this cylinder is embedded inside the conductor.



The electric field on the surface of the conductor

Since electric field is normal to the surface of the conductor, the curved part of the cylinder has zero electric flux. Also inside the conductor, the electric field is zero. Hence the bottom flat part of the Gaussian surface has no electric flux.

Therefore the top flat surface alone contributes to the electric flux. The electric field is parallel to the area vector and the total charge inside the surface is σA. By applying Gaus's law,

$$EA = \frac{\sigma A}{\epsilon_0}$$

In vector form,

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$$

Where $\hat{n}$ represents the unit vector outward normal to the surface of the conductor. Suppose $\sigma < 0$, then electric field points inward perpendicular to the surface.

(iv) The electrostatic potential has the same value on the surface and inside of the conductor.

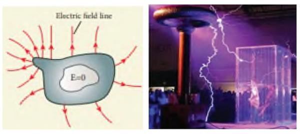
We know that the conductor has no parallel electric component on the surface which means that charges can be moved on the surface without doing any work. This is possible only if the electrostatic potential is constant at all points on the surface and there is no potential difference between any two points on the surface.

Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

Electrostatic shielding

Using Gauss law, we can prove that the electric field inside the charged spherical shell is zero, Further, we can show that the electric field inside both hollow and solid conductors is zero. It is a very interesting property which has an important consequence.

Consider a cavity inside the conductor as shown in Figure (a). Whatever be the charges at the surfaces and whatever be the electrical disturbances outside, the electric field inside the cavity is zero. A sensitive electrical instrument which is to be protected from external electrical disturbance can be kept inside this cavity. This is called electrostatic shielding.



(a) Electric field inside the cavity (b) Faraday cage

Faraday cage is an instrument used to demonstrate this effect. It is made up of metal bars as shown in Figure (b). If an artificial lightning jolt is created outside, the person inside is not affected.

During lightning accompanied by a thunderstorm, it is always safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, since the electric field inside is zero. During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

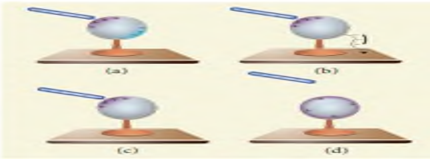
Electrostatic induction

In section 1.1, we have learnt that an object can be charged by rubbing using an appropriate material. Whenever a charged rod is touched by another conductor, charges start to flow from charged rod to the conductor. Is it possible to charge a conductor without any contact? The answer is yes. This type of charging without actual contact is called electrostatic induction.

(i) Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it, as shown in Figure (a).

The negative charge of the rod repels the electrons in the conductor to the opposite side. As a result, positive charges are induced near the region of the charged rod while negative charges on the farther side.

Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero. Once the charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod. But the total charge is zero.



Various steps in electrostatic induction

(ii) Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding. Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere. Note that positive charges will not flow to the ground because they are attracted by the negative charges of the rod (Figure (b)).

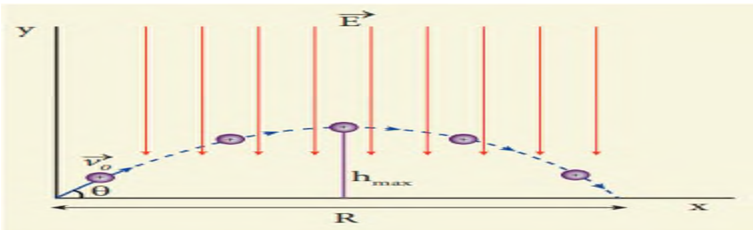
(iii) When the grounding wire is removed from the conductor, the positive charges remain near the charged rod (Figure (c))

(iv) Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor (Figure (d)). By this process, the neutral conducting sphere becomes positively charged.

For an arbitrary shaped conductor, the intermediate steps and conclusion are the same except the final step. The distribution of positive charges is not uniform for arbitrarily-shaped conductors. Why is it not uniform? The reason for it is discussed in the section

EXAMPLE

A small ball of conducting material having a charge +q and mass m is thrown upward at an angle θ to horizontal surface with an initial speed v_o as shown in the figure. There exists an uniform electric field E downward along with the gravitational field g. Calculate the range, maximum height and time of flight in the motion of this charged ball. Neglect the effect of air and treat the ball as a point mass.



Solution

If the conductor has no net charge, then its motion is the same as usual projectile motion of a mass m which we studied in Kinematics (unit 2, vol-1 XI physics). Here, in this problem, in addition to downward gravitational force, the charge also will experience a downward uniform electrostatic force.

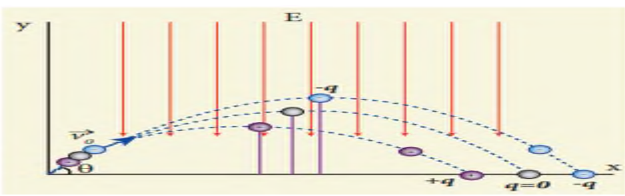
The acceleration of the charged ball due to gravity = $-g\hat{j}$

The acceleration of the charged ball due to uniform electric field = $-\frac{qE}{m}\hat{j}$

It is important here to note that the acceleration depends on the mass of the object. Galileo's conclusion that all objects fall at the same rate towards the Earth is true only in a uniform gravitational field. When a uniform electric field is included, the acceleration of a charged object depends on both mass and charge. But still the acceleration $a=\left(g+\frac{qE}{m}\right)$ is constant throughout the motion. Hence we use kinematic equations to calculate the range, maximum height and time of flight. In fact we can simply replace g by $g+\frac{qE}{m}$ in the usual expressions of range, maximum height and time of flight of a projectile.

	Without charge	With the charge +q
Time of flight T	$\frac{2v_0\sin\theta}{g}$	$\frac{2v_0\sin\theta}{\left(g+\frac{qE}{m}\right)}$
Maximum height h_{max}	$\frac{v_0^2\sin^2\theta}{2g}$	$\frac{v_0^2\sin^2\theta}{2\left(g+\frac{qE}{m}\right)}$
Range R	$\frac{v_0^2\sin 2\theta}{g}$	$\frac{v_0^2\sin 2\theta}{\left(g+\frac{qE}{m}\right)}$

Note that the time of flight, maximum height, range are all inversely proportional to the acceleration of the object. Since $\left(g+\frac{qE}{m}\right)>g$ for charge +q, the quantities T, h_{max}, and R will decrease when compared to the motion of an object of mass m and zero net charge. Suppose the charge is –q, then $\left(g-\frac{qE}{m}\right)<g$, and the quantities T, h_{max} and R will increase. Interestingly the trajectory is still parabolic as shown



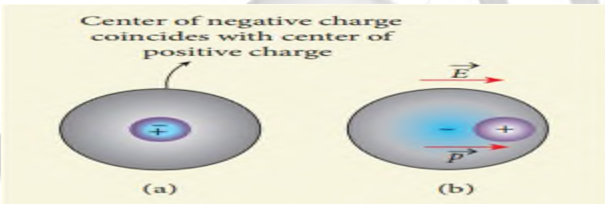
Dielectrics or insulators

A dielectric is a non-conducting material and has no free electrons. The electrons in a dielectric are bound within the atoms. Ebonite, glass and mica are some examples of dielectrics. When an external electric field is applied, the electrons are not free to move anywhere but they are realigned in a specific way. A dielectric is made up of either polar molecules or nonpolar molecules.

Non-polar molecules

A non-polar molecule is one in which centres of positive and negative charges coincide. As a result, it has no permanent dipole moment. Examples of non-polar molecules are hydrogen (H_2), oxygen (O_2), and carbon dioxide (CO_2) etc.

When an external electric field is applied, the centres of positive and negative charges are separated by a small distance which induces dipole moment in the direction of the external electric field. Then the dielectric is said to be polarized by an external electric field. This is shown

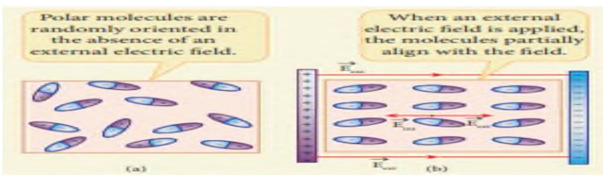


Non polar molecules (a) without external field (b) with the external field

Polar molecules

In polar molecules, the centres of the positive and negative charges are separated even in the absence of an external electric field. They have a permanent dipole moment. Due to thermal motion, the direction of each dipole moment is oriented randomly (Figure (a)). Hence the net dipole moment is zero in the absence of an external electric field. Examples of polar molecules are H_2O, N_2O, HCl, NH_3.

When an external electric field is applied, the dipoles inside the material tend to align in the direction of the electric field. Hence a net dipole moment is induced in it. Then the dielectric is said to be polarized by an external electric field (Figure (b)).



(a) Randomly oriented polar molecules (b) Align with the external electric field

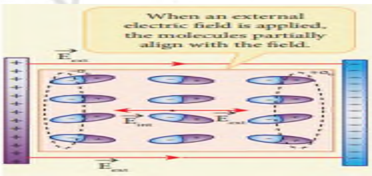
Polarisation

In the presence of an external electric field, the dipole moment is induced in the dielectric material. Polarisation $\vec{p}$ is defined as the total dipole moment per unit volume of the dielectric. For most dielectrics (linear isotropic), the Polarisation is directly proportional to the strength of the external electric field. This is written as $\vec{p} = c_e \vec{E}_{ext}$

Induced Electric field inside the dielectric

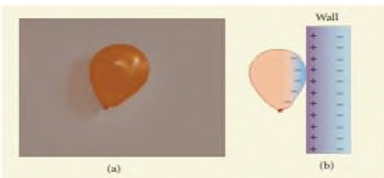
When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which tends to cancel the external electric field. But in the case of a dielectric, which has no free electrons, the external electric field only realigns the charges so that an internal electric field is produced. The magnitude of the internal electric field is smaller than that of external electric field. Therefore the net electric field inside the dielectric is not zero but is parallel to an external electric field with magnitude less than that of the external electric field. For example, let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor) as shown.

The uniform electric field between the plates acts as an external electric field $\vec{E}_{ext}$ which polarizes the dielectric placed between plates. The positive charges are induced on one side surface and negative charges are induced on the other side of surface. But inside the dielectric, the net charge is zero even in a small volume. So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in conductors. This is shown in the Figure



Induced electric field lines inside the dielectric

For example, the charged balloon after rubbing sticks onto a wall. The reason is that the negatively charged balloon is brought near the wall, it polarizes opposite charges on the surface of the wall, which attracts the balloon. This is shown



(a) Balloon sticks to the wall (b) Polarisation of wall due to the electric field created by the balloon

Dielectric strength

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown. The maximum electric field the dielectric can withstand before it breaksdown is called dielectric strength. For example, the dielectric strength of air is 3 × 106 V m–1.If the applied electric field increases beyond this, a spark is produced in the air. The dielectric strengths of some dielectrics are given in the Table

Dielectric strength

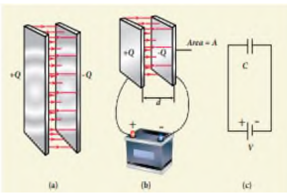
Substance	Dielectric strength (Vm^{-1})
Mica	100×10^{6}
Teflon	60×10^{6}
Paper	16×10^{6}
Air	3×10^{6}
Pyrex glass	14×10^{6}

CAPACITORS AND CAPACITANCE

Capacitors

Capacitor is a device used to store electric charge and electrical energy. It consists of two conducting objects (usually plates or sheets) separated by some distance. Capacitors are widely used in many electronic circuits and have applications in many areas of science and technology.

A simple capacitor consists of two parallel metal plates separated by a small distance as shown



(a) Parallel plate capacitor (b) Capacitor connected with a battery (c) Symbolic representation of capacitor.

When a capacitor is connected to a battery of potential difference V, the electrons are transferred from one plate to the other plate by battery so that one plate becomes negatively charged with a charge of
–Q and the other plate positively charged with +Q. The potential difference between the plates is equivalent to the battery's terminal voltage. This is shown in Figure 1.52 (b). If the battery voltage is increased, the amount of charges stored in the plates also increase. In general, the charge stored in the capacitor is proportional to the potential difference between the plates.

$$Q \propto V$$

so that Q = CV

Where the C is the proportionality constant called capacitance. The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between them.

$$C = \frac{Q}{V}$$

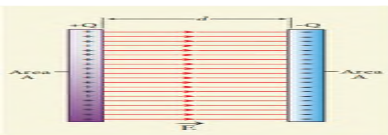
The SI unit of capacitance is coulomb per volt or farad (F) in honor of Michael Faraday. Farad is a larger unit of capacitance. In practice, capacitors are available in the range of microfarad ($1\mu F = 10^{-6}$ F) to picofarad ($1pF = 10^{-12}$ F). A capacitor is represented by the symbol or . Note that the total charge stored in the capacitor is zero (Q – Q = 0). When we say the capacitor stores charges, it means the amount of charge that can be stored in any one of the plates.

Nowadays there are capacitors available in various shapes (cylindrical, disk) and types (tantalum, ceramic and electrolytic), as shown. These capacitors are extensively used in various kinds of electronic circuits.

Capacitance of a parallel plate capacitor

Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d as shown.

The electric field between two infinite parallel plates is uniform and is given by $E = \frac{\sigma}{\varepsilon_0}$ where σ is the surface charge density on either plates $\left(\sigma = \frac{Q}{A}\right)$. If the separation distance d is very much smaller than the size of the plate ($d^2 << A$), then the above result can be used even for finite–sized parallel plate capacitor.



Capacitance of a parallel plate capacitor

The electric field between the plates is

$$E = \frac{Q}{A\varepsilon_0}$$

Since the electric field is uniform, the electric potential difference between the plates having separation d is given by

$$V = Ed = \frac{Qd}{A\epsilon_0}$$

Therefore the capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\epsilon_0}\right)} = \frac{A\epsilon_0}{d}$$

From equation, it is evident that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates. This can be understood from the following.

(i) If the area of cross-section of the capacitor plates is increased, more charges can be distributed for the same potential difference. As a result, the capacitance is increased.

(ii) If the distance d between the two plates is reduced, the potential difference between the plates (V = Ed) decreases with E constant. As a result, voltage difference between the terminals of the battery increases which in turn leads to an additional flow of charge to the plates from the battery, till the voltage on the capacitor equals to the battery's terminal voltage. Suppose the distance is increased, the capacitor voltage increases and becomes greater than the battery voltage. Then, the charges flow from capacitor plates to battery till both voltages becomes equal.

EXAMPLE

A parallel plate capacitor has square plates of side 5 cm and separated by a distance of 1 mm.

(a) Calculate the capacitance of this capacitor.
(b) If a 10 V battery is connected to the capacitor, what is the charge stored in any one of the plates?
(The value of ϵ_0 = 8.85×10^{-12} $N^{-1}m^{-2}C^2$)

Solution

(a) The capacitance of the capacitor is

$$C = \frac{A\epsilon_0}{d} = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{1 \times 10^{-3}}$$

$$= 221.2 \times 10^{-13} F$$

$$C = 22.12 \times 10^{-12} F = 22.12 pF$$

(b) The charge stored in any one of the plates is Q = CV, Then

$$Q = 22.12 \times 10^{-12} \times 10 = 221.2 \times 10^{-12} C$$

$$C = 221.2 pC$$

Energy stored in the capacitor

Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge –Q are transferred from one plate to the other plate. To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor.

To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

dW = V dQ

Where $V = \frac{Q}{C}$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C} dQ = \frac{Q^2}{2C}$$

This work done is stored as electrostatic potential energy (U_E) in the capacitor.

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} CV^2 \qquad (\because Q = CV)$$

Where Q = CV is used. This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor.

But where is this energy stored in the capacitor? To understand this question, the equation (1.87) is rewritten as follows using the results

$C = \frac{A\epsilon_0}{d}$ and V = Ed

$$U_E = \frac{1}{2}\left(\frac{A\epsilon_0}{d}\right)(Ed)^2 = \frac{1}{2}\epsilon_0 (Ad) E^2$$

Where Ad = volume of the space between the capacitor plates. The energy stored per unit volume of space is defined as energy density $U_E = \frac{U}{Volume}$. From the equation we get,

$$U_E = \frac{1}{2}\epsilon_0 E^2$$

From equation, we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.

It is important to note that the energy density depends only on the electric field and not on the size of the plates of the capacitor. In fact, expression (1.89) is true for the electric field due to any type of charge configuration.

Applications of capacitors

Capacitors are used in various electronics circuits. A few of the applications.

(a) Flash capacitors are used in digital cameras for taking photographs. The flash which comes from the camera when we take photographs is due to the energy released from the capacitor, called a flash capacitor
(b) During cardiac arrest, a device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function.
(c) Capacitors are used in the ignition system of automobile engines to eliminate sparking
(d) Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

However, capacitors have disadvantage as well. Even after the battery or power supply is removed, the capacitor stores charges and energy for some time. For example if the TV is switched off, it is always advisable to not touch the back side of the TV panel

Effect of dielectrics in capacitors

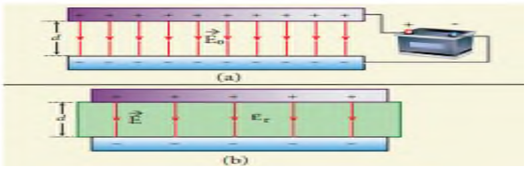
In earlier discussions, we assumed that the space between the parallel plates of a capacitor is either empty or filled with air. Suppose dielectrics like mica, glass or paper are introduced between the plates, then the capacitance of the capacitor is altered. The dielectric can be inserted into the plates in two different ways. (i) when the capacitor is disconnected from the battery. (ii) when the capacitor is connected to the battery.

(i) When the capacitor is disconnected from the battery

Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V_0 and the charge stored is Q_0. The capacitance of the capacitor without the dielectric is

$$C_0 = \frac{Q_0}{V_0}$$

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates.



(a) Capacitor is charged with a battery (b) Dielectric is inserted after the battery is disconnected

The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified electric field is given by

$$E = \frac{E_0}{e_r}$$

where E_0 is the electric field inside the capacitors when there is no dielectric and e_r is the relative permittivity of the dielectric or simply known as the dielectric constant. Since $e_r > 1$, the electric field $E < E_0$.

As a result, the electrostatic potential difference between the plates (V = Ed) is also reduced. But at the same time, the charge Q_0 will remain constant once the battery is disconnected.

Hence the new potential difference is

$$V = Ed = \frac{E_0}{e_r} d = \frac{V_0}{e_r}$$

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases.

Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q_0}{V} = e_r \frac{Q_0}{V_0} = e_r C_0$$

Since $\varepsilon_r > 1$, we have C > Co. Thus insertion of the dielectric increases the capacitance.

Using equation

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

Where $\varepsilon = \varepsilon_r \varepsilon_0$ is the permittivity of the dielectric medium.

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$$

After the dielectric is inserted, the charge Q_0 remains constant but the capacitance is increased. As a result, the stored energy is decreased.

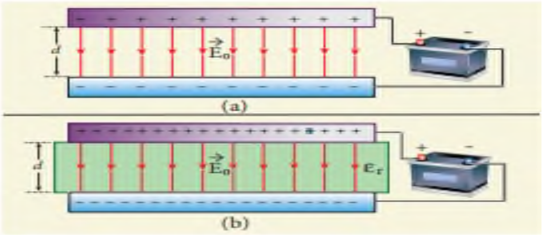
$$U = \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\frac{Q_0^2}{\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r}$$

Since $\varepsilon_r > 1$ we get $U < U_0$. There is a decrease in energy because, when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor

Let us now consider what happens when the battery of voltage V_0 remains connected to the capacitor when the dielectric is inserted into the capacitor.

The potential difference V_0 across the plates remains constant. But it is found experimentally (first shown by Faraday) that when dielectric is inserted, the charge stored in the capacitor is increased by a factor ε_r.



(a) Capacitor is charged through a battery (b) Dielectric is inserted when the battery is connected.

$$Q = \varepsilon_r Q_0$$

Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q}{V_0} = e_r \frac{Q_0}{V_0} = e_r C_0$$

However the reason for the increase in capacitance in this case when the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

Now, $C_0 = \frac{A e_0}{d}$

And $C = \frac{eA}{d}$

The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2} C_0 V_0^2$$

Note that here we have not used the expression $U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$ because here, both charge and capacitance are changed, whereas in equation (1.100), V_0 remains constant.

After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2} e_r C_0 V_0^2 = e_r U_0$$

Since $e_r > 1$ we have $U > U_0$.

It may be noted here that since voltage between the capacitor V_0 is constant, the electric field between the plates also remains constant.

The energy density is given by

$$u = \frac{1}{2} e E_0^2$$

Where e is the permittivity of the given dielectric material.

The results of the above discussions are summarised in the following Table

Table 1.2 Effoect of dielectrics in capacitors

S. No	Dielectric is nserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

EXAMPLE

A parallel plate capacitor filled with mica having ε_r= 5 is connected to a 10 V battery. The area of each parallel plate is 6 cm2 and separation distance is 6 mm. (a) Find the capacitance and stored charge.

(b) After the capacitor is fully charged, the battery is disconnected and the dielectric is removed carefully.

Calculate the new values of capacitance, stored energy and charge.

Solution

(a) The capacitance of the capacitor in the presence of dielectric is

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{5 \times 8.85 \times 10^{-12} \times 6 \times 10^{-4}}{6 \times 10^{-3}}$$
$$= 44.25 \times 10^{-13} F = 4.425\ pF$$

The stored charge is

$$Q = CV = 44.25 \times 10^{-13} \times 10$$
$$= 44.25 \times 10^{-13} C = 44.25\ pC$$

The stored energy is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 44.25C \times 10^{-13} \times 100$$
$$= 2.21 \times 10^{-10}\ J$$

(b) After the removal of the dielectric, since the battery is already disconnected the total charge will not change. But the potential difference between the plates increases. As a result, the capacitance is decreased.

New capacitance is

$$C_0 = \frac{C}{\varepsilon_r} = \frac{4.425 \times 10^{-12}}{5}$$

$$= 0.885 \times 10^{-12} F = 0.885 pF$$

The stored charge remains same and 44.25 pC. Hence newly stored energy is

$$U_0 = \frac{Q^2}{2C_0} = \frac{Q^2 \varepsilon_r}{2C} = \varepsilon_r U$$

$$= 5 \times 2.21 \times 10^{-10} J = 11.05 \times 10^{-10} J$$

The increased energy is

$\Delta U = (11.05 – 2.21) \times 10^{-10}$ J = 8.84×10^{-10} J

When the dielectric is removed, it experiences an inward pulling force due to the plates. To remove the dielectric, an external agency has to do work on the dielectric which is stored as additional energy. This is the source for the extra energy 8.84 × 10–10 J.

Capacitor in series and parallel

(i) Capacitor in series

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V as shown in the Figure 1.58 (a).

As soon as the battery is connected to the capacitors in series, the electrons of charge

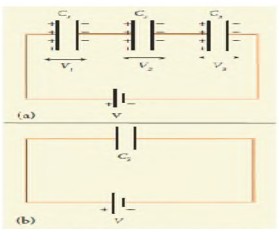


Figure 1.58 (a) Capacitors connected in series (b) Equivalent capacitors C_s

–Q are transferred from negative terminal to the right plate of C_3 which pushes the electrons of same amount –Q from left plate of C_3 to the right plate of C_2 due to electrostatic induction. Similarly, the left plate of C_2 pushes the charges of –Q to the right plate of C_1 which induces the positive charge +Q on the left plate of C_1. At the same time, electrons of charge –Q are transferred from left plate of C_1 to positive terminal of the battery.

By these processes, each capacitor stores the same amount of charge Q. The capacitances of the capacitors are in general different, so that the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively.

The sum of the voltages across the capacitor must be equal to the voltage of the battery.
V = V1 + V2 + V3 (1.103)

Since, Q = CV, we have V = $\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}$

$$= Q\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)$$

If three capacitors in series are considered to form an equivalent single capacitor Cs shown in Figure 1.58(b), then we have V = $\frac{Q}{C_S}$ Substituting this expression into equation (1.104), we get

$$\frac{Q}{C_S} = Q\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)$$

$$\frac{1}{C_S} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

Thus, the inverse of the equivalent capacitance C_S of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance C_S is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V as shown in Figure 1.59 (a).

Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage. Since capacitances of the capacitors are different

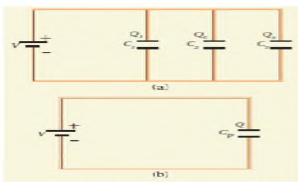


Figure 1.59 (a) capacitors in parallel (b) equivalent capacitance with the same total charge

the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q_1, Q_2, and Q_3 respectively. According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$$Q = Q_1 + Q_2 + Q_3$$

Since Q = CV, we have

$$Q = C_1V + C_2V + C_3V$$

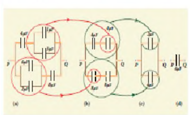
If these three capacitors are considered to form a single equivalent capacitance CP which stores the total charge Q as shown in the Figure 1.59(b), then we can write Q = CPV. Substituting this in equation (1.107), we get

$$C_PV = C_1V + C_2V + C_3V$$
$$C_P = C_1 + C_2 + C_3$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances. The equivalent capacitance CP in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

EXAMPLE

Find the equivalent capacitance between P and Q for the configuration shown below in the figure (a).



Solution

The capacitors 1 μF and 3 μF are connected in parallel and 6 μF and 2 μF are also separately connected in parallel. So these parallel combinations reduced to equivalent single capacitances in their respective positions, as shown in the figure (b).

$$C_{eq} = 1 + 3 = 4\ \mu F$$
$$C_{eq} = 6 + 2 = 8\ \mu F$$

From the figure (b), we infer that the two 4 μF capacitors are connected in series and the two 8 μF capacitors are connected in series. By using formula for the series, we can reduce to their equivalent capacitances as shown in figure (c).

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} + \frac{1}{2} \quad \Rightarrow \quad C_{eq} = 2\mu F$$

and

$$\frac{1}{C_{eq}} = \frac{1}{8}+\frac{1}{8}+\frac{1}{4} \quad \Rightarrow \quad C_{eq} = 4\mu F$$

From the figure (c), we infer that 2 μF and 4 μF are connected in parallel. So the equivalent capacitance is given in the figure (d).
C_{eq} = 2 + 4 = 6 μF

Thus the combination of capacitances in figure (a) can be replaced by a single capacitance 6 μF.

DISTRIBUTION OF CHARGES IN A CONDUCTOR AND ACTION AT POINTS

Distribution of charges in a conductor

Consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a thin conducting wire as shown in the Figure 1.60. The distance between the spheres is much greater than the radii of either spheres.

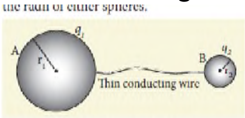


Figure 1.60 Two conductors are connected through conducting wire

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium. Let q_1 be the charge residing on the surface of sphere A and q_2 is the charge residing on the surface of sphere B such that $Q = q_1 + q_2$. The charges are distributed only on the surface and there is no net charge inside the conductor.

The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

VA = VB or $\frac{q_1}{r_1} = \frac{q_2}{r_2}$

Let the charge density on the surface of sphere A be σ_1 and that on the surface of sphere B be σ_2. This implies that $q_1 = 4\pi r_1^2 \sigma_1$ and
$q_2 = 4\pi r_2^2 \sigma_2$. Substituting these values into equation (1.112), we get

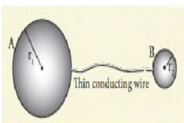
$\sigma_1 r_1 = \sigma_2 r_2$ (1.113)

from which we conclude that
σr = constant

Thus the surface charge density σ is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

EXAMPLE

Two conducting spheres of radius r_1 = 8 cm and r_2 = 2 cm are separated by a distance much larger than 8 cm and are connected by a thin conducting wire as shown in the figure. A total charge of Q = +100 nC is placed on one of the spheres. After a fraction of a second, the charge Q is redistributed and both the spheres attain electrostatic equilibrium.



(a) Calculate the charge and surface charge density on each sphere.
(b) Calculate the potential at the surface of each sphere.

Solution

(a) The electrostatic potential on the surface of the sphere A is

$$V_A = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1}$$

The electrostatic potential on the surface of the sphere B is $V_B = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$

Since $V_A = V_B$. We have

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow q_1 = \left(\frac{r_1}{r_2}\right) q_2$$

But from the conservation of total charge, $Q = q_1 + q_2$, we get $q_1 = Q - q_2$. By substituting this in the above equation

$$Q - q_2 = \left(\frac{r_1}{r_2}\right) q_2$$

So that $q_2 = Q\left(\frac{r_2}{r_1+r_2}\right)$

Therefore,

$q_2 = 100\times10^{-9}\times\left(\frac{2}{10}\right) = 20\,nC$ and $q_1 = Q - q_2 = 80nC$

The electric charge density on sphere A is $\sigma_1 = \frac{q_1}{4\pi r_1^2}$

The electric charge density on sphere B is $\sigma_2 = \frac{q_2}{4\pi r_2^2}$

Therefore,

$\sigma_1 = \frac{80\times10^{-9}}{4\pi\times64\times10^{-4}} = 0.99\times10^{-6}\,Cm^{-2}$ and $\sigma_2 = \frac{20\times10^{-9}}{4\pi\times4\times10^{-4}} = 3.9\times10^{-6}\,Cm^{-2}$

Note that the surface charge density is greater on the smaller sphere compared to the larger sphere ($\sigma_2 \approx 4\sigma_1$) which confirms the result $\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$

The potential on both spheres is the same. So we can calculate the potential on any one of the spheres.

$$V_A = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = \frac{9\times10^{9}\times80\times10^{-9}}{8\times10^{-2}} = 9KV$$

Action of points or Corona discharge

Consider a charged conductor of irregular shape as shown in Figure 1.61 (a).

We know that smaller the radius of curvature, the larger is the charge density. The end of the conductor which has larger curvature (smaller radius) has a large charge accumulation as shown in Figure 1.61 (b).

As a result, the electric field near this edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action of points or corona discharge.

1.9.3 Lightning arrester or lightning conductor

This is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge.

This device consists of a long thick copper rod passing from top of the building to the ground. The upper end of the rod has a sharp spike or a sharp needle as shown in Figure 1.62 (a) and (b).

The lower end of the rod is connected to copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building, it induces a positive charge on the spike. Since the induced charge density on thin sharp spike is large, it results in a corona discharge. This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud. The negative charge pushed to the spikes passes through the copper rod andis safely diverted to the Earth. The lightning arrester does not stop the lightning; rather it diverts the lightning to the ground safely

Van de Graaff Generator

In the year 1929, Robert Van de Graaff designed a machine which produces a large amount of electrostatic potential difference, up to several million volts (107 V). This Van de Graff generator works on the principle of electrostatic induction and action at points.

A large hollow spherical conductor is fixed on the insulating stand as shown in Figure 1.63. A pulley B is mounted at the centre of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb shaped metallic conductors E and D are fixed near the pulleys.

The comb D is maintained at a positive potential of 104 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

Due to the high electric field near comb D, air between the belt and comb D gets ionized by the action of points. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges on the belt reach the point near the comb E, the comb E acquires negative charge and the sphere acquires positive charge due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductorthe positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.

When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of 107 which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can

be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications

E X A M P L E 1.23

Dielectric strength of air is 3 × 106 V m–1. Suppose the radius of a hollow sphere in the Van de Graff generator is R = 0.5 m, calculate the maximum potential difference created by this Van de Graaff generator.
Solution

The electric field on the surface of the sphere is given by (by Gauss law)

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$$

The potential on the surface of the hollow metallic sphere is given by

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = ER$$

Since $V_{max} = E_{max}R$
Here $E_{max}=3\times10^6$ Vm^{-1}. 1So the maximum potential difference created is given by

$V_{max} = 3 \times 10^6 \times 0.5$
$= 1.5 \times 10^6$ V (or) 1.5 million volt
Like charges repel and unlike charges attract

12th Physics
2nd lesson
CURRENT ELECTRICITY

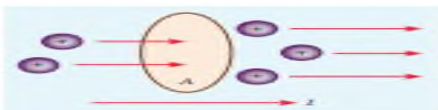
ELECTRIC CURRENT

Matter is made up of atoms. Each atom consists of a positively charged nucleus with negatively charged electrons moving around the nucleus. Atoms in metals have one or more electrons which are loosely bound to the nucleus. These electrons are called free electrons and can be easily detached from the atoms. The substances which have an abundance of these free electrons are called conductors. These free electrons move randomly throughout the conductor at a given temperature. In general due to this random motion, there is no net transfer of charges from one end of the conductor to other end and hence no current in the conductor. When a potential difference is applied by the battery across the ends of the conductor, the free electrons drift towards the positive terminal of the battery, producing a net electric current. This is easily understandable from the analogy given

Water current and Electric current

In the XI Volume 2, unit 6, we studied, that the mass move from higher gravitational potential to lower gravitational potential. Likewise, positive charge flows from region of higher electric potential to region of lower electric potential and negative charge flows from region of lower electric potential to region of higher electric potential. So battery or electric cell simply creates potential difference across the conductor.

The electric current in a conductor is defined as the rate of flow of charges through a given cross-sectional area A. It is shown



Charges flow across the area A

If a net charge Q passes through any cross section of a conductor in time t, then the current is defined as I=Q/t. But charge flow is not always constant. Hence current can more generally be defined as

$$I_{avg} = \Delta Q / \Delta t$$

Where ΔQ is the amount of charge that passes through the conductor at any cross section during the time interval Δt.If the rate at which charge flows changes with time, the current also changes. The instantaneous current I is defined as the limit of the average current, as $\Delta t \to 0$

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

$$1A = \frac{1C}{1s}$$

That is, 1A of current is equivalent to 1 coulomb of charge passing through a perpendicular cross section in a conductor in one second. The electric current is a scalar quantity.

EXAMPLE

Compute the current in the wire if a charge of 120 C is flowing through a copper wire in 1 minute.

Solution

The current (rate of flow of charge) in the wire is

$$I = \frac{Q}{t} = \frac{120}{60} = 2A$$

Conventional Current

Direction of conventional current and electron flow

In an electric circuit, arrow heads are used to indicate the direction of flow of current. By convention, this flow in the circuit should be from the positive terminal of the battery to the negative terminal. This current is called the conventional current or simply current and is in the direction in which a positive test charge would move. In typical circuits the charges that flow are actually electrons, from the negative terminal of the battery to the positive terminal. As a result, the flow of electrons and the direction of conventional current point in opposite direction as shown. Mathematically, a transfer of positive charge is thesame as a transfer of negative charge in the opposite direction.

- Electric current is not only produced by batteries. In nature, lightning bolt produces enormous electric current in a short time. During lightning, very high potential difference is created between the clouds and ground and hence charges flow between the clouds and ground.

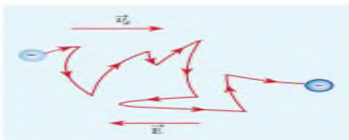
Drift velocity

In a conductor the charge carriers are free electrons. These electrons move freely through the conductor and collide repeatedly with the positive ions. If there is no electric field, the electrons move in random directions, and hence their velocities are also randomly oriented. On an average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction. As a result, there is no net flow of electrons in any direction and hence there will not be any current.

Suppose a potential difference is set across the conductor by connecting a battery, an electric field $\vec{E}$ is created in the conductor. This electric field exerts a force on the electrons, producing a current. The electric field accelerates the electrons, while ions scatter the electrons and change their direction of motion. Thus, we see zigzag motion of electrons. In addition to the zigzag motion due to the collisions, the electrons move slowly along the conductor in a direction opposite to that of $\vec{E}$ as shown.

Ions

Any material is made up of neutral atoms with equal number of electrons and protons. If the outermost electrons leave the atoms, they become free electrons and are responsible for electric current. The atoms after losing their outer most electrons will have more positive charges and hence are called positive ions. These ions will not move freely within the material like the free electrons. Hence the positive ions will not give rise to current.



Zig-zag motion and drift velocity

This velocity is called drift velocity $\vec{V_d}$. The drift velocity is the average velocity acquired by the electrons inside the conductor whenit is subjected to an electric field. The average time between two successive collisions is called the mean free time denoted by τ. The acceleration $\vec{a}$ experienced by the electron in an electric field $\vec{E}$ is given by

$$\vec{a} = \frac{-e\vec{E}}{m} \quad (\text{since } \vec{F} = -e\vec{E})$$

The drift velocity $\vec{V_d}$ is given by

$$\vec{V_d} = \vec{a}\tau$$

$$\vec{V_d} = -\frac{e\tau}{m}\vec{E}$$

$$\vec{V_d} = -\mu\vec{E}$$

EXAMPLE

If an electric field of magnitude 570 N C^{-1}, is applied in the copper wire, find the acceleration experienced by the electron.

Solution

$E = 570$ N C^{-1}, $e = 1.6 \times 10^{-19}$ C,

$m = 9.11 \times 10^{-31}$ kg and a = ?

F = ma = eE

$$a = \frac{eE}{m} = \frac{570 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}$$

$$= \frac{912 \times 10^{-19} \times 10^{31}}{9.11}$$

$$= 1.001 \times 10^{14} ms^{-2}$$

Misconception

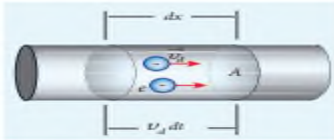
i. There is a common misconception that the battery is the source of electrons. It is not true. When a battery is connected across the given wire, the electrons in the closed circuit resulting the current. Battery sets the potential difference (electrical energy) due to which these electrons in the conducting wire flow in a particular direction. The resulting electrical energy is used by electric bulb, electric fan etc. Similarly the electricity board is supplying the electrical energy to our home.

ii. We often use the phrases like 'charging the battery in my mobile' and 'my mobile phone battery has no charge' etc. These sentences are not correct.

When we say 'battery has no charge', it means, that the battery has lost ability to provide energy or provide potential difference to the electrons in the circuit. When we say 'mobile is charging', it implies that the battery is receiving energy from AC power supply and not electrons.

Microscopic model of current

Consider a conductor with area of cross section A and let an electric field $\vec{E}$ be applied to it from right to left. Suppose there are n electrons per unit volume in the conductor and assume that all the electrons move with the same drift velocity $\vec{V}_d$ as shown.



Microscopic model of current

The drift velocity of the electrons = v_d

If the electrons move through a distance dx within a small interval of dt, then

$$V_d = \frac{dx}{dt}; dx = v_d dt$$

Since A is the area of cross section of the conductor, the electrons available in the volume of length dx is

= volume × number of electrons per unit volume

$$= Adx \times n$$

Substituting for dx from equation (2.7) in (2.8)

= (A $v_d$dt) n

Total charge in the volume element dQ = (charge) × (number of electrons in the volume element)

dQ = (e)(A$v_d$dt)n

Hence the current $I = \frac{dQ}{dt}$

$$I = \frac{dQ}{dt}$$

Current density (J)

The current density (J) is defined as the current per unit area of cross section of the conductor.

$$J = \frac{I}{A}$$

The S.I unit of current density is A / m2 (or) A m^{-2}

$$J = \frac{neAV_d}{A}$$

$$J = neV_d$$

The above expression is valid only when the direction of the current is perpendicular to the area A. In general, the current density is a vector quantity and it is given by

$$\vec{J} = ne\vec{V_d}$$

Substituting $\vec{V_d}$ from equation (2.4)

$$\vec{J} = -\frac{n.e^2\tau}{m}\vec{E}$$

$$\vec{J} = -\sigma\vec{E}$$

But conventionally, we take the direction of (conventional) current density as the direction of electric field. So the above equation becomes

$$\vec{J} = \sigma\vec{E}$$

Where $\sigma = \frac{n.e^2\tau}{m}$ is called conductivity. The equation (2.12) is called microscopic form of ohm's law.

The inverse of conductivity is called resistivity (ρ) [Refer section 2.2.1]

$$\rho = \frac{1}{\sigma} = \frac{n.e^2\tau}{m}$$

EXAMPLE

A copper wire of cross-sectional area 0.5 mm^2 carries a current of
0.2 A. If the free electron density of copper is 8.4×10^{28} m^{-3} then compute the drift velocity of free electrons.

Solution

The relation between drift velocity of electrons and current in a wire of crosssectional area A is

$$V_d = \frac{I}{neA} = \frac{0.2}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.5 \times 10^{-6}}$$

$$V_d = 0.03 \times 10^{-3} ms^{-1}$$

EXAMPLE

Determine the number of electrons flowing per second through a conductor, when a current of 32 A flows through it.

Solution

I = 32 A, t = 1 s
Charge of an electron, e = 1.6×10^{-19} C
The number of electrons flowing per second, n =?

$$I = \frac{q}{t} = \frac{ne}{t}$$

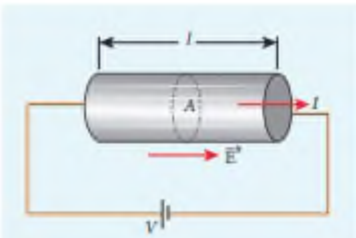
$$n = \frac{It}{e}$$

$$n = \frac{32 \times 1}{1.6 \times 10^{-19} C}$$

$$n = 20 \times 10^{19} = 2 \times 10^{20} electrons$$

OHM'S LAW

The ohm's law can be derived from the equation J = σE. Consider a segment of wire of length l and cross sectional area A as shown in Figure 2.7.



Current through the conductor

When a potential difference V is applied across the wire, a net electric field is created in the wire which constitutes the current in the wire. For simplicity, we assume that the electric field is uniform in the entire length of the wire, then the potential difference (voltage V) can be written as

$$V = El$$

As we know, the magnitude of current density

$$J = \sigma E = \sigma \frac{V}{l}$$

But $J = \frac{I}{A}$, so we write the equation

$$\frac{I}{A} = \sigma \frac{V}{l}$$

By rearranging the above equation, we get

$$V = I\left(\frac{l}{\sigma A}\right)$$

The quantity $\frac{l}{\sigma A}$ is called resistance of the conductor and it is denoted as R. Note that the resistance is directly proportional to the length of the conductor and inversely proportional to area of cross section.

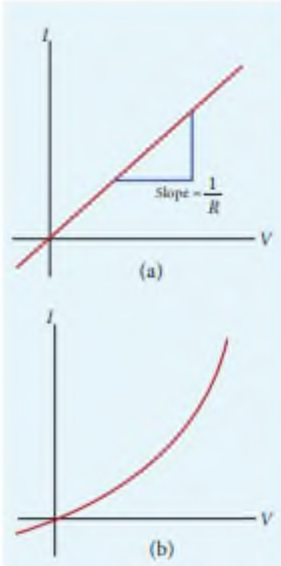
Therefore, the macroscopic form of ohm's law can be stated as

$$V = IR$$

From the above equation, the resistance is the ratio of potential difference across the given conductor to the current passing through the conductor.

$$R = \frac{V}{I}$$

The SI unit of resistance is ohm (Ω). From the equation (2.16), we infer that the graph between current versus voltage isstraight line with a slope equal to the inverse of resistance R of the conductor. It is shown in the Figure 2.8 (a).



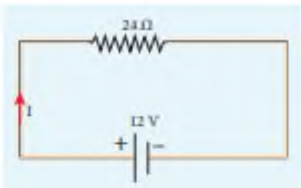
Current against voltage for
(a) a conductor which obeys Ohm's law and
(b) for a non-ohmic device (Diode given in XII physics, unit 9 is an example of a non-ohmic device)

Materials for which the current versus voltage graph is a straight line through the origin, are said to obey Ohm's law and their behaviour is said to be ohmic as shown in Figure 2.8(a). Materials or devices that do not follow Ohm's law are said to be nonohmic. These materials have more complex relationships between voltage and current. A plot of I versus V for a non-ohmic material is non-linear and they do not have a constant resistance (Figure 2.8(b)).

E X AM P LE

A potential difference across 24 Ω resistor is 12 V. What is the current through the resistor?

Solution



V = 12 V and R = 24 Ω
Current, I = ?
From Ohm's law, $I = \frac{V}{R} = \frac{12}{24} = 0.5A$

Resistivity

In the previous section, we have seen that the resistance R of any conductor is given by

$$R = \frac{l}{s\,A}$$

Where σ is called the conductivity of the material and it depends only on the type of the material used and not on its dimension.

The resistivity of a material is equal to the reciprocal of its conductivity.

$$r = \frac{1}{s}$$

Now we can rewrite equation (2.18) using equation (2.19)

$$R = r\,\frac{l}{A}$$

The resistance of a material is directly proportional to the length of the conductor and inversely proportional to the area of cross section of the conductor. The proportionality constant ρ is called the resistivity of the material.

If l = 1 m and A = 1 m^2, then the resistance R = ρ. In other words, the electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section. The SI unit of ρ is ohm-metre (Ω m). Based on the resistivity, materials are classified as conductors, insulators and semiconductors. The conductors have lowest resistivity, insulators have highest resistivity and semiconductors have resistivity greater than conductors but less than insulators. The typical resistivity values of some conductors, insulators and semiconductors are given in the Table 2.1

Resistivity for various materials

Material	Resistivity, **ρ** (**Ω** m) at 20 ° C
Insulators	
Pure Water	2.5×10^5
Glass	$10^{10} – 10^{14}$
Hard Rubber	$10^{13} – 10^{16}$
NaCl	10^{14}
Fused Quartz	10^{16}
Semiconductors	
Germanium	0.46
Silicon	640
Conductors	
Silver	1.6×10^{-8}
Copper	1.7×10^{-8}

Aluminium	2.7×10^{-8}
Tungsten	5.6×10^{-8}
Iron	10×10^{-8}

EXAMPLE

The resistance of a wire is 20 Ω. What will be new resistance, if it is stretched uniformly 8 times its original length?

Solution

R1 = 20 Ω, R2 = ?

Let the original length of the wire (l_1) be l.
New length, $l_2 = 8l_1$ (i.,e) $l_2 = 8l$

Original resistance, $R_1 = r \frac{l_1}{A_1}$

New resistance $R_2 = r \frac{l_2}{A_2} = \frac{r\,(8l)}{A_2}$

Though the wire is stretched, its volume remains unchanged.

Initial volume = Final volume

$A_1 l_1 = A_2\, l_2$, $A_1 l = A_2\,(8l)$

$$\frac{A_1}{A_2} = \frac{8l}{l} = 8$$

By dividing equation for R_2 by equation for R_1, we get

$$\frac{R_2}{R_1} = \frac{r\,(8l)}{A_2} \times \frac{A_1}{r\,l}$$

$$\frac{R_2}{R_1} = \frac{A_1}{A_2} \times 8$$

Substituting the value of $\frac{A_1}{A_2}$, we get

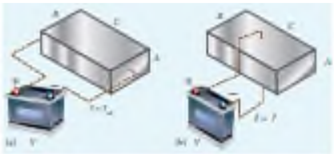
$$\frac{R_2}{R_1} = 8 \times 8 = 64$$

R2 = 64 × 20=1280 Ω

Hence, stretching the length of the wire has increased its resistance.

EXAMPLE

Consider a rectangular block of metal of height A, width B and length C as shown in the figure.



If a potential difference of V is applied between the two faces A and B of the block (figure (a)), the current I AB is observed. Find the current that flows if the same potential difference V is applied between the two faces B and C of the block (figure (b)). Give your answers in terms of I AB.

Solution

In the first case, the resistance of the block

$$R_{AB} = r\,\frac{length}{Area} = r\,\frac{C}{AB}$$

The current $I_{AB} = \frac{V}{R_{AB}} = \frac{V}{r}.\frac{AB}{C}$

In the second case, the resistance of the block $R_{BC} = r\,\frac{A}{BC}$

The current $I_{BC} = \frac{V}{R_{BC}} = \frac{V}{r}.\frac{BC}{A}$

To express I_{BC}interms of I_{AB}, we multiply and divide equation (2) by AC, we get

$$I_{BC} = \frac{V}{r}.\frac{BC}{A}\frac{AB}{C} = \left(\frac{V}{r}.\frac{AB}{C}\right)\frac{C^2}{A^2} = \frac{C^2}{A^2}.I_{AB}$$

Since C > A, the current I_{BC}> I_{AB}

v The human body contains a large amount of water which has low resistance of around 200 Ω and the dry skin has high resistance of around 500 k Ω. But when the skin is wet, the resistance is reduced to around1000 Ω. This is the reason why repairing the electrical connection with the wet skin is always dangerous.

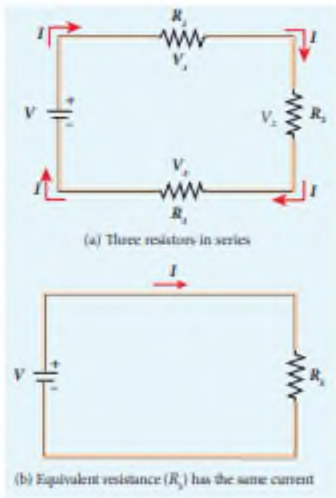
Resistors in series and parallel

An electric circuit may contain a number of resistors which can be connected in different ways. For each type of circuit, we can calculate the equivalent resistance produced by a group of individual resistors.

Resistors in series

When two or more resistors are connected end to end, they are said to be in series. The resistors could be simple resistors or bulbs or heating elements or other devices. Figure 2.9 (a) shows three resistors R1 , R2 and R3 connected in series.

The amount of charge passing through resistor R_1 must also pass through resistors R_2 and R_3 since the charges cannot accumulate anywhere in the circuit. Due to this reason, the current I passing through all the three resistors is the same. According to Ohm's law, if same current pass through different resistors of different values, then the potential difference across each resistor must be different.



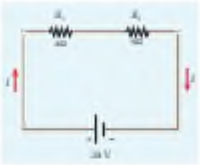
Resistors in series

If V_1, V_2 and V_3 be the potential differences (voltage) across each of the resistors R_1, R_2 and R_3 respectively, then we can write $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$. But the supply voltage V must be equal to the sum of voltages(potential differences) across each resistor.

$V = V_1 + V_2 + V_3 = IR1 + IR2 + IR3$ (2.21)
$V = I (R1 + R2 + R3)$
$V = IRS$

When several resistors are connected in series, the total or equivalent resistance is the sum of the individual resistances as shown in the Figure 2.9 (b). Note: The value of equivalent resistance in series connection will be greater than each individual resistance. EXAMPLE

2.8 Calculate the equivalent resistance for the circuit which is connected to 24 V battery and also find the potential difference across each resistors in the circuit.



Solution Since the resistors are connected in series, the effective resistance in the circuit = 4 Ω + 6 Ω = 10 Ω current I in the circuit

current I in the circuit $= \frac{V}{R_{eq}} = \frac{24}{10} = 2.4\,\text{A}$

Voltage across 4Ω resistor

$$V_1 = IR_1 = 2.4\,\text{A} \times 4\Omega = 9.6\,\text{V}$$

Voltage across 6 Ω resistor

$$V_2 = IR_2 = 2.4\,\text{A} \times 6\Omega = 14.4\,\text{V}$$

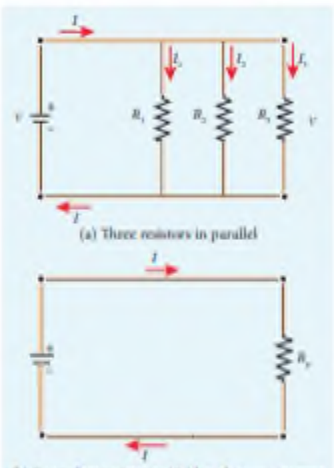
Resistors in parallel Resistors are in parallel when they are connected across the same potential difference as shown in Figure 2.10 (a). In this case, the total current I that leaves the battery is split into three separate components. Let I 1 , I 2 and I 3 be the current through the resistors R1 , R2 and R3 respectively. Due to the conservation of charge, total current in the circuit I is equal to sum of the currents through each of the three resistors. I = I1 + I2 + I3 (2.24) Since the voltage across each resistor is the same, applying Ohm's law to each resistor, we have

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \quad (2.25)$$

Substituting these values in equation (2.24), we get

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$I = \frac{V}{R_p}$$



(a) Three resistors in parallel

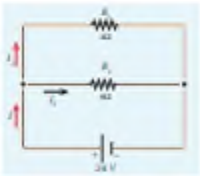
(b) Equivalent resistance (R_p) has the same current

Resistors in parallel

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Here RP is the equivalent resistance of the parallel combination of the resistors. Thus, when a number of resistors are connected in parallel, the sum of the reciprocals of resistance of the individual resistors is equal to the reciprocal of the effective resistance of the combination as shown in the Figure 2.10 (b). Note: The value of equivalent resistance in parallel connection will be lesser than each individual resistance. House hold appliances are always connected in parallel so that even if one is switched off, the other devices could function properly.

EXAMPLE 2.9 Calculate the equivalent resistance in the following circuit and also find the values of current I, I1 and I2 in the given circuit.



Solution Since the resistances are connected in parallel, the equivalent resistance in the circuit is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{R_P} = \frac{5}{12}\Omega \quad \text{or } R_P = \frac{12}{5}\Omega$$

The resistors are connected in parallel, the potential diffrence (voltage) across them is the same.

$$I_1 = \frac{V}{R_1} = \frac{24\text{V}}{4\Omega} = 6\text{A}$$

$$I_2 = \frac{V}{R_2} = \frac{24}{6} = 4\text{A}$$

The current I is the sum of the currents in the two branches. Then, I = I1 + I2 = 6 A + 4 A = 10 A

EXAMPLE 2.10 Two resistors when connected in series and parallel, their equivalent resistances are 15 Ω and 56 15Ω respectively. Find the values of the resistances.

Solution

$$R_s = R_1 + R_2 = 15\,\Omega \qquad (1)$$

$$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{56}{15}\,\Omega \qquad (2)$$

From equation (1) substituting for $R_1 + R_2$ in equation (2)

$$\frac{R_1 R_2}{15} = \frac{56}{15}\,\Omega$$

$$\therefore R_1 R_2 = 56$$

$$R_2 = \frac{56}{R_1}\,\Omega \qquad (3)$$

Substituting for R_2 in equation (1) from equation (3)

$$R_1 + \frac{56}{R_1} = 15$$

Then, $\frac{R_1^2 + 56}{R_1} = 15$

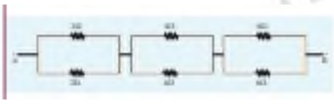
$$R_1^2 + 56 = 15\,R_1$$

R1

2

+ 56 = 15 R1
R1
2
-15 R1
+ 56 = 0
The above equation can be solved using
factorisation.
R1 = 8 Ω (or) R1 = 7 Ω
If R1
= 8 Ω
Substituting in equation (1)
8 + R2 = 15
R2
= 15 – 8 = 7 Ω ,
R2
= 7 Ω i.e , (when R1
= 8 Ω ; R2
= 7 Ω)
If R1
= 7 Ω
Substituting in equation (1)
7 + R2 = 15
R2
= 8 Ω ,i.e , (when R1
= 7 Ω ; R2
= 8 Ω)

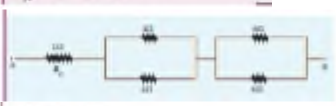
EXAMPLE 2.11 Calculate the equivalent resistance between A and B in the given circuit



Solution In all the sections, the resistors are connected in parallel. Section 1

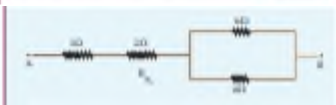
$$\frac{1}{R_{P_1}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{P_1}} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \quad R_{P_1} = 1\Omega$$



Section II

$$\frac{1}{R_{P_2}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}, \ \frac{1}{R_{P_2}} = \frac{1}{2}, \ R_{P_2} = 2\Omega$$



Section III

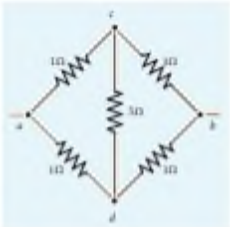
$$\frac{1}{R_{P_3}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$\frac{1}{R_{P_3}} = \frac{1}{3}, \ R_{P_3} = 3\,\Omega$$

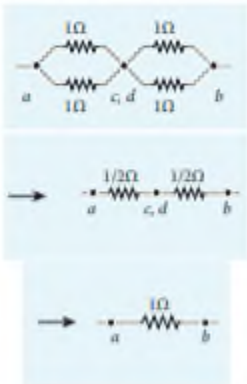
Equivalent resistance is given by R R = +p p R R + p 1 2 3 R = 1 Ω + 2 Ω + 3 Ω = 6 Ω The circuit becomes,

Equivalent resistance between A and B is

EXAMPLE 2.12 Five resistors are connected in the configuration as shown in the figure. Calculate the equivalent resistance between the points a and b.



Solution Case (a) To find the equivalent resistance between the points a and b, we assume that a current is entering the junction at a. Since all the resistances in the outside loop are the same (1Ω), the current in the branches ac and ad must be equal. Hence the points C and D are at the same potential and no current through 5 Ω. It implies that the 5 Ω has no role in determining the equivalent resistance and it can be removed. So the circuit is simplified as shown in the figure.



The equivalent resistance of the circuit between a and b is R eq = 1 Ω

Colour code for Carbon resistors

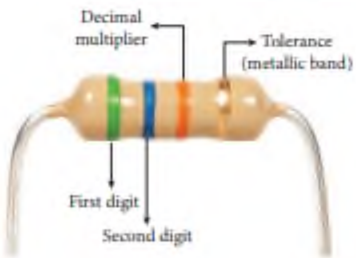
Resistances used in laboratory

Carbon resistors consists of a ceramic core, on which a thin layer of crystalline carbon is deposited as shown in Figure 2.11. These resistors are inexpensive, stable and compact in size. Colour rings are used to indicate the value of the resistance according to the rules given in the Table 2.2. Three coloured rings are used to indicate the values of a resistor: the first two rings are significant figures of resistances, the third ring indicates the decimal multiplier after them. The fourth colour, silver or gold,

Table 2.2 Colour Coding for Resistors

Colour	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Sliver		10^{-2}	10%
Colourless			20%

shows the tolerance of the resistor at 10% or 5% as shown in the Figure 2.12 .If there is no fourth ring, the tolerance is 20%. For the resistor shown in Figure 2.12, the first digit = 5 (green), the second digit = 6 (blue), decimal multiplier = 103 (orange) and tolerance = 5% (gold). The value of resistance = 56 × 103 Ω or 56 kΩ with the tolerance value 5%.



Resistor colour coding

Temperature dependence of resistivity The resistivity of a material is dependent on temperature. It is experimentally found that for a wide range of temperatures, the resistivity of a conductor increases with increase in temperature according to the expression,

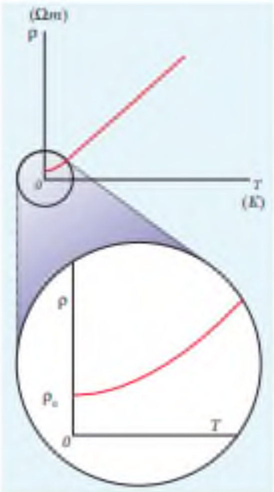
$$\rho_T = \rho_\circ [1 + \alpha (T - T_\circ)]$$

whereρT is the resistivity of a conductor at T o C, ρo is the resistivity of the conductor at some reference temperature To (usually at 20o C) and α is the temperature coefficient of resistivity. It is defined as the ratio of increase in resistivity per degree rise in temperature to its resistivity at To . From the equation (2.27), we can write

$$\rho_T - \rho_\circ = \alpha \rho_\circ (T - T_\circ)$$

$$\therefore \alpha = \frac{\rho_T - \rho_\circ}{\rho_\circ (T - T_\circ)} = \frac{\Delta\rho}{\rho_\circ \Delta T}$$

where Δρ = ρT – ρo is change in resistivity for a change in temperature ΔT = T – To . Its unit is per o C. α of conductors For conductors α is positive. If the temperature of a conductor increases, the average kinetic energy of electrons in the conductor increases. This results in more frequent collisions and hence the resistivity increases. The graph of the equation (2.27) is shown in Figure 2.13. Even though, the resistivity of conductors like metals varies linearly for wide range of temperatures, there also exists a nonlinear region at very low temperatures. The resistivity approaches some finite value as the temperature approaches absolute zero as shown in Figure 2.13(b).



(a) Temperature dependence of resistivity for a conductor (b) Non linear region at low temperature

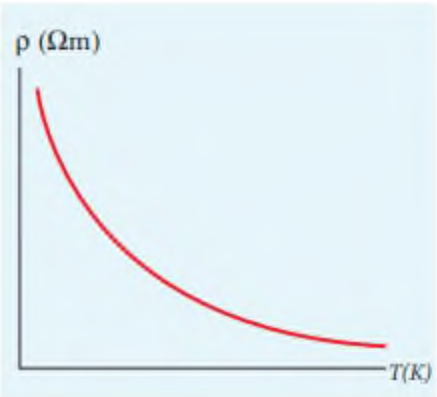
Using the equation ρ= R A l in equation (2.27), we get the expression for the resistance of a conductor at temperature T o C as R R T = +□ ( ) T T − □ □ □ oo 1 α (2.28) The temperature coefficient of resistivity can also be obtained from the equation (2.28),

$$R_T - R_o = \alpha R_o (T - T_o)$$

$$\therefore \alpha = \frac{R_T - R_o}{R_o(T - T_o)} = \frac{1}{R_o}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{R_o}\frac{\Delta R}{\Delta T} \quad (2$$

where ΔR R = −T Ro is change in resistance during the change in temperature ΔT T = −To α of semiconductors For semiconductors, the resistivity decreases with increase in temperature. As the temperature increases, more electrons will be liberated from their atoms (Refer unit 9 for conduction in semi conductors).



Temperature dependence of resistivity for a semiconductor

Hence the current increases and therefore the resistivity decreases as shown in Figure 2.14. A semiconductor with a negative temperature coefficient of resistivity is called a thermistor. The typical values of temperature coefficients of various materials are given in table 2.3.

Table 2.3

Material	Temperature Coefficient of resistivity α $[(°C)^{-1}]$
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.92×10^{-3}
Lead	3.9×10^{-3}
Nichrome	0.4×10^{-3}
Carbon	-0.5×10^{-3}
Germanium	-48×10^{-3}
Silicon	-75×10^{-3}

We can understand the temperature dependence of resistivity in the following way. In section 2.1.3, we have shown that the electrical conductivity, σ τ = ne m 2 . As the resistivity is inverse of σ, it can be written as

$$\rho = \frac{m}{ne^2\tau}$$

The resistivity of materials is i) inversely proportional to the number density (n) of the electrons ii) inversely proportional to the average time between the collisions (τ). In metals, if the temperature increases, the average time between the collision (τ) decreases and n is independent of temperature. In semiconductors when temperature increases, n increases and τ decreases, but increase in n is dominant than decreasing τ, so that overall resistivity decreases.

- v The resistance of certain materials become zero below certain temperature Tc. This temperature is known as critical temperature or transition temperature. The materials which exhibit this property are known as superconductors. This phenomenon was first observed by KammerlinghOnnes in 1911. He found that mercury exhibits superconductor behaviour at 4.2 K. Since R = 0, current once induced in a superconductor persists without any potential difference.

EXAMPLE 2.13 If the resistance of coil is 3 Ω at 200 C and α = 0.004/ 0 C then determine its resistance at 100 0 C.

Solution

$R_0 = 3\,\Omega, \quad T = 100^0C, \quad T_0 = 20^0C$

$\alpha = 0.004/^0C, \quad R_T = ?$

$R_T = R_0(1 + \alpha(T - T_0))$

$R_{100} = 3(1 + 0.004 \times 80)$

$R_{100} = 3.96\,\Omega$

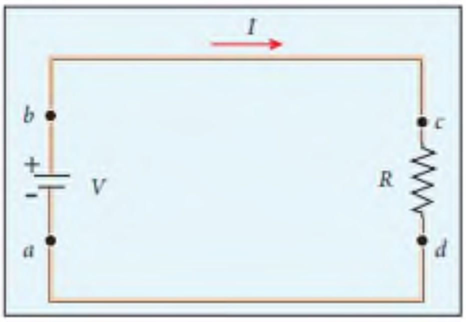
EXAMPLE 2.14 Resistance of a material at 200 C and 400 C are 45 Ω and 85 Ω respectively. Find its temperature coefficient of resistivity. Solution T0 = 20 0 C, T = 40 0 C, R0 = 45 Ω , R= 85 Ω

$$\alpha = \frac{1}{R_o}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{45}\left(\frac{85-45}{40-20}\right) = \frac{1}{45}(2)$$

$$\alpha = 0.044 \text{ per}^\circ C$$

ENERGY AND POWER IN ELECTRICAL CIRCUITS When a battery is connected between the ends of a conductor, a current is established. The battery is supplying energy to the device which is connected in the circuit. Consider a circuit in which a battery of voltage V is connected to the resistor as shown in Figure 2.15. Assume that a positive charge of dQ moves from point a to b through the battery and moves from point c to d through the resistor and back to point a. When the charge



Energy given by the battery

moves from point a to b, it gains potential energy dU = V.dQ and the chemical potential energy of the battery decreases by the same amount. When this charge dQ passes through resistor it loses the potential energy dU = V.dQ due to collision with atoms in the resistor and again reaches the point a. This process occurs continuously till the battery is connected in the circuit. The rate at which the charge loses its electrical potential energy in the resistor can be calculated. The electrical power P is the rate at which the electrical potential energy is delivered,

$$P = \frac{dU}{dt} = \frac{(V.dQ)}{dt} = V\frac{dQ}{dt}$$

Since the electric current I dQdt= , the equation (2.31) can be rewritten as P = VI (2.32) This expression gives the power delivered by the battery to any electrical system, where I is the current passing through it and V is the potential difference across it. The SI unit of electrical power is watt (1W = 1 Js–1). Commercially, the electrical bulbs used in houses come with the power and voltage rating of 5W-220V, 30W-220V, 60W-220V etc. (Figure 2.16).

Usually these voltage rating refers AC RMS voltages. For a given bulb, if the voltage drop across the bulb is greater than voltage rating, the bulb will fuse. Using Ohm's law, power delivered to the resistance R is expressed in other forms

$$P = IV = I\,(IR) = I^2R$$

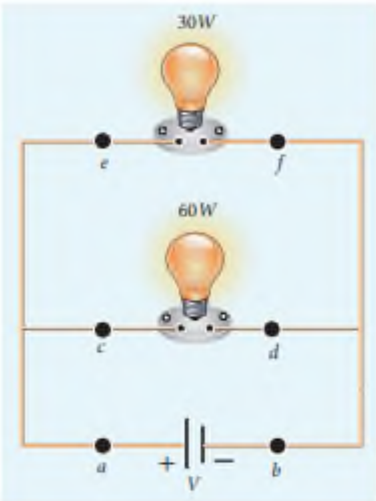
$$P = IV = \frac{V}{R}V = \frac{V^2}{R}$$

- v The electrical power produced (dissipated) by a resistor is I2 R. It depends on the square of the current. Hence, if current is doubled, the power will increase by four times. Similar explanation holds true for voltage also.

The total electrical energy used by any device is obtained by multiplying the power and duration of the time when it is ON. If the power is in watts and the time is in seconds, the energy will be in joules. In practice, electrical energy is measured in kilowatt hour (kWh). 1 kWh is known as 1 unit of electrical energy. (1 kWh = 1000 Wh = (1000 W) (3600 s) = 3.6 × 106 J)

- v The Tamilnadu Electricity Board is charging for the amount of energy you use and not for the power. A current of 1A flowing through a potential difference of 1V produces a power of 1W

EXAMPLE 2.15 A battery of voltage V is connected to 30 W bulb and 60 W bulb as shown in the figure. (a) Identify brightest bulb (b) which bulb has greater resistance? (c) Suppose the two bulbs are connected in series, which bulb will glow brighter?



Solution (a) The power delivered by the battery P = VI. Since the bulbs are connected in parallel, the voltage drop across each bulb is the same. If the voltage is kept fixed, then the power is directly proportional to current (P ∝ I). So 60 W bulb draws twice as much as current as 30 W and it will glow brighter than 30 W bulb.

(b) To calculate the resistance of the bulbs, we use the relation P V R = 2 . In both the bulbs, the voltage drop is the same. So the power is inversely proportional to the resistance or resistance is inversely proportional to the power R P ∝ □ 1 □ □ □ □ □ □ □ □ □ . It implies that, the 30W has twice as much as resistance as 60 W bulb. (c) When the bulbs are connected in series, the current passing through each bulb is the same. It is equivalent to two resistors connected in series. The bulb which has higher resistance has higher voltage drop. So 30W bulb will glow brighter than 60W bulb. So the higher power rating does not always imply more brightness and it depends whether bulbs are connected in series or parallel. EXAMPLE 2.16 Two electric bulbs marked 20 W – 220 V and 100 W – 220 V are connected in series to 440 V supply. Which bulb will get fused? Solution To check which bulb will get fused, the voltage drop across each bulb has to be calculated. The resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{(\text{Rated voltage})^2}{\text{Rated power}}$$

For 20W-220V bulb,

$$R_1 = \frac{(220)^2}{20}\Omega = 2420\Omega$$

(c)

For 100W-220V bulb,

$$R_2 = \frac{(220)^2}{100}\Omega = 484\Omega$$

Both the bulbs are connected in series. So same current will pass through both the bulbs. The current that passes through the circuit, I V Rtot= .

$$R_{tot} = (R_1 + R_2)$$

$$R_{tot} = (484 + 2420)\Omega = 2904\Omega$$

$$I = \frac{440V}{2904\Omega} \approx 0.151\,\text{A}$$

The voltage drop across the 20W bulb is

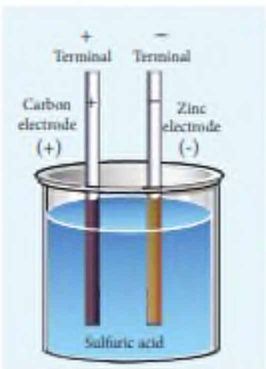
$$V_1 = IR_1 = \frac{440}{2904} \times 2420 \approx 366.6\text{ V}$$

The voltage drop across the 100W bulb is

$$V_2 = IR_2 = \frac{440}{2904} \times 484 \approx 73.3\text{ V}$$

The 20 W bulb will get fused because the voltage across it is more than the voltage rating.

ELECTRIC CELLS AND BATTERIES An electric cell converts chemical energy into electrical energy to produce electricity. It contains two electrodes (carbon and zinc) immersed in an electrolyte (sulphuric acid) as shown in Figure 2.17. Several electric cells connected together form a battery. When a cell or battery is connected to a circuit, electrons flow from the negative terminal to the positive terminal through the circuit. By using chemical reactions, a battery

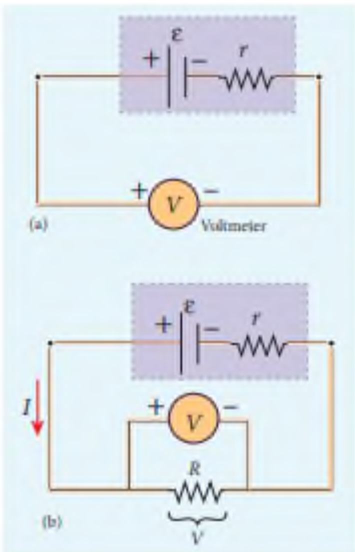


Simple electric cell

produces potential difference across its terminals. This potential difference provides the energy to move the electrons through the circuit. Commercially available electric cells and batteries are shown in Figure 2.18.

Electromotive force and internal resistance A battery or cell is called a source of electromotive force (emf). The term 'electromotive force' is a misnomer since it does not really refer to a force but describes a potential difference in volts. The emf of a battery or cell is the voltage provided by the battery when no current flows in

the external circuit. It is shown in Figure 2.19. Electromotive force determines the amount of work a battery or cell has to do
move a certain amount of charge around the circuit. It is denoted by the symbol ε. An ideal battery has zero internal resistance and the potential difference (terminal voltage) across the battery equals to its emf. In reality, the battery is made of electrodes and electrolyte, there is resistance to the flow of charges within the battery. This resistance is called internal resistance r. For a real battery, the terminal voltage is not equal to the emf of the battery. A freshly prepared cell has low internal resistance and it increases with ageing. 2.4.2 Determination of internal resistance The circuit connections are made as shown in Figure 2.20. The emf of cell ε is measured by connecting a high resistance voltmeter across it without connecting the external resistance R as shown in Figure 2.20(a). Since the voltmeter draws very little current for deflection, the circuit may be considered as open. Hence the voltmeter reading gives the emf of the cell. Then, external resistance R is included in the circuit and current I is established in the circuit. The potential difference across



Internal resistance of the cell

R is equal to the potential difference across the cell (V) as shown in Figure 2.20(b). The potential drop across the resistor R is V = IR (2.35) Due to internal resistance r of the cell, the voltmeter reads a value V, which is less than the emf of cell ε. It is because, certain amount of voltage (Ir) has dropped across the internal resistance r.

Then $V = \varepsilon - Ir$

$$Ir = \varepsilon - V \qquad (2.36)$$

Dividing equation (2.36) by equation (2.35), we get

$$\frac{Ir}{IR} = \frac{\varepsilon - V}{V}$$

$$r = \left[\frac{\varepsilon - V}{V}\right] R \qquad (2.37)$$

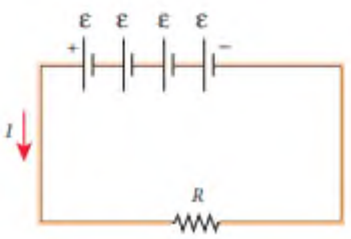
Since ε, V and R are known, internal resistance r can be determined. We can also find the total current that flows in the circuit. Due to this internal resistance, the power delivered to the circuit is not equal to power rating mentioned in the battery. For a battery of emf ε, with an internal resistance r, the power delivered to the circuit of resistance R is given by P = Iε = I (V + Ir) (from equation 2.36) Here V is the voltage drop across the resistance R and it is equal to IR. Therefore, P = I (IR +Ir)

P = I2 R + I2 r (2.38) Here I 2 r is the power delivered to the internal resistance and I 2 R is the power delivered to the electrical device (here it is the resistance R). For a good battery, the internal resistance r is very small, then I 2 r << I 2 R and almost entire power is delivered to the external resistance. EXAMPLE 2.17 A battery has an emf of 12 V and connected to a resistor of 3 Ω. The current in the circuit is 3.93A. Calculate (a) terminal voltage and the internal resistance of the battery (b) power delivered by the battery and power delivered to the resistor Solution The given values I = 3.93 A, ε = 12 V, R = 3 Ω (a) The terminal voltage of the battery is equal to voltage drop across the resistor V = IR = 3.93 × 3 = 11.79 V

The internal resistance of the battery

$$r = \left[\frac{\varepsilon - V}{V}\right] R = \left[\frac{12 - 11.79}{11.79}\right] \times 3 = 0.05\,\Omega$$

(b) The power delivered by the battery P = Iε = 3.93 × 12 = 47.1 W The power delivered to the resistor = I 2 R = 46.3 W The remaining power P = (47.1 – 46.3) = 0.8 W is delivered to the internal resistance and cannot be used to do useful work. (It is equal to I 2 r). 2.4.3 Cells in series Several cells can be connected to form a battery. In series connection, the negative terminal of one cell is connected to the positive terminal of the second cell, the negative terminal of second cell is connected to the positive terminal of the third cell and so on. The free positive terminal of the first cell and the free negative terminal of the last cell become the terminals of the battery. Suppose n cells, each of emf ε volts and internal resistance r ohms are connected in series with an external resistance R as shown in Figure 2.21



cells in series

The total emf of the battery = nεThe total resistance in the circuit = nr + R By Ohm's law, the current in the circuit is

$$I = \frac{\text{total emf}}{\text{total resistance}} = \frac{n\varepsilon}{nr + R} \quad (2.39)$$

Case (a) If $r << R$, then,

$$I = \frac{n\varepsilon}{R} \approx nI_1 \quad (2.40)$$

where, I_1 is the current due to a single cell

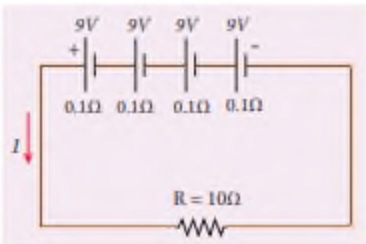
$$\left(I_1 = \frac{\varepsilon}{R}\right)$$

Thus, if r is negligible when compared to R the current supplied by the battery is n times that supplied by a single cell.

Case (b) If $r >> R$, $I = \frac{n\varepsilon}{nr} \approx \frac{\varepsilon}{r}$

It is the current due to a single cell. That is, current due to the whole battery is the same as that due to a single cell and hence there is no advantage in connecting several cells. Thus series connection of cells is advantageous only when the effective internal resistance of the cells is negligibly small compared with R.

EXAMPLE 2.18 From the given circuit,



Find i) Equivalent emf of the combination ii) Equivalent internal resistance iii) Total current iv) Potential difference across external resistance v) Potential difference across each cell Solution i) Equivalent emf of the combination ε eq = nε = 4 × 9 = 36 V ii) Equivalent internal resistance r eq = nr = 4 × 0.1 = 0.4 Ω

iii) Total current $I = \frac{n\varepsilon}{R + nr}$

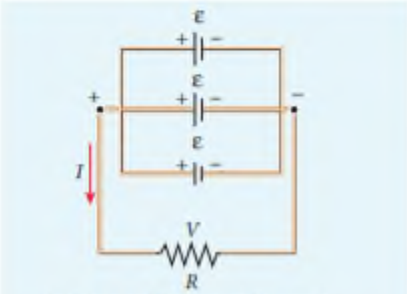
$$= \frac{4 \times 9}{10 + (4 \times 0.1)}$$

$$= \frac{4 \times 9}{10 + 0.4} = \frac{36}{10.4}$$

$$I = 3.46 \text{ A}$$

iv) Potential difference across external resistance V = IR = 3.46 × 10 = 34.6 V. The remaining 1.4 V is dropped across the internal resistances of cells. v) Potential difference across each cell V n = = 34 6 4 8 65 . . V

Cells in parallel In parallel connection all the positive terminals of the cells are connected to one point and all the negative terminals to a second point. These two points form the positive and negative terminals of the battery. Let n cells be connected in parallel between the points A and B and a resistance R is connected between the points A and B as shown in Figure 2.22. Let ε be the emf and r the internal resistance of each cell.



Cells in parallel (circuit diagram)

Cells in parallel

The equivalent internal resistance of the battery is 1 1 1 1 r rrr n n r eq = + + = ... ( ) terms . So r r n eq = and the total resistance in the circuit = R r n + . The total emf is the potential difference between the points A and B, which is equal to ε. The current in the circuit is given by

$$I = \frac{\varepsilon}{\frac{r}{n} + R}$$

$$I = \frac{n\varepsilon}{r + nR} \qquad (2.42)$$

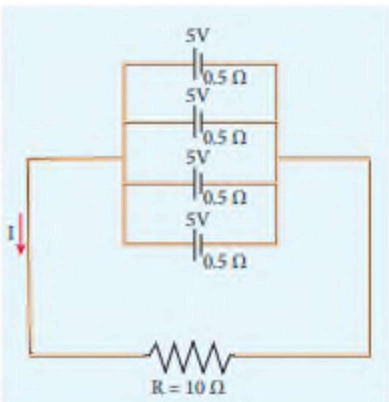
Case (a) If $r >> R$, $I = \frac{n\varepsilon}{r} = nI_1$ (2.43)

where I1 is the current due to a single cell ε r □ □ □ □ □ □ □ □ □ □ when R is negligible. Thus, the current through the external resistance due to the whole battery is n times the current due to a single cell.

Case (b) If $r << R$, $I = \frac{\varepsilon}{R}$

The above equation implies that current due to the whole battery is the same as that due to a single cell. Hence it is advantageous to connect cells in parallel when the external resistance is very small compared to the internal resistance of the cells.

EXAMPLE 2.19 For the given circuit



Find i) Equivalent emf ii) Equivalent internal resistance iii) Total current (I) iv) Potential difference across each cell v) Current from each cell Solution i) Equivalent emf ε eq = 5 V ii) Equivalent internal resistance,

$$R_{eq} = \frac{r}{n} = \frac{0.5}{4} = 0.125\Omega$$

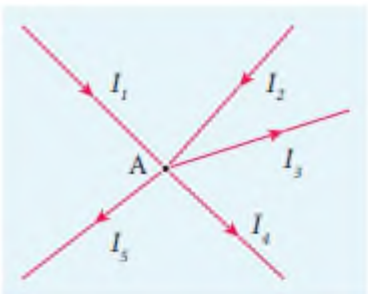
iii) total current, $I = \frac{\varepsilon}{R + r/n}$

$$I = \frac{5}{10 + 0.125} = \frac{5}{10.125}$$

$I = 0.5$ A

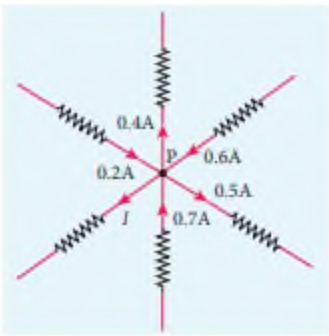
iv) Potential difference across each cell V = IR = 0.5 × 10 = 5 V v) Current from each cell, I′ = I n I′ = = 0 5 4 0 125 . . A

KIRCHHOFF'S RULES Ohm's law is useful only for simple circuits. For more complex circuits, Kirchhoff 's rules can be used to find current and voltage. There are two generalized rules: i) Kirchhoff 's current rule ii) Kirchhoff 's voltage rule. 2.5.1 Kirchhoff's first rule (Current rule or Junction rule) It states that the algebraic sum of the currents at any junction of a circuit is zero. It is a statement of law of conservation of electric charge. The charges that enter a given junction in a circuit must leave that junction since charge cannot build up or disappear at a junction. By convention, current entering the junction is taken as positive and current leaving the junction is taken as negative.



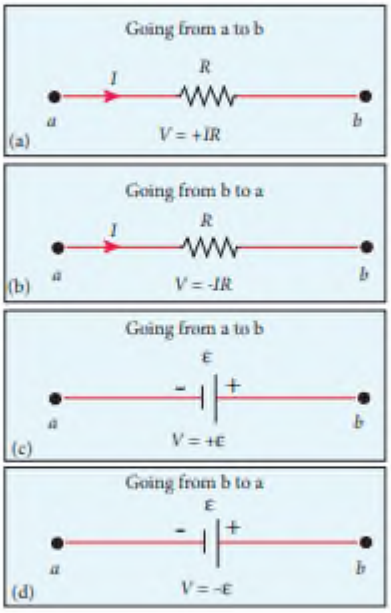
Kirchhoff 's current rule

Applying this law to the junction A in Figure 2.23 I1 + I2 – I3 – I4 – I5 = 0 (or) I1 + I2 = I3 + I4 + I5 EXAMPLE 2.20 For the given circuit find the value of I.



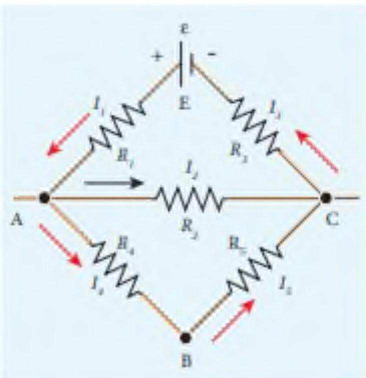
Solution Applying Kirchhoff 's rule to the point P in the circuit, The arrows pointing towards P are positive and away from P are negative. Therefore, 0.2A – 0.4A + 0.6A – 0.5A + 0.7A – I = 0 1.5A – 0.9A – I = 0 0.6A – I = 0 I = 0.6 A

Kirchhoff's Second rule (Voltage rule or Loop rule) It states that in a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. This rule follows from the law of conservation of energy for an isolated system (The energy supplied by the emf sources is equal to the sum of the energy delivered to all resistors). The product of current and resistance is taken as positive when the direction of the current is followed. Suppose if the direction of current is opposite to the direction of the loop, then product of current and voltage across the resistor is negative. It is shown in Figure 2.24 (a) and (b). The emf is considered positive when proceeding from the negative to the positive terminal of the cell. It is shown in Figure 2.24 (c) and (d).



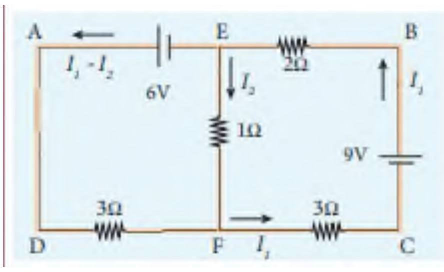
Kirchhoff voltage rule

Kirchhoff voltage rule has to be applied only when all currents in the circuit reach a steady state condition (the current in various branches are constant). EXAMPLE 2.21 The following figure shows a complex network of conductors which can be divided into two closed loops like EACE and ABCA. Apply Kirchhoff's voltage rule (KVR),

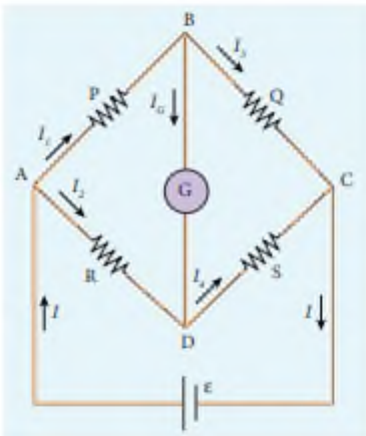


Solution Thus applying Kirchhoff 's second law to the closed loop EACE $I1\ R1 + I2\ R2 + I3\ R3 = \varepsilon$ and for the closed loop ABCA $I4\ R4 + I5\ R5 - I2\ R2 = 0$

Solution



We can denote the current that flows from 9V battery as I1 and it splits up into I2 and (I1 – I2) at the junction E according Kirchhoff 's current rule (KCR). Now consider the loop EFCBE and apply KVR, we get 1I2 + 3I1 + 2I1 = 9 5I1 + I2 = 9 (1) Applying KVR to the loop EADFE, we get 3 (I1 – I2) – 1I2 = 6 3I1 – 4I2 = 6 (2) Solving equation (1) and (2), we get I1 = 1.83 A and I2 = –0.13 A It implies that the current in the 1 ohm resistor flows from F to E. 2.5.3 Wheatstone's bridge An important application of Kirchhoff 's rules is the Wheatstone's bridge. It is used to compare resistances and in determining the unknown resistance in electrical network. The bridge consists of four resistances P, Q, R and S connected as shown in Figure 2.25. A galvanometer G is connected between the points B and D. The battery is connected between the points A and C. The current



Wheatstone's bridge

through the galvanometer is I G and its resistance is G. Applying Kirchhoff 's current rule to junction B and D respectively. I1 – IG – I3 = 0 (2.45) I2 + IG – I4 = 0 (2.46) Applying Kirchhoff 's voltage rule to loop ABDA, I1 P + IGG – I2 R = 0 (2.47) Applying Kirchhoff 's voltage rule to loop ABCDA, I1 P + I3 Q – I4 S – I2 R = 0 (2.48) When the points B and D are at the same potential, the bridge is said to be balanced. As there is no potential difference between B and D, no current flows through galvanometer (IG = 0). Substituting IG = 0 in equation (2.45), (2.46) and (2.47), we get I1 = I3

I2 = I4 (2.50) I1 P = I2 R (2.51) Using equation (2.51) in equation (2.48) I3 Q = I4 S (2.52) Dividing equation (2.52) by equation (2.51), we get P Q R S = (2.53) This is the condition for bridge balance. Only under this condition, galvanometer shows null deflection. Suppose we know the values of two adjacent resistances, the other two resistances can be compared. If three of the resistances are known, the value of unknown resistance (fourth one) can be determined.

EXAMPLE 2.23 In a Wheatstone's bridge P = 100 Ω, Q = 1000 Ω and R = 40 Ω. If the galvanometer shows zero deflection, determine the value of S.

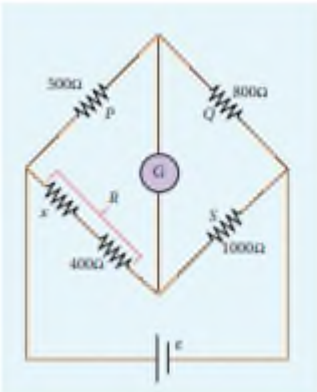
Solution

$$\frac{P}{Q} = \frac{R}{S}$$

$$S = \frac{Q}{P} \times R$$

$$S = \frac{1000}{100} \times 40 \quad S = 400\ \Omega$$

EXAMPLE 2.24 What is the value of x when the Wheatstone's network is balanced? P = 500 Ω, Q = 800 Ω, R = x + 400, S = 1000 Ω



Solution

$$\frac{P}{Q} = \frac{R}{S}, \text{ when the network is balanced}$$

$$\frac{500}{800} = \frac{x+400}{1000}$$

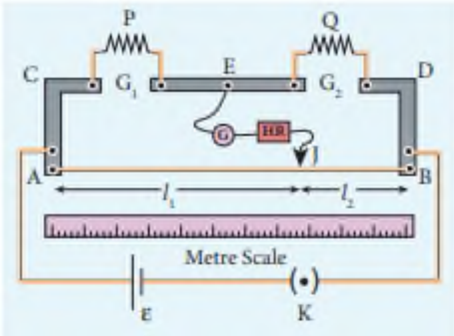
$$x + 400 = \frac{5}{8} \times 1000$$

$$x + 400 = 625$$

$$x = 625 - 400$$

$$x = 225\ \Omega$$

Meter bridge The meter bridge is another form of Wheatstone's bridge. It consists of a uniform wire of manganin AB of one meter length. This wire is stretched along a metre scale on a wooden board between two copper strips C and D. Between these two copper strips another copper strip E is mounted to enclose two gaps G1 and G2 as shown in Figure 2.26. An unknown resistance P is connected in G1 and a standard resistance Q is connected in G2 . A jockey (conducting wire-contact maker) is connected to the terminal E on the central copper strip through a galvanometer (G) and a high resistance (HR). The exact position of jockey on the wire can be read on the scale. A Lechlanche cell and a key (K) are connected between the ends of the bridge wire.



Meter bridge

The position of the jockey on the wire is adjusted so that the galvanometer shows zero deflection. Let the position of jockey at the wire be at J. The resistances corresponding to AJ and JB of the bridge wire form the resistances R and S of the Wheatstone's bridge. Then for the bridge balance

$$\frac{P}{Q}=\frac{R}{S}=\frac{r.AJ}{r.JB}$$

where r is the resistance per unit length of wire.

$$\frac{P}{Q}=\frac{AJ}{JB}=\frac{l_1}{l_2}$$

$$P=Q\frac{l_1}{l_2}$$

The bridge wire is soldered at the ends of the copper strips. Due to imperfect contact, some resistance might be introduced at the contact. These are called end resistances. This error can be eliminated, if another set of readings is taken with P and Q interchanged and the average value of P is found. To find the specific resistance of the material of the wire in the coil P, the radius a and length l of the wire are measured. The specific resistance or resistivity ρ can be calculated using the relation.

$$\text{Resistance}=\rho\frac{l}{A}$$

By rearranging the above equation, we get

$$\rho=\text{Resistance}\times\frac{A}{l} \quad (2.57)$$

If P is the unknown resistance equation (2.57) becomes,

$$\rho=P\frac{\pi a^2}{l}$$

EXAMPLE 2.25 In a meter bridge experiment with a standard resistance of 15 Ω in the right gap, the ratio of balancing length is 3:2. Find the value of the other resistance. Solution Q = 15 Ω, l 1 :l2 = 3:2

$$\frac{l_1}{l_2}=\frac{3}{2}$$

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

$$P = Q\,\frac{l_1}{l_2}$$

$$P = 15 \times \frac{3}{2} = 22.5\ \Omega$$

EXAMPLE 2.26 In a meter bridge experiment, the value of resistance in the resistance box connected in the right gap is 10 Ω. The balancing length is l 1 = 55 cm. Find the value of unknown resistance. Solution Q = 10 Ω

$$\frac{P}{Q} = \frac{l_1}{100 - l_1} = \frac{l_1}{l_2}$$

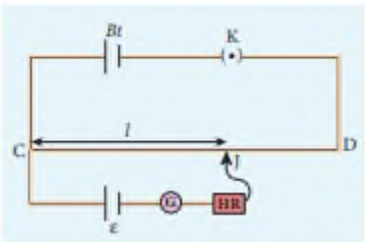
$$P = Q \times \frac{l_1}{100 - l_1}$$

$$P = \frac{10 \times 55}{100 - 55}$$

$$P = \frac{550}{45} = 12.2\ \Omega$$

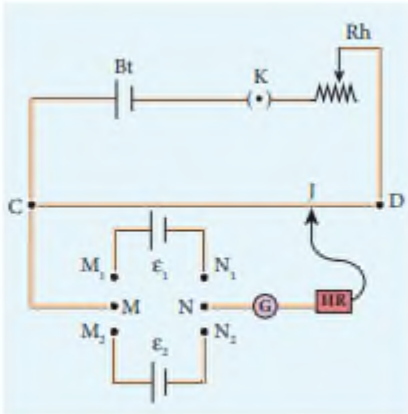
Potentiometer Potentiometer is used for the accurate measurement of potential differences, current and resistances. It consists of ten meter long uniform wire of manganin or constantan stretched in parallel rows each of 1 meter length, on a wooden board. The two free ends A and B are brought to the same side and fixed to copper strips with binding screws. A meter scale is fixed parallel to the wire. A jockey is provided for making contact. The principle of the potentiometer is illustrated in Figure 2.27. A steady current is maintained across the wire CD by a battery Bt.

The battery, key and the potentiometer wire connected in series form the primary circuit. The positive terminal of a primary cell of emf ε is connected to the point C and negative terminal is connected to the jockey through a galvanometer G and a high resistance HR. This forms the secondary circuit.



Potentiometer

Let the contact be made at any point J on the wire by jockey. If the potential difference across CJ is equal to the emf of the cell ε, then no current will flow through the galvanometer and it will show zero deflection. CJ is the balancing length l. The potential difference across CJ is equal to Irl where I is the current flowing through the wire and r is the resistance per unit length of the wire. Hence $\varepsilon = Irl$ (2.58) Since I and r are constants, $\varepsilon \propto l$. The emf of the cell is directly proportional to the balancing length. 2.5.6 Comparison of emf of two cells with a potentiometer To compare the emf of two cells, the circuit connections are made as shown in Figure 2.28. Potentiometer wire CD is connected to a battery Bt and a key K in series. This is the primary circuit. The end C of the wire is connected to the terminal M of a DPDT (Double Pole Double Throw) switch and the other terminal N is connected to a jockey through a galvanometer G and a high resistance HR. The cells whose emf ε1 and ε2 to be compared are connected to the terminals M1 ,N1 and M2 ,N2 of the DPDT switch. The positive terminals of Bt, ε1 and ε2 should be connected to the same end C.

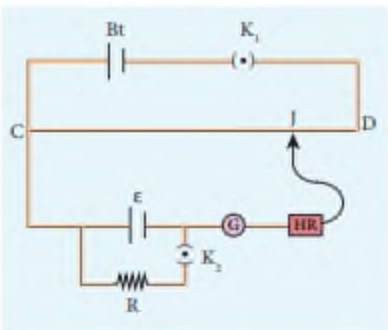


Comparison of emf of two cells

The DPDT switch is pressed towards M1 , N1 so that cell ε1 is included in the secondary circuit and the balancing length l 1 is found by adjusting the jockey for zero deflection. Then the second cell ε2 is included in the circuit and the balancing length l 2 is determined. Let r be the resistance per unit length of the potentiometer wire and I be the current flowing through the wire. we have ε1 = Irl1 (2.59) ε2 = Irl2 (2.60) By dividing equation (2.59) by (2.60)

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

By including a rheostat (Rh) in the primary circuit, the experiment can be repeated several times by changing the current flowing through it. 2.5.7 Measurement of internal resistance of a cell by potentiometer To measure the internal resistance of a cell, the circuit connections are made as shown in Figure 2.29. The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and the negative terminal of the battery is connected to the end D through a key K1 . This forms the primary circuit.



measurement of internal resistance

The positive terminal of the cell of emf ε whose internal resistance is to be determined is also connected to the end C of the wire. The negative terminal of the cell ε is connected to a jockey through a galvanometer and a high resistance. A resistance box R and key K2 are connected across the cell ε. With K2 open, the balancing point J is obtained and the balancing length CJ = l 1 is measured. Since the cell is in open circuit, its emf is ε ∝ l 1

A suitable resistance (say, 10 Ω) is included in the resistance box and key K2 is closed. Let r be the internal resistance of the cell. The current passing through the cell and the resistance R is given by

$$I = \frac{\varepsilon}{R+r}$$

The potential difference across R is

$$V = \frac{\varepsilon R}{R+r}$$

When this potential difference is balanced on the potentiometer wire, let l 2 be the balancing length.

$$\text{Then } \frac{\varepsilon R}{R+r} \propto l_2 \qquad (2.63)$$

From equations (2.62) and (2.63)

$$\frac{R+r}{R} = \frac{l_1}{l_2} \qquad (2.64)$$

$$1 + \frac{r}{R} = \frac{l_1}{l_2};$$

$$r = R\left[\frac{l_1}{l_2} - 1\right]$$

$$\therefore r = R\left(\frac{l_1 - l_2}{l_2}\right) \qquad (2.65)$$

Substituting the values of the R, l 1 and l 2, the internal resistance of the cell is determined. The experiment can be repeated for different values of R. It is found that the internal resistance of the cell is not constant but increases with increase of external resistance connected across its terminals. 2.6 HEATING EFFECT OF ELECTRIC CURRENT When current flows through a resistor, some of the electrical energy delivered to the resistor is converted into heat energy and it is dissipated. This heating effect of current is known as Joule's heating effect. Just as current produces thermal energy, thermal energy may also be suitably used to produce an electromotive force. This is known as thermoelectric effect. 2.6.1 Joule's law If a current I flows through a conductor kept across a potential difference V for a time t, the work done or the electric potential energy spent is W = VIt (2.66) In the absence of any other external effect, this energy is spent in heating the conductor. The amount of heat(H) produced is H = VIt (2.67) For a resistance R, H = I2 Rt (2.68) This relation was experimentally verified by Joule and is known as Joule's law of heating. It states that the heat developed in an electrical circuit due to the flow of current varies directly as (i) the square of the current (ii) the resistance of the circuit and (iii) the time of flow. EXAMPLE 2.27 Find the heat energy produced in a resistance of 10 Ω when 5 A current flows through it for 5 minutes.

Solution

R = 10 Ω, I = 5 A, t = 5 minutes = 5 × 60 s

H = I 2
Rt
= 52
× 10 × 5 × 60
= 25 × 10 × 300
= 25 × 3000
= 75000 J (or) 75 kJ

Application of Joule's heating effect 1. Electric heaters Electric iron, electric heater, electric toaster shown in Figure 2.30 are some of the home appliances that utilize the heating effect of current. In these appliances, the heating elements are made of nichrome, an alloy of nickel and chromium. Nichrome has a high specific resistance and can be heated to very high temperatures without oxidation.

EXAMPLE 2.28 An electric heater of resistance 10 Ω connected to 220 V power supply is immersed in the water of 1 kg. How long the electrical heater has to be switched on to increase its temperature from 30°C to 60°C. (Specific heat capacity of water is s = 4200 J kg–1 K–1)

Solution According to Joule's heating law H = I 2 Rt The current passed through the electrical heater = 220 10 22 V A Ω = Heat produced in one second by the electrical heater H = I2 R Heat produced in one second H = (22)2 × 10 = 4840 J = 4.84 k J. In fact the power rating of this electrical heater is 4.84 k W. The amount of heat energy to increase the temperature of 1kg water from 30°C to 60°C is

Q = ms ∆T (Refer XI physics vol 2, unit 8) Here m = 1 kg, s = 4200 J kg–1 K–1, ∆T = 30 K, so Q = 1 × 4200 × 30 = 126 kJ The time required to produce this heat energy t Q I R = = × 2 ≈ 3 126 10 4840 26 03 .s

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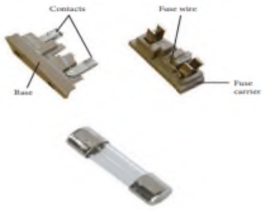
3 126 10
4840
26 03 .s

2. Electric fuses Fuses as shown in Figure 2.31, are connected in series in a circuit to protect the electric devices from the heat developed by the passage of excessive current. It is a short length of a wire made of a low melting point material. It melts and breaks the circuit if current exceeds a certain value. An alloy of lead - tin is used for fuses when current rating is below 15 A and when current rating is above 15 A, copper fuse wires are used. The only disadvantage with the above fuses is that once fuse wire is burnt due to excessive current, they need to be replaced. Nowadays in houses, circuit breakers

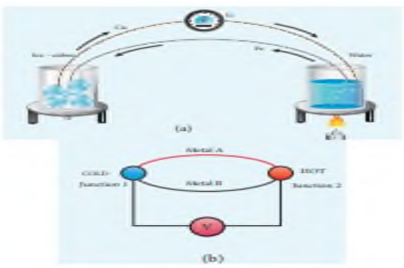


Electric Fuse

. Electric furnace Furnaces as shown in Figure 2.33 are used to manufacture a large number of technologically important materials such as steel, silicon carbide, quartz, gallium arsenide, etc. To produce temperatures up to 1500°C, molybdenum-nichrome wire wound on a silica tube is used. Carbon arc furnaces produce temperatures up to 3000°C.

4. Electrical lamp It consists of a tungsten filament (melting point 33800 C) kept inside a glass bulb and heated to incandescence by current. In incandescent electric lamps only about 5% of electrical energy is converted into light and the rest is wasted as heat. Electric discharge lamps, electric welding and electric arc also utilize the heating effect of current as shown in Figure 2.34

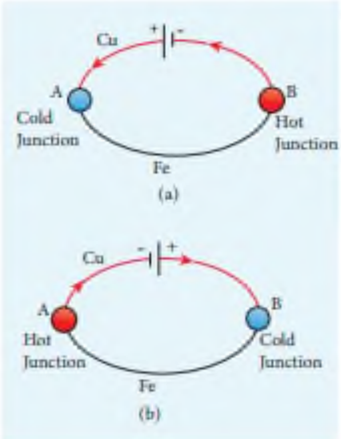
THERMOELECTRIC EFFECT Conversion of temperature differences into electrical voltage and vice versa is known as thermoelectric effect. A thermoelectric device generates voltage when there is a temperature difference on each side. If a voltage is applied, it generates a temperature difference.
2.7.1 Seebeck effect Seebeck discovered that in a closed circuit consisting of two dissimilar metals, when the junctions are maintained at different temperatures an emf (potential difference) is developed. The current that flows due to the emf developed is called thermoelectric current. The two dissimilar metals connected to form two junctions is known as thermocouple (Figure 2.35).



Seebeck effect (Thermocouple)

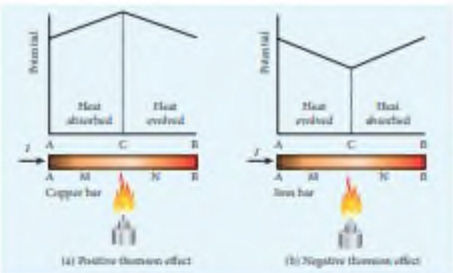
If the hot and cold junctions are interchanged, the direction of current also reverses. Hence the effect is reversible. The magnitude of the emf developed in a thermocouple depends on (i) the nature of the metals forming the couple and (ii) the temperature difference between the junctions. Applications of Seebeck effect 1. Seebeck effect is used in thermoelectric generators (Seebeck generators).These thermoelectric generators are used in power plants to convert waste heat into electricity.
2. This effect is utilized in automobiles as automotive thermoelectric generators for increasing fuel efficiency. 3. Seebeck effect is used in thermocouples and

thermopiles to measure the temperature difference between the two objects. 2.7.2 Peltier effect In 1834, Peltier discovered that when an electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and absorbed at the other junction. This is known as Peltier effect.



Peltier effect: Cu – Fe thermocouple

In the Cu-Fe thermocouple the junctions A and B are maintained at the same temperature. Let a current from a battery flow through the thermocouple (Figure 2.36 (a)). At the junction A, where the current flows from Cu to Fe, heat is absorbed and the junction A becomes cold. At the junction B, where the current flows from Fe to Cu heat is liberated and it becomes hot. When the direction of current is reversed, junction A
gets heated and junction B gets cooled as shown in the Figure 2.36(b). Hence Peltier effect is reversible. 2.7.3 Thomson effect Thomson showed that if two points in a conductor are at different temperatures, the density of electrons at these points will differ and as a result the potential difference is created between these points. Thomson effect is also reversible.



7 (a) Positive Thomson effect (b) Negative Thomson effect
If current is passed through a copper bar AB which is heated at the middle point C, the point C will be at higher potential. This indicates that the heat is absorbed along AC and evolved along CB of the conductor as shown in Figure 2.37(a). Thus heat is transferred due to the current flow in the direction of the current. It is called positive Thomson effect. Similar effect is observed in metals like silver, zinc, and cadmium. When the copper bar is replaced by an iron bar, heat is evolved along CA and absorbed along BC. Thus heat is transferred due to the current flow in the direction opposite to the direction of current. It is called negative Thomson effect as shown in the Figure 2.37(b). Similar effect is observed in metals like platinum, nickel, cobalt, and mercury.